Wavy Instability in Liquid-Fluidized Beds

Maxime Nicolas,[†] **John Hinch,**[‡] **and Élisabeth Guazzelli***[,†]

Laboratoire de Physique et Mécanique des Milieux Hétérogènes, CNRS UMR 7636, ESPCI, 10 rue Vauquelin, 75231 Paris Cedex 05, France, and Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, U.K.

Experiments on the primary wavy instability in a liquid-fluidized bed are designed to test the two-phase flow governing equations. The wave is shown to saturate along the bed. The saturated wave can be well described as a cnoidal wave. The solid-phase viscosity and pressure which are unknown in the two-phase model are deduced from the shape of the saturated wave. The validity of the two-phase Newtonian model is then questioned.

1. Introduction

To fluidize a bed of particles, a fluid is pumped upward at the bottom of the bed through a porous plate. At low flow rates (below minimum fluidization), the bed is packed. As the flow is increased, the drag force on the particles increases until it is sufficient to balance their buoyant weight. The particles then become free to move, and the bed is said to be fluidized. When the flow is increased above minimum fluidization, the bed expands to accommodate the upward flow.

Uniform and homogeneous fluidized beds are rarely realized in practice. Fluidized beds present usually a variety of complex flow regimes above minimum fluidization. Gas-fluidized beds are very unstable and rapidly attain a bubbling regime. In this regime, bubbles, which are regions essentially devoid of particles, rise through the bed.[1−3] Liquid-fluidized beds exhibit voidage instability waves.[4−7] The instability remains one-dimensional only in narrow beds. The measured disturbances were shown to grow exponentially upward along the bed height and eventually to lead to a saturated finite amplitude.[4,5] In wider beds, there is a secondary gravitational overturning instability. A conjecture is that bubbles originate from the later evolution of this secondary instability.[8,9]

Most of what is known about fluidized beds comes from experiments. A complete theory for fluidized beds is not yet fully established. Two-phase modeling has been used extensively over the past 2 decades with some success; see, in particular, the pioneering work of Anderson and Jackson.[10,11] The fluidized bed is assumed to be one-dimensional and infinite, and the conservation equations for the mass and the momentum are written for the two phases, the particles and the fluid. To close these equations, it is then necessary to postulate expressions for the interphase force and the stress tensors associated with the fluid and particle phases. An empirical correlation, the Richardson−Zaki law, can be used for the drag force in terms of the relative motion between the phases. This law depends on the particle volume fraction, ϕ. A Newtonian fluid form is usually adopted for the stress tensors. It is not clear, however, how to describe the pressure, p_s, and the viscosity, μ_s, of the solid phase. These parameters may increase with an increase of the particle volume fraction.

One of the most pressing problems in the fundamental understanding of fluidized beds is the uncertainties in the governing equations. The objective of the present paper is to test the individual terms by using an experimental investigation of the primary wavy instability of fluidized beds. This primary wavy instability is an interesting problem in itself and is also the first step in the understanding of bubble formation.

2. Convective Nature of the Instability

From a hydrodynamic point of view, a fluidized bed is an open flow; i.e., fluid particles are not confined in the fluidization column. Instabilities evolving in such flows can be classified in two categories. If the instability is sensitive to external perturbations (noise), it is said to be *convective*. Perturbations are then amplified while convected by the mean flow, and the flow can be described as a noise amplifier. On the contrary, if the instability has an intrinsic behavior, it is said to be *absolute*. The flow then behaves as an oscillator.[12]

Experimentally, the behavior of the fluidized bed can be determined by investigating the response of the suspension to a localized and controlled perturbation. The nature of the wavy instability was examined experimentally for a liquid-fluidized bed. A fluidized suspension of glass spheres (with diameter $d = 685 \pm 28$ μm and density $\rho_s = 4.0 \pm 0.1$ g cm^{-3}) in water was contained in a thin vertical glass tube with a minimum fluidization velocity of $q_{mf} = 0.82 \pm 0.07$ cm s^{-1} (see Figure 1). The typical Froude number of the present experiment was 0.06. The suspension was held by a porous piston which can be moved with a sinusoidal motion at a given frequency and with an amplitude of 1.5d. This piston-type distributor was used to study the response of the suspension to a local harmonic forcing.

Above this special distributor, the wave propagation can be visualized by backlighting the column with a neon light. A CCD camera captures a one-dimensional image of the tube, and a digital imaging system is able to construct spatiotemporal plots from successive 1D images. Figure 2 shows such plots and demonstrates the convective nature of the instability. When a sinusoidal perturbation (larger than the “natural noise” of the distributor) is applied, the waves are periodic and follow the forcing, as can be seen for two different frequencies in Figure 2. Further details on the spatial

[†] ESPCI. Present address: IUSTI, CNRS UMR 6595, Université de Provence, Technopôle de Château Gombert, 5 rue Enrico Fermi, 13453 Marseille Cedex 13, France.

[‡] University of Cambridge.

10.1021/ie980401b CCC: $18.00
Published on Web 01/15/1999

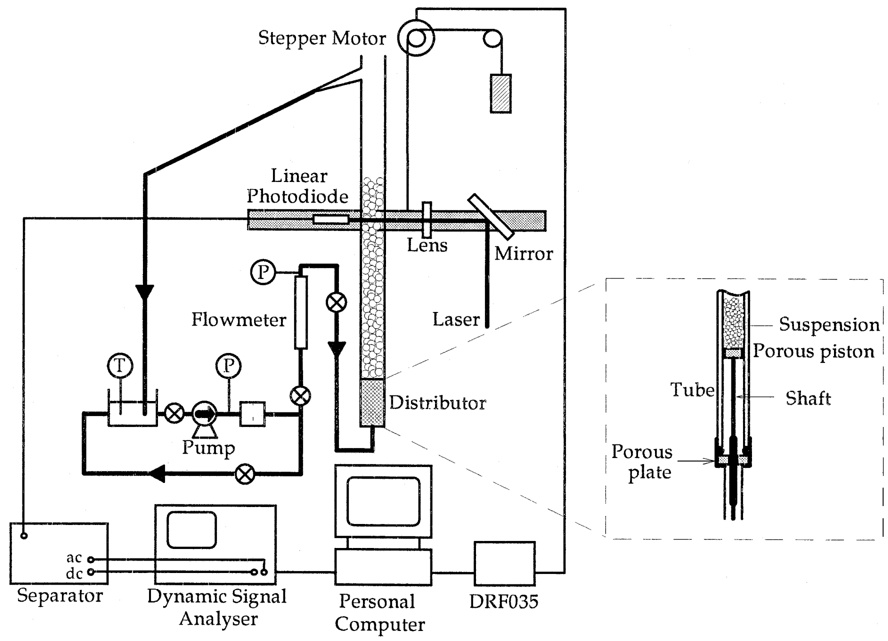

Figure 1. Schematic of the fluidized bed.

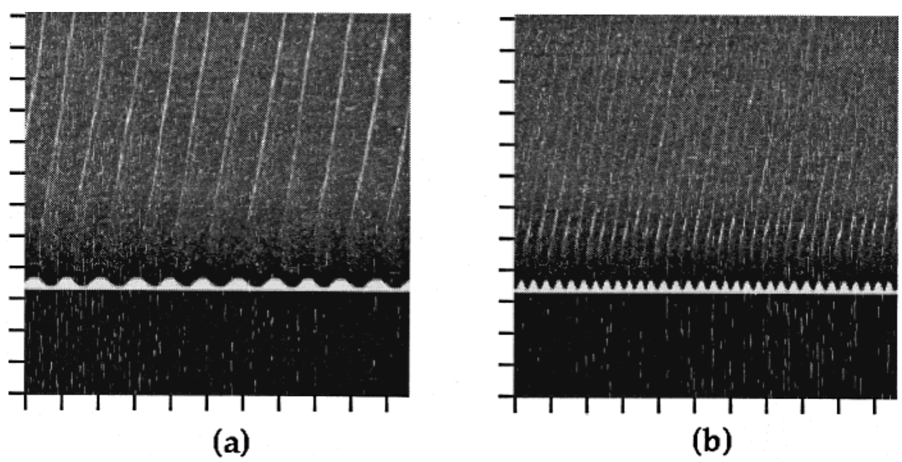

Figure 2. Spatiotemporal plots: waves forced at 0.5 (a) and 1.5 Hz (b). The mean particle volume fraction is 0.500 ± 0.001. The inclined white lines in these plots correspond to low concentration regions of the suspension moving upward with nearly constant velocity. The sinusoidal motion of the porous piston can also be seen.

stability of the two-phase equations as well as on the experimental results may be found in refs 13 and 14.

Though not formulated in the absolute/convective framework, the spatial evolution of the wave was noted in the early work of Anderson and Jackson.[4] The power spectra of the waves were found to be very broad (without forcing). There was clear evidence, however, of a dominant low-frequency mode. Its amplitude was found to increase along the bed and eventually to saturate.[5−7]

3. Nonlinear Saturation of the Wave

Because the instability is convective in nature, any perturbation at the bottom of the bed (natural noise or controlled perturbation) is amplified up the suspension. It is then fundamental to investigate the spatial evolution of each mode independently. To this end, a more quantitative experiment was designed in which the particle volume fraction was measured with a light attenuation technique, using a He−Ne laser beam as a light source. A synchronized average method was used to separate the forcing mode from other modes (noise), using the moving piston as a reference signal.

The amplitudes of the unstable modes were found to saturate. As can be seen in Figure 3, which is typically of many further observations, the saturated wave can be described as a succession of concentration dips and plateaus. Within experimental errors, the wave seems to be symmetric about the minimum of concentration. It can be well fitted by a cnoidal wave (a periodic solution of the Korteweg−de Vries equation[15]):

$$\phi(t) = \phi_{\max} - (\phi_{\max} - \phi_{\min})\mathrm{cn}^2[\tilde{\omega}(t-t_0)|m] \qquad (1)$$

where $\phi_{\max}$ and $\phi_{\min}$ are maximum and minimum concentrations, $\tilde{\omega}$ is the frequency, and t is time. The parameter m (≈0.99 in the present case) is a hidden parameter of the cnoidal function cn (a Jacobian elliptic function[16]). It is interesting to mention that such a cnoidal solution has been found in weakly nonlinear analysis of the two-phase equations.[17]

4. Questioning the Two-Phase Model

Using the experimental measurement of the shape of the saturated wave, it is then possible to estimate the

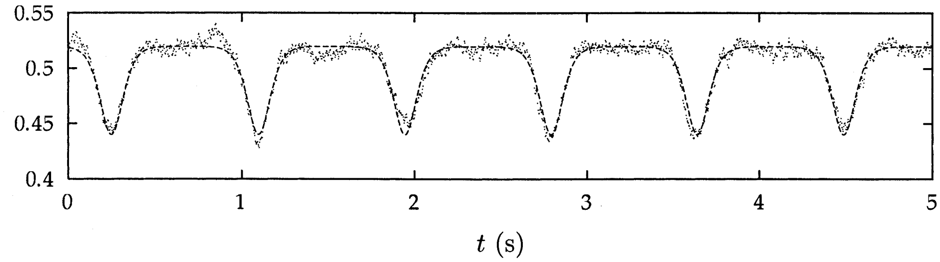

Figure 3. Example of a saturated wave (points) fitted by a cnoidal function (dashed line) measured 5 cm above the distributor. The coefficients of (1) are $\phi_0 = 0.503 \pm 0.02$, $f_0 = 1.17 \pm 0.03$ Hz, $\phi_{max} = 0.5199$, $\phi_{min} = 0.44$, $\tilde{\omega} = 11.30$ s^{-1}, $m = 0.9989$, and $t_0 = 1.0968$ s.

viscous and pressure terms (μ_s and p_s), which are unknown parameters in the two-phase model. For the purpose of the present work, we choose to consider the model of Anderson and Jackson.[10,11] The fluidized bed is assumed to be one-dimensional and infinite, and the conservation equations for the mass and the momentum are written for the two phases in the following forms:

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}(\phi v) = 0 \tag{2}$$

$$u = \frac{q - \phi v}{1 - \phi} \tag{3}$$

$$\rho_s \phi \left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = -\frac{\partial p_s}{\partial x} + \beta(u - v) - \phi(\rho_s - \rho_f)g + \frac{4}{3}\frac{\partial}{\partial x}\left(\mu_s \frac{\partial v}{\partial x}\right) + \rho_f \phi \tilde{C}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \frac{\partial v}{\partial t} - v\frac{\partial v}{\partial x}\right) + \rho_f \phi\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) \tag{4}$$

In this formulation, u and v are the fluid and particle velocities, q is a constant mixture velocity (equivalent to fluidizing flow rate), and ρ_f and ρ_s are the fluid and solid densities, respectively. The parameter $\tilde{C}$ is the added-mass coefficient, which cannot be neglected in liquid fluidization and depends a priori on the particle volume fraction ϕ. The drag on the solid phase is represented by the coefficient β, which can be related to the useful Richardson–Zaki law.[9]

These equations are rewritten in a reference frame moving with the saturated wave at velocity c_{sat}, where the spatial coordinate is $X = x - c_{sat}t$. After elimination of u and v with (2) and (3), a single equation can be derived from (4) which describes the shape of the saturated wave:

$$\frac{4}{3}\frac{c_{sat}\phi^*}{\phi^2}\left[\mu_s(\phi)\,\phi_{XX} + \left(\mu'_s(\phi) - 2\frac{\mu_s(\phi)}{\phi}\right)\phi_X^{\,2}\right] + F_1(\phi) + [F_2(\phi) - p'_s(\phi)]\phi_X = 0 \tag{5}$$

with

$$\mu'_s(\phi) = \mathrm{d}\mu_s/\mathrm{d}\phi, \quad p'_s(\phi) = \mathrm{d}p_s/\mathrm{d}\phi,$$

$$F_1(\phi) = \phi\rho_s \frac{g(\rho_s - \rho_f)}{\rho_s}\left[\frac{q - c_{sat}(1 - \phi^*/\phi)}{v_t(1 - \phi)^n} - 1\right] \tag{6}$$

$$F_2(\phi) = \rho_f\phi(1 + \tilde{C})\frac{[q + c_{sat}(\phi^* - 1)]^2}{(1 - \phi)^3} + (\rho_s + \rho_f\tilde{C})\left(\frac{c_{sat}\phi^*}{\phi}\right)^2 \tag{7}$$

The Richardson–Zaki parameters are v_t, the terminal velocity of a single particle, and n, an exponent depending on the Reynolds number. The volume fraction ϕ^* is theoretically equal to ϕ_0, the mean particle volume fraction, but may differ slightly because of experimental errors in the measurement. The saturated wave velocity was observed not to vary much with the forcing frequency ($c_{sat} = 3.8 \pm 0.2$ cm s^{-1}).

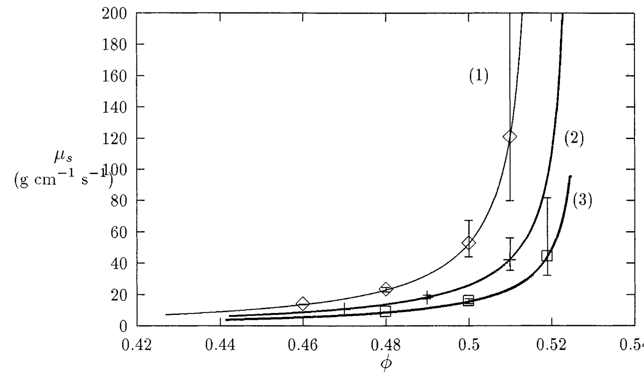

Figure 4. Solid-phase viscosity plotted against concentration for three different waves: $f_0 = 1$ (1), $f_0 = 1.4$ (2), and $f_0 = 2.1$ Hz (3). The volume fraction is the same ($\phi_0 = 0.5$) for these waves. The viscosity increases with decreasing frequencies.

Theoretical studies[9,17] assume some expressions for the viscosity $\mu_s(\phi)$ and pressure $p_s(\phi)$ and then use (5) to solve for the shape of the steady wave of $\phi(X)$. Here we do the opposite. From the experimental observations of the shape of the wave, we solve (5) to find the unknown viscosity and pressure functions.

For a symmetric wave such as that found experimentally, the pressure term balances the inertial terms and therefore

$$p'_s = F_2(\phi) \tag{8}$$

The viscous term balances the gravity and drag terms [$F_1(\phi)$]. The solid viscosity can then be found numerically. Further details can be found in ref 18.

The deduced viscosity increases with increasing volume fraction and seems to diverge at large volume fractions as can be seen in Figure 4. The existence of the three separated curves means that the "viscosity" depends also upon the frequency of the forcing mode and of the mean volume fraction of the suspension. This assertion assumes that the quasi-static Richardson–Zaki drag law can still be applied, which we believe to be true because the fluid typically passes a particle in 10 ms, a time much shorter than the period of waves. This finding suggests that a Newtonian description of the stress tensor of the solid phase may not be valid. However, these results are strongly dependent on the assumed cnoidal form of the wave. Further work is necessary to confirm and fully understand these results.

Literature Cited

(1) Clift, R.; Grace, J. R. Continuous bubbling and slugging. In *Fluidization*; Davidson, J. F., Clift, R., Harrison, D., Eds.; Academic: London, 1985.

(2) Homsy, G. M. Aspects of flow and mixing behavior in fluidized beds. In *Disorder and Mixing*; Guyon, E., Nadal, J.-P., Pomeau, Y., Eds.; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1988.

(3) Davidson, J. F. Bubbles in fluidized beds. In *Mobile Particulate Systems*; Guazzelli, É., Oger, L. Eds.; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1995.

(4) Anderson, T. B.; Jackson, R. A fluid mechanical description of fluidized beds—Comparison of theory and experiment. *Ind. Eng. Chem. Fundam.* **1969**, *8*, 137.

(5) El-Kaissy, M. M.; Homsy G. M. Instability waves and the origin of bubbles in fluidized beds. Part I. Experiments. *Int. J. Multiphase Flow* **1976**, *2*, 379.

(6) Didwania, A. K.; Homsy, G. M. Flow regime and flow transitions in liquid-fluidized beds. *Int. J. Multiphase Flow* **1981**, *7*, 563.

(7) Ham, J. M.; Thomas, S.; Guazzelli, É.; Homsy, G. M.; Anselmet, M.-C. An experimental study of the stability of liquid-fluidized beds. *Int. J. Multiphase Flow* **1990**, *16*, 171.

(8) Batchelor, G. K. Secondary instability of a gas-fluidized bed. *J. Fluid Mech.* **1993**, *257*, 359.

(9) Anderson, K.; Sundaresan, S.; Jackson, R. Instabilities and the formation of bubbles in fluidized beds. *J. Fluid Mech.* **1995**, *303*, 327.

(10) Anderson, T. B.; Jackson, R. A fluid mechanical description of fluidized beds—Equations of motion. *Ind. Eng. Chem. Fundam.* **1967**, *6*, 527.

(11) Anderson, T. B.; Jackson, R. A fluid mechanical description of fluidized beds—Stability of the state of uniform fluidization. *Ind. Eng. Chem. Fundam.* **1968**, *7*, 12.

(12) Huerre, P.; Monkewitz, P. A. Local and global instabilities in spatially developing flows. *Annu. Rev. Fluid Mech.* **1990**, *22*, 473.

(13) Nicolas, M.; Chomaz, J.-M.; Guazzelli, É. Absolute and convective instabilities of fluidized beds. *Phys. Fluids* **1994**, *6*, 3936.

(14) Nicolas, M.; Chomaz, J.-M.; Vallet, D.; Guazzelli, É. Experimental investigations on the nature of the first wavy instability in liquid-fluidized beds. *Phys. Fluids* **1996**, *8*, 1987.

(15) Drazin, P. G.; Johnson, R. S. *Solitons: an introduction*; Cambridge University Press: Cambridge, U.K., 1989.

(16) Abramowitz, M.; Stegun, I. A. *Handbook of mathematical functions*; National Bureau of Standards: Washington, DC, 1972.

(17) Harris, S. E. The growth of periodic waves in gas-fluidized beds. *J. Fluid Mech.* **1996**, *325*, 261.

(18) Nicolas, M. Instabilité en onde plane dans un lit fluidisé liquide-solide. Thèse de l'Université Paris 6, 1997.

Received for review June 23, 1998
Revised manuscript received September 17, 1998
Accepted September 22, 1998

IE980401B