Higher Spins & Strings

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based mainly on

MRG, R. Gopakumar, 1406.6103 and 1501.07236

Motivation

One way to get quantitative handle on AdS/CFT correspondence is to consider regime where gauge theory is weakly coupled but still at large N:

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g_{\rm YM}^2 \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g_{\rm YM}^2 N = \lambda$$

large

small

[Sundborg '01]
[Witten '01]
[Sezgin,Sundell '01]

$l_{\rm s} \to \infty$ `tensionless strings'

Higher spin theory

Resulting theory has an infinite number of massless higher spin fields, which generate a very large gauge symmetry.

$\rightarrow$ effective description in terms of Vasiliev Higher Spin Theory.

maximally unbroken phase of string theory

Leading Regge trajectory

On the dual CFT side, the traces of bilinears of elementary Yang-Mills fields form closed subsector in free theory.

This subsector is believed to correspond to the leading Regge trajectory from the string point of view:

vector-like HS -- CFT duality

State of the Art

In the past this idea was taken as a general motivation to consider dualities relating

Vasiliev HS theory on AdS	$\longleftrightarrow$	vector-like weakly coupled CFT

However, recently interesting progress about how these dualities fit actually into stringy AdS/CFT correspondence has been made....

[Chang, Minwalla, Sharma, Yin '12]
[MRG, Gopakumar '14]

3d proposal

Concrete duality of this kind (somewhat similar to Klebanov & Polyakov proposal for AdS4/CFT3)

[MRG,Gopakumar '10]

AdS3:
higher spin theory with a complex scalar of mass M

$\longleftrightarrow$

2d CFT:
$\mathcal{W}_{N,k}$ minimal models in large N 't Hooft limit with coupling λ

where $\lambda = \dfrac{N}{N+k}$ and $M^2 = -(1-\lambda^2)$

3d proposal

This version of the duality is bosonic, but can nevertheless be tested in quite some detail.

In particular, the unique quantisation (finite c) of the asymptotic symmetry algebra of the hs theory agrees precisely with the W-algebra of the dual CFT. This is even true at finite N and k.

[MRG,Gopakumar '12]

Spectrum

Furthermore, the 1-loop thermal partition function of the hs theory with one massive complex scalar field agrees, in the 't Hooft limit, with the `perturbative' half of the CFT partition function

$$Z_{\text{pert}} = \sum_{\Lambda} \left|\chi_{(\Lambda,0)}\right|^2 \,.$$

[MRG,Gopakumar '10]
[MRG,Gopakumar,Hartman,Raju '11]
[Giombi, Klebanov '13]

$\mathcal{N} = 4$ Supersymmetry

In order to relate this bosonic hs duality to stringy dualities, consider situation with $\mathcal{N} = 4$ supersymmetry.

Best studied example:

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times M_4 \qquad \text{with } M_4 = \mathbb{T}^4 \text{ or } M_4 = \mathrm{K3}$$

Then dual CFT is believed to have small $\mathcal{N} = 4$ superconformal symmetry.

Dual CFT

At one point in moduli space, the dual CFT of string theory is described by the symmetric orbifold theory

$$\mathrm{Sym}_{N+1}(\mathbb{T}^4) \equiv \left(\mathbb{T}^{4(N+1)}\right)/S_{N+1}$$

This CFT is essentially free --- could this be the

CFT dual in the tensionless limit?

Small $\mathcal{N} = 4$ Supersymmetry

Unfortunately, no coset CFTs with small $\mathcal{N} = 4$ (that would naturally appear in a hs--CFT duality) are known....

$\boxed{\text{`tensionless'}\ \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4}$

hs theory

$\updownarrow$

coset dual ??

string theory

$\updownarrow$

symmetric orbifold

$\mathrm{Sym}_{N+1}(\mathbb{T}^4)$

Large $\mathcal{N} = 4$ Supersymmetry

We shall therefore first explore the situation with large $\mathcal{N} = 4$ superconformal symmetry --- this is the symmetry algebra of the CFT dual to string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

Towards String Theory

Indeed, dual CFT is expected to have

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

Large $\mathcal{N} = 4$

$$\mathrm{Vir} \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$$

with 4 supercharges

[Boonstra, Peeters, Skenderis ’98; Elitzur, Feinerman, Giveon, Tsabar ’99;
de Boer, Pasquinucci, Skenderis ’99; Gukov, Martinec, Moore, Strominger ’04; ...]

Large $\mathcal{N} = 4$

Since there are two current algebras, the superconformal algebra is actually characterised by two parameters: in addition to the central charge

$$c = \frac{6k^+k^-}{k^+ + k^-}$$

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, Theodoridis '88-'90; Goddard, Schwimmer '88]

have parameter

$$\gamma = \frac{k^-}{k^+ + k^-} \ .$$

($k^{\pm}$: size of the two S3s.)

Dual CFT

In this case the situation is reversed: the dual CFT of this string background is not known.

[Gukov, Martinec, Moore, Strominger '04]
see however [Tong '14]

However, one family of $\mathcal{N}=4$ coset CFTs is known: it is based on the Wolf symmetric spaces

$$\frac{\mathfrak{su}(N+2)^{(1)}_{k+N+2}}{\mathfrak{su}(N)^{(1)}_{k+N+2}\oplus\mathfrak{u}(1)^{(1)}_{\kappa}}\oplus\mathfrak{u}(1)^{(1)}_{\kappa}\cong\frac{\mathfrak{su}(N+2)_k\oplus\mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2}\oplus\mathfrak{u}(1)_\kappa}\oplus\mathfrak{u}(1)_\kappa\ .$$

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, Theodoridis '88-'90]

Dual CFT

They contain large $\mathcal{N} = 4$ algebra with

$$\begin{aligned} k^+ &= k+1 \\ k^- &= N+1 \end{aligned} \quad \Rightarrow \quad \boxed{\gamma = \frac{N+1}{N+k+2}}$$

as well as certain higher spin currents.

Thus it is natural to look for a hs dual of these cosets in 't Hooft limit.

[MRG,Gopakumar '13]

$\mathcal{N} = 4$ hs duality

The duality turned out to be

higher spin theory based on $\mathrm{shs}_2[\lambda]$ $\longleftrightarrow$ Wolf symmetric space cosets

with

$$\lambda = \frac{N+1}{N+k+2} = \gamma$$

[MRG,Gopakumar '13]

Further checks

Further evidence for this proposal: asymptotic symmetry algebra of hs theory matches precisely with the W-algebra of the Wolf space cosets in the 't Hooft limit.

[MRG, Peng '14]

In fact, the most general quantum W-algebra with this spin spectrum is uniquely determined by the levels of the two su(2)'s.

[Beccaria, Candu, MRG '14]

[This holds both for the `linear' as well as the `non-linear' version of the W-algebra.]

Spectrum

Also, the 1-loop thermal partition function of the hs theory with one complex scalar multiplet was successfully matched with the `perturbative half' of the CFT partition function of the Wolf space cosets in the 't Hooft limit.

$$Z_{\rm pert} = \sum_{\Lambda} \left| \chi_{(0;\Lambda)} \right|^2 .$$

[Creutzig, Hikida, Roenne '13]
[Candu, Vollenweider '13]

Basic situation

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$

hs theory based on

$\mathrm{shs}_2[\lambda]$

$\updownarrow$

Wolf space cosets

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$

string theory

$\updownarrow$

symmetric orbifold

$\mathrm{Sym}_{N+1}(\mathbb{T}^4)$

Contraction

While we cannot compare these two dualities directly, the large superconformal symmetry contracts to the small superconformal symmetry in the limit in which one of the two levels, say k^+, goes to infinity,

$$k^+ \to \infty \qquad \left(\lambda = \gamma = \frac{k^-}{k^+ + k^-} \to 0\right)$$

Indeed, this just describes the case where the radius of the corresponding 3-sphere goes to infinity, and hence the sphere approximates flat space.

Basic idea

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1 \quad \xrightarrow{\lambda \to 0} \quad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

hs theory based on $\mathrm{shs}_2[\lambda]$ — string theory

$$\updownarrow \qquad\qquad\qquad \updownarrow$$

Wolf space cosets $\overset{?}{\subset}$ symmetric orbifold

The free limit

Thus we should analyse the

$$k \to \infty \qquad\qquad (k^+ = k+1)$$

limit of the Wolf space cosets --- and then compare to the symmetric orbifold.

[MRG, Gopakumar '14]

Wolf space cosets

What happens to the Wolf space cosets in this limit?

$$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \oplus \mathfrak{u}(1)_\kappa \ .$$

[MRG, Suchanek '11]

As in bosonic case, the `perturbative' part of the spectrum can be identified with the U(N)-singlet sector

$$\mathcal{H}_{\rm pert} = \bigoplus_{\Lambda} (0;\Lambda) \otimes (0;\Lambda^*) = \left(\begin{array}{ll} 4(N+1) & \text{free bosons} \\ 4(N+1) & \text{free fermions} \end{array} \right) / \mathrm{U}(N)$$

[MRG, Gopakumar '14]

Untwisted sector

Here the free bosons and fermions transform as

$$
\begin{array}{ll}
\text{bosons:} & 2\cdot(\mathbf{N},\mathbf{1})\oplus 2\cdot(\bar{\mathbf{N}},\mathbf{1})\oplus 4\cdot(\mathbf{1},\mathbf{1}) \\
\text{fermions:} & (\mathbf{N},\mathbf{2})\oplus(\bar{\mathbf{N}},\mathbf{2})\oplus 2\cdot(\mathbf{1},\mathbf{2})
\end{array}
$$

$$\mathrm{U}(\mathrm{N}) \quad \mathfrak{su}(2)$$

The other coset representations can be interpreted as twisted sectors (and descendants) of this continuous orbifold --- actually, can give very concrete identification....

[MRG, Gopakumar '14]
[MRG, Kelm '14]

Comparison

This now looks very similar to the untwisted sector of the symmetric orbifold

$$\mathrm{Sym}_{N+1}(\mathbb{T}^4) \equiv \left(\mathbb{T}^{4(N+1)}\right)/S_{N+1}$$

Indeed, this sector is generated by free bosons and fermions in

$$\begin{aligned}
\text{bosons:} \qquad & 4\cdot(\mathbf{N+1},\mathbf{1}) = 4\cdot(\mathbf{N},\mathbf{1})\oplus 4\cdot(\mathbf{1},\mathbf{1}) \\
\text{fermions:} \qquad & 2\cdot(\mathbf{N+1},\mathbf{2}) = 2\cdot(\mathbf{N},\mathbf{2})\oplus 2\cdot(\mathbf{1},\mathbf{2})
\end{aligned}$$

S_{N+1} $\quad \mathfrak{su}(2)$

Branching rule

In fact

$$S_{N+1} \subset \mathrm{U}(N)$$

and under this embedding, we have the branching rules

$$\mathbf{N}_{\mathrm{U}(N)} \to \mathbf{N}_{S_{N+1}} \qquad \bar{\mathbf{N}}_{\mathrm{U}(N)} \to \mathbf{N}_{S_{N+1}}$$

Comparison

Wolf coset:

$$\text{bosons:} \qquad 2\cdot(\mathbf{N},\mathbf{1})\oplus 2\cdot(\bar{\mathbf{N}},\mathbf{1})\oplus 4\cdot(\mathbf{1},\mathbf{1})$$

$$\text{fermions:} \qquad (\mathbf{N},\mathbf{2})\oplus(\bar{\mathbf{N}},\mathbf{2})\oplus 2\cdot(\mathbf{1},\mathbf{2})$$

Symmetric orbifold:

$$\text{bosons:} \qquad 4\cdot(\mathbf{N},\mathbf{1})\oplus 4\cdot(\mathbf{1},\mathbf{1})$$

$$\text{fermions:} \qquad 2\cdot(\mathbf{N},\mathbf{2})\oplus 2\cdot(\mathbf{1},\mathbf{2})$$

Thus the action of the permutation group on the free bosons and fermions is induced from the U(N) action!

Subtheory

It therefore follows that [MRG, Gopakumar '14]

U(N) invariant states of free theory $\subset$ untwisted sector of sym. orbifold i.e. S_{N+1} invariant states of free theory

$\uparrow$

perturbative part of CFT dual of hs theory for $\lambda \to 0$

$\uparrow$

(part of) CFT dual of string theory in this limit

hs theory is closed subsector of string theory!

Stringy symmetry

From the hs point of view, the symmetric orbifold (i.e. the stringy CFT dual) is characterised by an extended chiral algebra.

The character of this stringy algebra equals

$$\mathrm{Z}_{\mathrm{stringy}}(q,y) = \sum_{\Lambda} D(\Lambda)\, \chi_{(0;\Lambda)}(q,y)$$

multiplicity of singlet representation of S_{N+1}

Stringy chiral algebra

Explicitly, we find

$$\begin{aligned} \mathrm{Z}_{\mathrm{stringy}}(q,y) \quad = \quad & \chi_{(0;0)}(q,y) + \chi_{(0;[2,0,\ldots,0])}(q,y) + \chi_{(0;[0,0,\ldots,0,2])}(q,y) \\ & + \chi_{(0;[3,0,\ldots,0,0])}(q,y) + \chi_{(0;[0,0,0,\ldots,0,3])}(q,y) \\ & + \chi_{(0;[2,0,\ldots,0,1])}(q,y) + \chi_{(0;[1,0,0,\ldots,0,2])}(q,y) \\ & + 2\cdot\chi_{(0;[4,0,\ldots,0,0])}(q,y) + 2\cdot\chi_{(0;[0,0,0,\ldots,0,4])}(q,y) \\ & + \chi_{(0;[0,2,0,\ldots 0,0])}(q,y) + \chi_{(0;[0,0,\ldots 0,2,0])}(q,y) \\ & + \chi_{(0;[3,0,\ldots,0,1])}(q,y) + \chi_{(0;[1,0,0,\ldots,0,3])}(q,y) \\ & + 2\cdot\chi_{(0;[2,0,0,\ldots,0,2])}(q,y) + \cdots \end{aligned}$$

This reproduces precisely vacuum character of symmetric orbifold (from DMVV).

K3 case

A similar analysis can be done for the K3 case, where

$$\mathcal{W}_\infty[0]^s \subset \mathrm{Sym}_N(\mathrm{K3})$$

subalgebra of $\mathcal{W}_\infty[0]$.

Symmetry of $\quad \mathrm{shs}_2[0]^s \longleftrightarrow \mathcal{W}_\infty[0]^s$

higher spin — CFT duality.

[Baggio, MRG, Peng, to appear]

Symmetry of String Theory

[MRG, Gopakumar '15]

Given that we have an explicit description of the stringy chiral algebra, we can ask what the symmetry underlying string theory is:

$$\begin{array}{ccc} \mathrm{shs}_2[0] & \xrightarrow{\text{asymptotic symmetry}} & \mathcal{W}_{\infty}^{(4)}[0] \\ \cap & & \cap \\ ??? & \xrightarrow{\text{asymptotic symmetry}} & \mathcal{W}_{\text{string}} \end{array}$$

Symmetry of String Theory

[MRG, Gopakumar ’15]

Given that we have an explicit description of this stringy chiral algebra, we can ask what the symmetry underlying string theory is:

$$
\begin{tikzcd}
\mathrm{shs}_2[0] \arrow[rr, "\text{asymptotic symmetry}"] & & \mathcal{W}_\infty^{(4)}[0] \arrow[ll, bend left] \\
\cap & & \cap \\
??? \arrow[rr, "\text{asymptotic symmetry}"] & & \mathcal{W}_{\mathrm{string}} \arrow[ll, bend left]
\end{tikzcd}
$$

restrict to `wedge’

Wedge algebra

Here the wedge algebra is generated by the modes that annihilate in- and out-vacuum

$$S_n^{(h)} \quad \text{with} \quad |n| \leq h - 1 \ .$$

These modes define a Lie algebra in the limit where the central charge goes to infinity — then non-linear terms drop out,

$$\text{e.g.} \quad \text{wedge}\,(\mathcal{W}_N) = \mathfrak{sl}(N) \ .$$

Generating fields

In first step, need to understand generating fields of the stringy algebra — for symmetric orbifold generating fields are simply counted by

$$(1-q)\,\mathrm{Z}_{\mathbb{T}^4}(q,y) = (1-q)\prod_{n=1}^{\infty}\frac{(1+yq^{n-1/2})^2\,(1+y^{-1}q^{n-1/2})^2}{(1-q^n)^4}$$

removes derivatives

[Analogy: for $\mathcal{W}_N$ these would be the fields of spin 2,3,…,N.]

Wedge decomposition

Single particle generators (& derivatives) should sit in wedge representations of $\mathcal{W}_{\infty}^{(4)}[0]$: in fact, we have the identity

[MRG, Gopakumar '15]

$$\mathrm{Z}_{\mathbb{T}^4}(q,y) = \sum_{m,n=0}^{\infty} \chi^{(\text{wedge})[\mathcal{N}=4]}_{(0;[m,0,\ldots,0,n])}(q,y)$$

generating fields

Generating fields

Thus we have a fairly explicit description of the generating fields and hence of their wedge modes, i.e., of the underlying vector space of the symmetry algebra.

What can we say about the Lie algebraic structure?

While we haven't yet managed to work this out in detail, can give fairly good description of simpler toy model.

Bosonic toy model

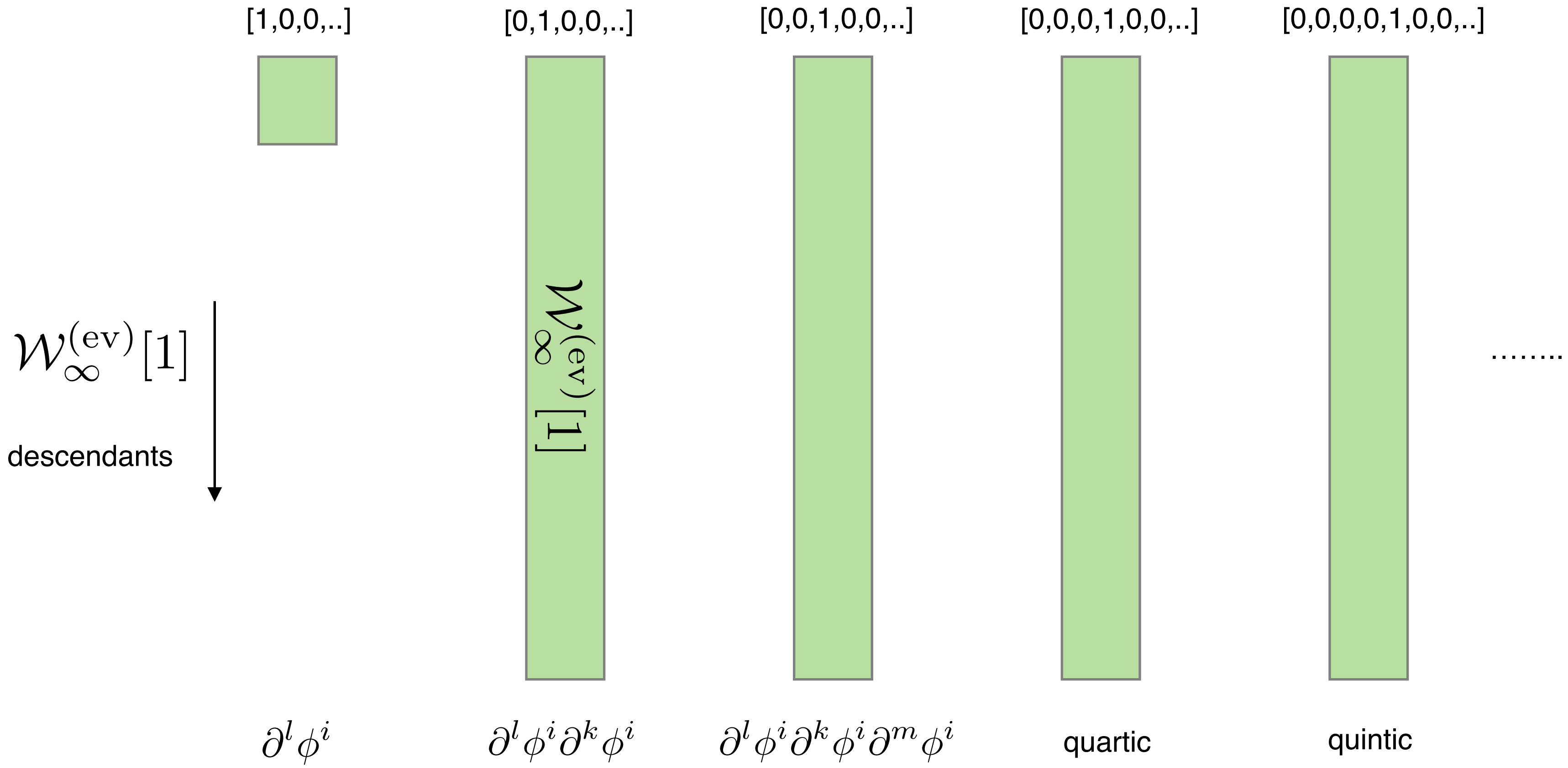
Consider the symmetric orbifold of a single real boson. (The $\mathcal{N}=4$ case consists of 4 bosons and fermions.) Its generating fields are counted by

$$\mathcal{W}^{(\mathrm{bos})}_{\mathrm{string}} : \qquad (1-q)\prod_{n=1}^{\infty}\frac{1}{(1-q^n)} = (1-q)\sum_{n=0}^{\infty}\chi^{(\mathrm{wedge})[\lambda=1]}_{([0^{n-1},1,0,\ldots,0];0)}(q)$$

removes derivatives

wedge characters of $\mathcal{W}^{(\mathrm{ev})}_{\infty}[1]$

Higher Spin Square



The other hs algebra

Because of bosonisation/fermionisation, the stringy algebra contains in this case a second higher spin algebra

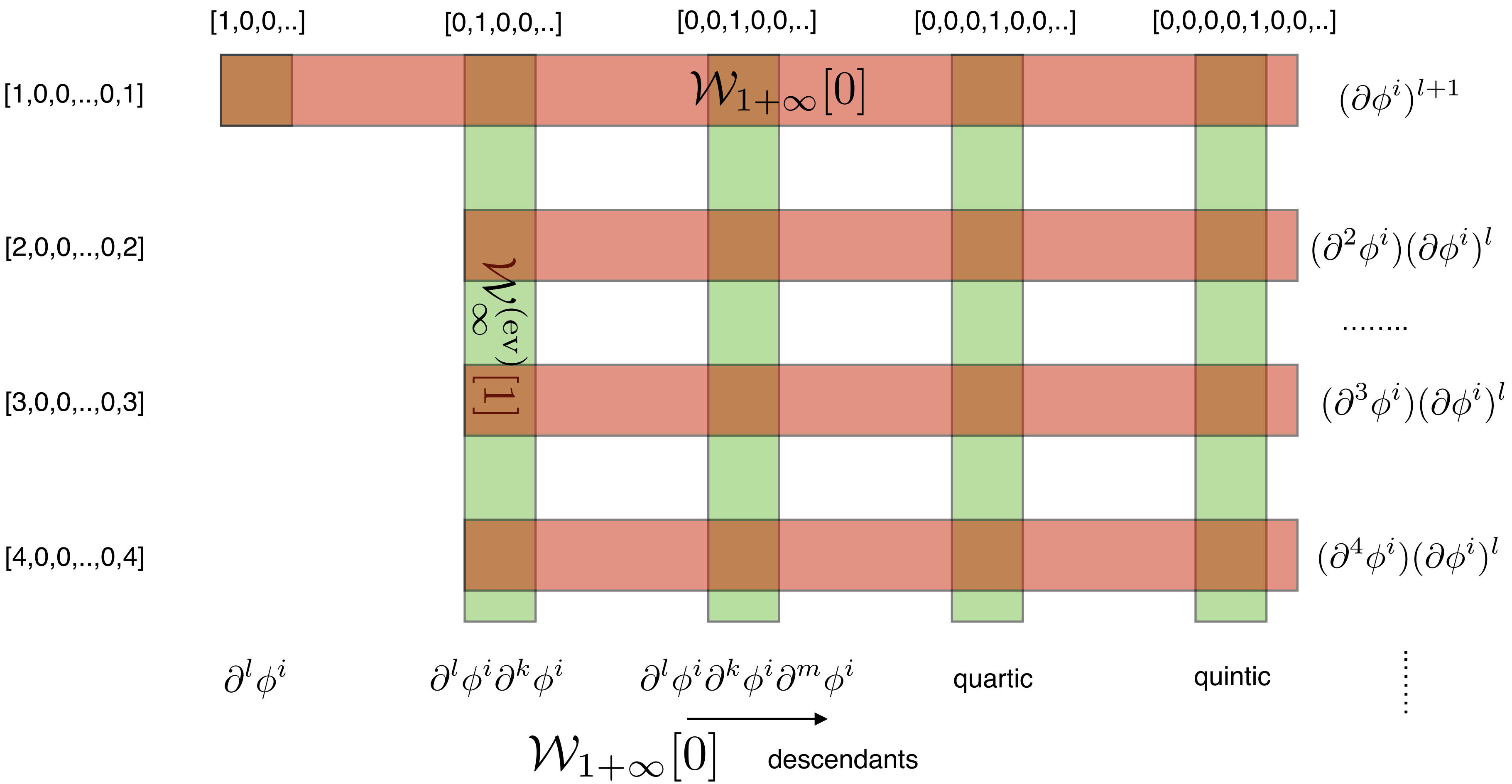
$\mathcal{W}_{1+\infty}[0]$ generated by $(\partial\phi^i)^l + \cdots$

Indeed, we have the identity

$$\prod_{n=1}^{\infty} \frac{1}{(1-q^n)} = \sum_{n=0}^{\infty} \chi^{(\text{wedge})\,[\lambda=0]}_{(0;[n,0,\ldots,0,n])}(q)$$

corresponding to horizontal decomposition.

Higher Spin Square



The Higher Spin Square

The full stringy algebra is then generated by successive commutators of the two algebras

$$\mathcal{W}_{\infty}^{(\mathrm{ev})}[1] \qquad \text{and} \qquad \mathcal{W}_{1+\infty}[0]$$

This is also true for the wedge subalgebra, i.e., can obtain the full wedge subalgebra in terms of successive commutators of

$$\mathrm{hs}^{(\mathrm{ev})}[1] \qquad \text{and} \qquad \mathrm{hs}[0]$$

Lie algebra structure

Thus the Lie algebra structure is fixed by this

Higher Spin Square

We expect that something similar happens in the $\mathcal{N}=4$ case. Also, the resulting structure is somewhat reminiscent of a Yangian symmetry….

Regge trajectories

In $\mathcal{N}=4$ case, it seems that different columns can be thought of as corresponding to different Regge trajectories:

first column = original higher spin fields

= leading Regge trajectory

2nd column = first subleading Regge trajectory

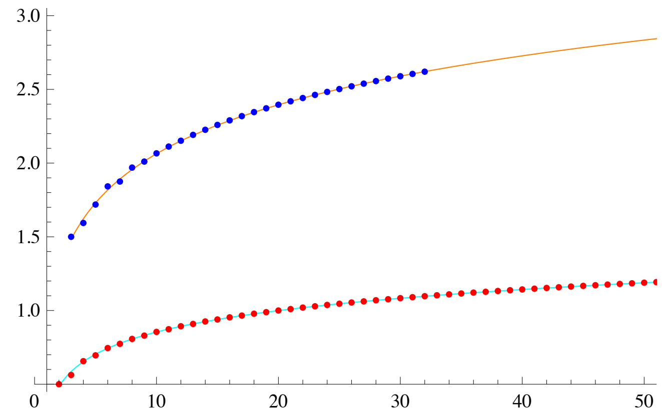
3rd column = 2nd subleading Regge trajectory

etc…

Tension perturbation

Evidence from analysis of perturbation by exactly marginal operator that corresponds to switching on string tension [MRG, Peng, Zadeh, to appear]

$$\frac{M^2}{\text{spin}}$$



2nd column

1st column

spin

Conclusion

- Explained evidence for large $\mathcal{N}=4$ version of minimal model holography.
- In this case found natural embedding of CFT dual of hs theory into CFT dual of string theory.
- Stringy algebra seems to have structure of a Higher Spin Square — generated by two $\mathcal{W}_\infty$ algebras.

Open problems & future directions

Open problems:

- Interpretation from D1-D5 viewpoint
- Relation to spin chain picture

[Babichenko, Stefanski, Zarembo ‘09]
[Borsato, Ohlsson Sax, Sfondrini, Stefanski ‘14]

HS viewpoint: new perspective on stringy CFT

- Find Lie algebra structure of stringy symmetry
- Find other stringy modular invariants
- higher dimensional analogue?

cf. [Chang, Minwalla, Sharma, Yin ’12]
cf. [Beisert,Bianchi,Morales,Samtleben ’04]