2 Consider the theory in 1+3 dimensions of a Dirac fermion ψ and a real scalar ϕ with Lagrangian density, where μ, m and λ are all real parameters and $\mu > 2m$:

$$\mathcal{L} = \bar{\psi}(i \not\partial - m)\psi + \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2}\mu^2 \phi^2 - \lambda \bar{\psi}\gamma^\mu \psi \partial_\mu \phi.$$

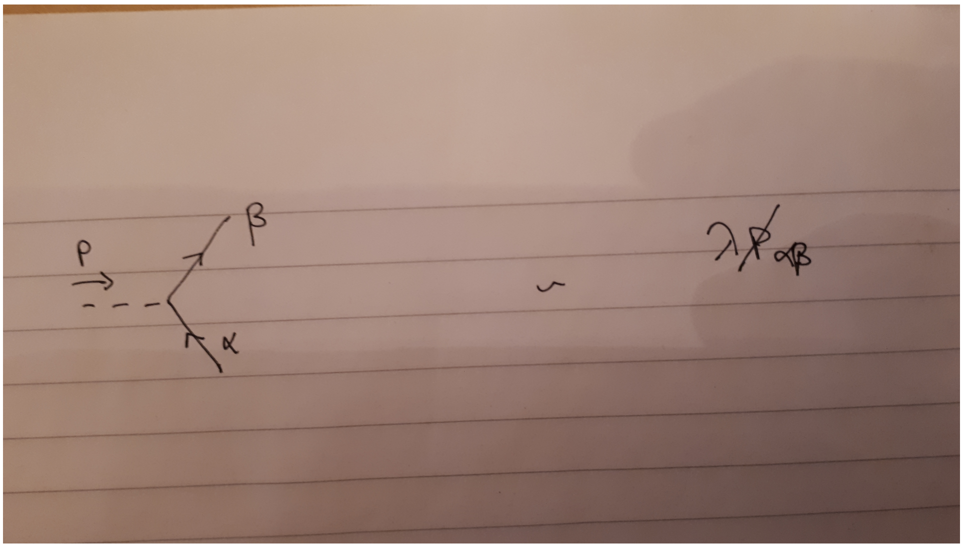
(i) Draw and write momentum-space Feynman rules for the *interactions* of the theory.

(ii) What is the mass dimension of λ? What implication does this have for the theory?

(iii) Consider the decay $\phi(p) \to \bar{\psi}(q_1)\psi(q_2)$. Draw a tree-level Feynman diagram for the modulus squared of the amplitude. Using Feynman rules, calculate the tree-level width of ϕ, deriving any properties of γ matrices from the Clifford algebra which you should quote.

(iv) What is the spin averaged/summed cross-section for $\psi\bar{\psi} \to \psi\psi$ (write your reasoning)?

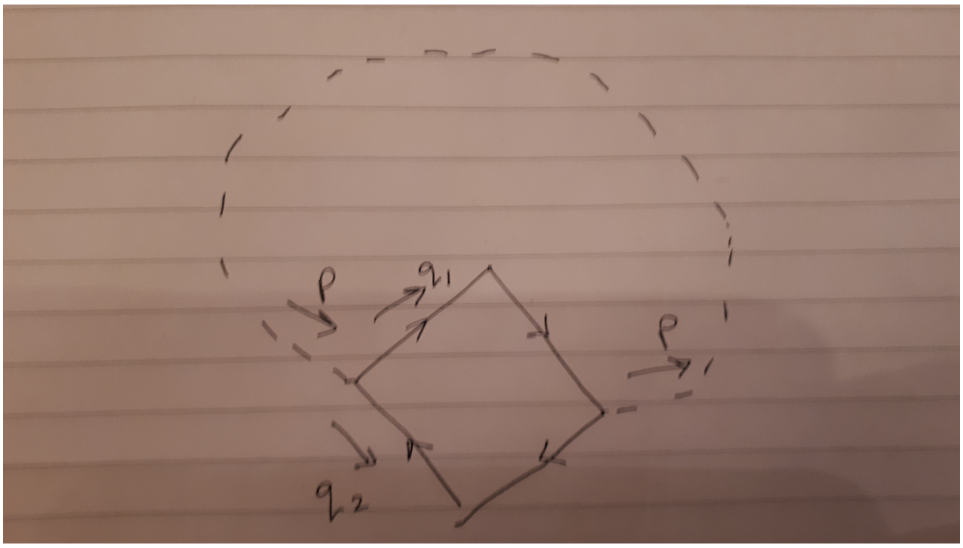
Marks **Comments**

(i) 

[2]

[1] *(ii)* $[\lambda] = -1$

[1] $\Rightarrow$ *theory is non-renormalisable.*

(iii) 

[2]

$$\begin{aligned}\overline{|\mathcal{M}|^2} &= |\lambda|^2 \mathrm{Tr}[(\not{q}_1 - m)\not{p}(\not{q}_2 + m)(-\not{p})] \\ \frac{\overline{|\mathcal{M}|^2}}{|\lambda|^2} &= \mathrm{Tr}[-\not{q}_1\not{p}\not{q}_2\not{p} + m^2\mu^2 I]\end{aligned}$$

[2]

[1] *since Clifford algebra* $\{\gamma^\mu,\ \gamma^\nu\} = 2\eta_{\mu\nu} I$

[1] $\Rightarrow \not{p}\not{p} = p_\mu p_\nu \{\gamma^\mu,\ \gamma^\nu\}/2 = p^2 I = \mu^2 I$, *and*

Use $\gamma_0^2 = I$ *so* $\mathrm{Tr}(\gamma_0^2\gamma^\mu) = \mathrm{Tr}(\gamma_0\gamma^\mu\gamma_0)$ *by cyclity of trace, but it is also equal to* $\mathrm{Tr}(-\gamma_0\gamma^\mu\gamma_0)$ *by Clifford algebra for* $\mu = 1, 2, 3$, *hence the initial trace is zero. For* $\mu = 0$, *do the same except with* $\gamma_i^2 = -I$. *Thus*
[2] $\Rightarrow \mathrm{Tr}(\gamma^\mu) = 0$.

and $\mathrm{Tr}(\not{p}\not{q}_2\not{p}) = \mathrm{Tr}(\not{p}\not{p}\not{q}_2) = \mu^2\mathrm{Tr}(\not{q}_2) = 0$,
[1] $\mathrm{Tr}(\not{q}_1\not{p}\not{p}) = \mu^2\mathrm{Tr}(\not{q}_1) = 0$.

so

$$\frac{\overline{|\mathcal{M}|^2}}{|\lambda|^2} = 4m^2\mu^2 - 4[2p\cdot q_1 p\cdot q_2 - q_1\cdot q_2\mu^2].$$

[1]

Centre of mass frame:

$$p^\mu = (\mu, \mathbf{0}), \qquad q_1^\mu = (\sqrt{\mathbf{q_1}^2 + m^2}, \mathbf{q_1}), \qquad q_2^\mu = (\sqrt{\mathbf{q_1}^2 + m^2}, -\mathbf{q_1}).$$

Conservation of energy and momentum: $\mu = 2\sqrt{\mathbf{q_1}^2 + m^2} \Rightarrow \mathbf{q_1}^2 = \mu^2/4 - m^2$, *so* $p\cdot q_1 = p\cdot q_2 = \mu^2/2$ *and* $q_1\cdot q_2 = \mu^2/2 - m^2$. *Hence*

$$\frac{\overline{|\mathcal{M}|^2}}{|\lambda|^2} = 4m^2\mu^2 - 4\left\{2\frac{\mu^4}{4} - \mu^2\left(\frac{\mu^2}{2} - m^2\right)\right\} = 0.$$

[4] *Hence* $\Gamma = 0$.

[1] *(iv)* $\sigma = 0$

[1] *because of conservation of global* $U(1)$ *charge*

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[20]