J Eng Math (2014) 84:41–48
DOI 10.1007/s10665-013-9632-y

Particles impacting on a granular bed

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Received: 30 July 2012 / Accepted: 7 February 2013 / Published online: 15 May 2013

Abstract An asymptotic analysis is made to find the penetration depth and the stopping time for a particle impacting a granular bed. Newton's equation is solved with a drag force with two terms, one term proportional to the square of the velocity and one term linear in the depth. The penetration depth is found to increase with the logarithm of the impact velocity, while the stopping time is found to decrease with the inverse of the square root of the logarithm of the impact velocity.

Keywords Asymptotic analysis · Granular medium · Impacting particle

1 Introduction

When a particle impacts a granular bed, how deep does it penetrate the bed, and how quickly does it stop? Answering these questions may help understand the formation of craters when asteroids and military objects impact the Earth. In the last 10 years, there have been a number of laboratory studies of particles impacting granular beds, along with a few simplified numerical simulations. For spheres of diameter D and density ρ_s falling freely a height H and then impacting a granular bed of density ρ_b, the depth of penetration δ has been found to scale as

$$\frac{\delta}{D} \propto \left(\frac{\rho_s}{\rho_b}\right)^{\alpha} \left(\frac{H}{D}\right)^{\beta},$$

with various values of the indices α and β reported: Uehara et al. [9], de Bryun and Walsh [2] and Ambroso et al. [1] found in experiments $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$; Tsimring and Volfson [8] found in two-dimensional simulations $\alpha = \beta = \frac{2}{5}$, which they suggested in three dimensions should be $\alpha = \beta = \frac{1}{3}$, and Goldman and Umbanhowar [4] found in experiments $\alpha = \beta = \frac{1}{2}$. The stopping time exhibits a curious behaviour of faster impacting particles stopping in a shorter time, tending to a non-zero plateau value at high impacting velocities, $H \gtrsim D\rho_s/\rho_b$. Ciammera et al. [3] saw only the plateau value in their experiments and two-dimensional simulations, whereas Goldman and Umbanhowar [4] suggest from experiments a plateau value scaling with $(\rho_s/\rho_b)^{1/4}(D/g)^{1/2}$, and Seguin et al. [7] suggest from simulations $1.7(\rho_s/\rho_b)^{1/2}(D/g)^{1/2}$. The precise scalings of the penetration depth and the stopping time thus remain unclear.

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On the other hand, a consensus has emerged on the form of the resistive force that a granular bed exerts on a moving particle. The force is the sum of two parts, and a fluid-like inertial part proportional to the square of the instantaneous velocity and a dry-solid friction part proportional to the instantaneous depth z in the bed, i.e.

$$F = -C_D \rho_b D^2 v^2 - \mu D^2 \rho_b g z, \tag{1}$$

where g is the acceleration due to gravity and C_D and μ are dimensionless constants. The two parts resist the motion and so would change sign if the velocity became negative. Moreover, for a stationary particle, the dry-solid friction term may be less than $\mu D^2 \rho_b g z$. The inertial part, first suggested from two-dimensional simulations by Tsimring and Volfson [8], is the force required to accelerate from rest to the velocity v a mass $\rho_b D^2 v$ per unit time. The quadratic variation with velocity was confirmed experimentally by Katsuragi and Durian [5] with a single granular bed and a single sphere; the same researchers also found that the dependence on depth of the dry-solid part was best approximated by a linear variation in depth, corresponding to a Coulombic coefficient of friction multiplied by the normal pressure force, which itself is proportional to depth. They gave values for the dimensionless coefficients in the force law (1) as $C_D = 0.8$ and $\mu = 9$. Goldman and Umbanhowar [4] found in experiments with four different granular beds and 15 different spheres of various densities and radii that at high velocities and shallow impacts the force varied quadratically with the velocity and quadratically with the diameter of the sphere and was independent of the mass of the sphere. Deeper into the bed and at lower velocities, there would have been a significant contribution from the dry-solid friction, which seems not to have been examined separately. In two-dimensional simulations of particles with zero tangential friction, Seguin et al. [7] demonstrated that the inertial part was proportional to the square of the velocity, the density of the bed and the diameter of the particle, and that the dry-solid friction part was proportional to the density of the bed and the diameter of the particle and linear in the depth in the bed. The linear variation with the diameter in two dimensions becomes the square of the diameter in three dimensions. Using a single bed of very light expanded polystyrene particles and a single hollow sphere whose mass could be varied to 18 different values, Pacheco-Vázquez et al. [6] found that the trajectories of the sphere were consistent with the force law (1) while the sphere penetrated the bed no further than one diameter of the container. Deeper into the bed, the dry-solid friction part of the force tended to a constant, due to the Janssen effect's causing the pressure to tend to a constant deep into the bed. Pacheco-Vázquez et al. [6] gave values for the dimensionless coefficients $C_D = 2.4$ and $\mu = 12$.

Notwithstanding the difference in the values of the dimensionless coefficients, force law (1) seems well established. In each of the studies, it predicts well the trajectories of the particles and so predicts well the penetration depth and the stopping time. This opens up the possibility of determining the correct scaling laws for the penetration depth and stopping times by an asymptotic analysis of the equation of motion using the established force law. That possibility is the subject of this article.

2 Governing equations

The motion of an impacting sphere is governed by Newton's law with the weight of the sphere and the resistive force (1). Let z be the distance downwards from the free surface of a bed. The mass of the sphere is $\rho_s \pi D^3/6$. It is convenient to non-dimensionalise the problem so that the coefficients of the two friction terms are unity. To make the coefficient of the inertial term unity, length is non-dimensionalised by

$$L = \frac{\pi}{6C_D} \frac{\rho_s}{\rho_b} D.$$

To make the coefficient of the dry-solid friction term unity, time is non-dimensionalised by

$$T = \sqrt{\frac{\pi}{6\mu} \frac{\rho_s}{\rho_b} \frac{D}{g}}.$$

The governing equation then becomes

$$\ddot{z} = k - z - \dot{z}^2, \tag{2}$$

where $k = C_D/\mu$ in the term which represents the weight of the particle, having made the two coefficients in the friction law equal to unity. The experiments of Katsuragi and Durian [5] give $k = 0.09$, while the more recent experiments by Pacheco-Vázquez et al. [6] give $k = 0.2$. For the purpose of plotting results in this article, the value $k = 0.2$ will be used.

The initial conditions are that the sphere starts at the surface with a velocity from falling freely through the height H, which with the aforementioned non-dimensionalisation become

$$z(0) = 0 \quad \text{and} \quad \dot{z}(0) = V_0 = \sqrt{\frac{12\mu}{\pi C_D^2}\frac{\rho_b}{\rho_s}\frac{H}{D}}. \tag{3}$$

We shall be interested in the asymptotic behaviour for large impact velocities $V_0 \gg 1$. The largest value in experiments is approximately $V_0 = 10$, corresponding to dropping a 2.5 cm sphere 1 m with the density of the sphere being twice that of the bed.

3 Direct integration

It is possible to integrate the governing equation twice to obtain expressions for the penetration distance and stopping time. The expressions can then be evaluated asymptotically in the limit of large impact velocities. The non-linear governing Eq. (2) can be simplified by making a Riccati-inspired transformation. Introducing

$$z = \ln x, \quad \text{so} \quad \dot{z} = \frac{\dot{x}}{x} \qquad \text{and} \qquad \ddot{z} = \frac{\ddot{x}}{x} - \frac{\dot{x}^2}{x^2},$$

the governing equation becomes

$$\ddot{x} = x(k - \ln x).$$

This has a first integral

$$\dot{x}^2 = V_0^2 + (k + \tfrac{1}{2})(x^2 - 1) - x^2 \ln x, \tag{4}$$

using the initial conditions $x = 1$ and $\dot{x} = V_0$ from (3).

The velocity vanishes at x_∞ given by

$$x_\infty^2(\ln x_\infty - k - \tfrac{1}{2}) = V_0^2 - k - \tfrac{1}{2}.$$

For $V_0 \gg 1$, one can solve this iteratively for

$$x_\infty \sim \frac{V_0}{\sqrt{\ln V_0}}\left[1 + \frac{1}{2\ln V_0}\left(\ln\sqrt{\ln V_0} + k + \tfrac{1}{2}\right)\right]. \tag{5}$$

Thus the penetration depth is

$$z_\infty \sim \ln V_0 - \ln\sqrt{\ln V_0} + \frac{1}{2\ln V_0}\left(\ln\sqrt{\ln V_0} + k + \tfrac{1}{2}\right). \tag{6}$$

We shall treat $\ln\sqrt{\ln V_0}$, which occurs in this expression and in many subsequent expressions, as an $O(1)$ quantity.

Figure 1 plots the penetration depth z_∞ as a function of impact velocity V_0, comparing the asymptotic approximation (6) with results of numerical solution of Eq. (2). Typical of expansions involving logarithms, the first approximation $z_\infty \sim \ln V_0$ is vaguely nearby when the small expansion parameter $1/\ln V_0 = 0.9$ ($V_0 = 3$) is quite large, but has a relative error of approximately 15 % at least to $V_0 = 1{,}000$. Adding the second term, $-\ln\sqrt{\ln V_0}$, leaves the relative error at 8 % for $V_0 = 20$, but the error does now decrease if only very slowly, becoming 4 % by $V_0 = 100$ and 2 % by $V_0 = 1{,}000$. Adding the final term dramatically improves the accuracy to around 0.2 % through this range.

The stopping time t_∞ requires an integration of the first integral (4):

$$t_\infty = \int_1^{x_\infty}\left[V_0^2 - k - \tfrac{1}{2} - x^2\left(\ln x - k - \tfrac{1}{2}\right)\right]^{-1/2} \mathrm{d}x.$$

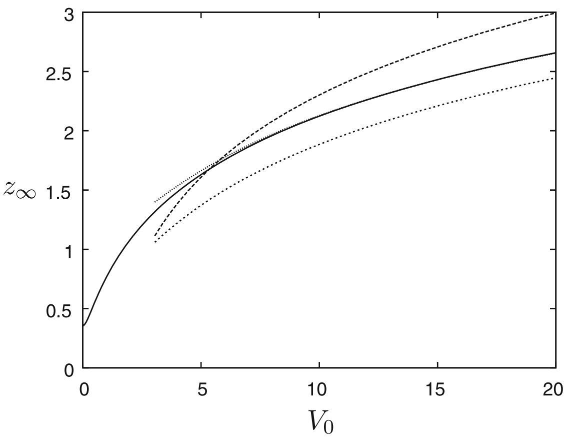

Fig. 1 Penetration depth z_∞ as a function of impact velocity V_0. The *continuous curve* is the result of a numerical integration of Eq. (2). The *long dashed curve* is the first term of the asymptotic result (6), while the *short dashed curve* is the first two terms and the *dotted curve* is all three terms

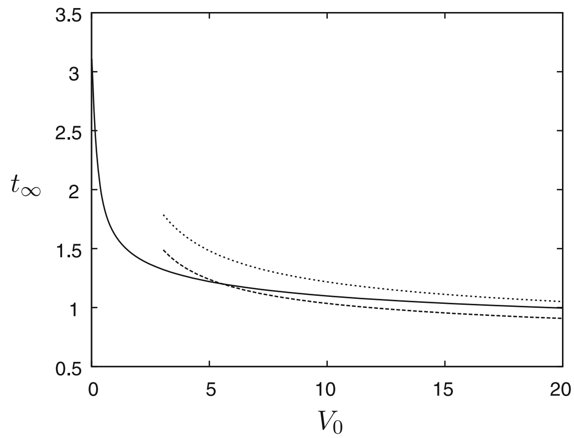

Fig. 2 Stopping time t_∞ as a function of impact velocity V_0. The *continuous curve* is the result of a numerical integration of Eq. (2). The *dashed curve* is the first term of the asymptotic result (7), while the *dotted curve* includes the correction

To evaluate this at large V_0, it is useful to substitute $x = x_\infty \xi$ to give

$$t_\infty = \frac{1}{\sqrt{\ln x_\infty - k - \frac{1}{2}}} \int_{1/x_\infty}^{1} \left[1 - \xi^2 \left(1 + \frac{\ln \xi}{\ln x_\infty - k - \frac{1}{2}} \right) \right]^{-1/2} \mathrm{d}\xi .$$

When $x_\infty \gg 1$, this gives

$$t_\infty \sim \frac{\pi/2}{\sqrt{\ln x_\infty - k - \frac{1}{2}}} \left(1 + \frac{\frac{1}{2}(1 - \ln 2)}{\ln x_\infty - k - \frac{1}{2}} \right).$$

Using our asymptotic expression for x_∞ (5), the stopping time in terms of V_0 is

$$t_\infty \sim \frac{\pi}{2\sqrt{\ln V_0}} \left[1 + \frac{1}{2 \ln V_0} \left(\ln \sqrt{\ln V_0} + k - \tfrac{1}{2} + \ln 2 \right) \right]. \tag{7}$$

Figure 2 plots the stopping time t_∞ as a function of impact velocity V_0, comparing the asymptotic approximation (7) with results of the numerical solution of Eq. (2). As with the penetration depth, the leading approximation $t_\infty \sim \pi/(2\sqrt{\ln V_0})$ is nearby throughout the range, but the error of approximately 0.1 does not decrease significantly. On the other hand, the second approximation gradually improves with an error of 0.06 at $V_0 = 20$, decreasing to 0.008 at $V_0 = 100$.

The asymptotic results for the penetration depth and the stopping time involve logarithms. The presence of logarithms explains the difficulty experienced in previous studies which tried to fit power-law scalings. We shall return to this issue in the discussion section.

The preceding direct integration yields expressions for the penetration depth and the stopping time but gives no insight into the form of the expressions, and in particular fails to explain why faster particles stop in a shorter time. To generate an understanding, we solve the problem with matched asymptotic expansions. There is first a fast phase when the velocity is $O(V_0)$ and the inertial part of the resistive force is dominant. The particle comes to rest in a second phase during which the two parts of the resistive force are comparable and the position changes little.

4 Fast initial phase

4.1 Scaling

During the first phase, the velocity is large, $\dot{z} = O(V_0)$, while the displacement is modest, $z = O(1)$, so the time scale must be short, $O(1/V_0)$. Hence we introduce a fast time scale:

$$\tau = V_0 t.$$

The governing problem then becomes

$$z_{\tau\tau} = -z_\tau^2 + \frac{1}{V_0^2}(k - z) \quad \text{with} \quad z_\tau(0) = 1.$$

In this initial phase the velocity is high, so that the inertial drag dominates.

The preceding form of the equation suggests an expansion

$$z(t, V_0) \sim \zeta_1(\tau) + \frac{1}{V_0^2}\zeta_2(\tau).$$

4.2 First approximation

The leading-order term is governed by

$$\zeta_{1\tau\tau} = -\zeta_{1\tau}^2, \quad \text{with} \quad \zeta_1(0) = 0, \quad \zeta_{1\tau}(0) = 1.$$

The solution is

$$\zeta_{1\tau} = \frac{1}{1+\tau} \quad \text{and} \quad \zeta_1 = \ln(1+\tau).$$

4.3 Correction

The correction is governed by

$$\zeta_{2\tau\tau} + 2\zeta_{1\tau}\zeta_{2\tau} = k - \zeta_1 \quad \text{with} \quad \zeta_2(0) = 0, \quad \zeta_{2\tau}(0) = 0.$$

The solution is

$$\zeta_2 = \tfrac{1}{6}(1+\tau)^2\left(-\ln(1+\tau) + k + \tfrac{5}{6}\right) + \tfrac{1}{3}\left(k + \tfrac{1}{3}\right)/(1+\tau) - \tfrac{1}{2}k - \tfrac{1}{4}.$$

4.4 Asymptoticity broken

The initial fast phase comes to an end when the increasing dry-solid friction term $-z$ in the governing equation becomes comparable with the decreasing inertial friction term $-\dot{z}^2$, i.e. when

$$\ln(1+\tau) = O\left(\frac{V_0^2}{\tau^2}\right),$$

i.e. at the large

$$\tau = O\left(\frac{V_0}{\sqrt{\ln V_0}}\right).$$

This large value of the initial fast time scale is the small time $t = O(1/\sqrt{\ln V_0})$. At this time the correction to the velocity $\zeta_{2\tau}/V_0^2$ becomes comparable to the first approximation to the velocity $\zeta_{1\tau}$ and

$$\dot{z} = O\left(\sqrt{\ln V_0}\right).$$

On the other hand, the displacement is still dominated by the first approximation ζ_1

$$z = O\left(\ln V_0\right).$$

5 Final stopping phase

5.1 Scalings

The breakdown of the initial fast phase sets the scalings of the final phase. We introduce a slower fast time,
$T = \sqrt{\ln V_0} t$,
and let Z be the $O(1)$ change in the large displacement established in the initial fast phase:
$z(t) = \ln V_0 + Z(T)$.
The governing equation then becomes
$$Z_{TT} = -1 - Z_T^2 + \frac{1}{\ln V_0}(k - Z).$$

5.2 Preparing to match backwards

Expressing the end of the initial phase at large τ in terms of the new slower fast time scale T
$$z \sim \ln V_0 - \ln \sqrt{\ln V_0} + \ln T - \tfrac{1}{6}T^2 + \frac{1}{6 \ln V_0}\left(\ln \sqrt{\ln V_0} - \ln T + k + \tfrac{5}{6}\right) \quad \text{as} \quad T \searrow 0.$$
We are treating the $\ln \sqrt{\ln V_0}$ as an $O(1)$ quantity. The governing equation and the matching both then suggest an expansion for the final stopping phase as
$$Z \sim Z_1 + \frac{1}{\ln V_0} Z_2.$$

5.3 First approximation

The first approximation is governed by
$Z_{1TT} = -1 - Z_{1T}^2$,
with
$Z_1 \sim -\ln \sqrt{\ln V_0} + \ln T - \tfrac{1}{6}T^2 \quad \text{as} \quad T \searrow 0$.
The solution is
$Z_{1T} = \cot T \quad \text{and} \quad Z_1 = -\ln \sqrt{\ln V_0} + \ln(\sin T)$,
with constants of integration set by the matching.

5.4 Correction

The correction is governed by
$Z_{2TT} + 2Z_{1T}Z_{2T} = k - Z_1$,
with
$Z_2 \sim \tfrac{1}{6}T^2\left(\ln \sqrt{\ln V_0} - \ln T + k + \tfrac{5}{6}\right) \quad \text{as} \quad T \searrow 0$.
The solution for Z_{2T} is
$$Z_{2T} = \csc^2 T \int_0^T \sin^2 s \left(\ln \sqrt{\ln V_0} + k - \ln(\sin s)\right) \mathrm{d}s,$$
with a constant of integration set by matching. Integrating,
$$Z_2 = \int_0^T (\cot s - \cot T) \sin^2 s \left(\ln \sqrt{\ln V_0} + k - \ln(\sin s)\right) \mathrm{d}s,$$
again with a constant of integration set by matching.

5.5 Stopping time

The velocity will vanish at the slow fast time

$$t_\infty = \frac{1}{\sqrt{\ln V_0}} T_\infty \quad \text{where} \quad Z_T(T_\infty) = 0.$$

Now for a first approximation for T_∞, we note

$$Z_{1T} = 0 \quad \text{at} \quad T = \frac{\pi}{2}.$$

Hence for a better approximation we pose

$$T_\infty \sim \frac{\pi}{2} + \frac{1}{\ln V_0} \Delta T.$$

Then

$$Z_T(T_\infty) \sim Z_{1T}(\tfrac{\pi}{2}) + \frac{1}{\ln V_0}\left(Z_{1TT}(\tfrac{\pi}{2})\Delta T + Z_{2T}(\tfrac{\pi}{2})\right).$$

Now $Z_{1TT} = -1 - Z_{1T}^2$ and $Z_{1T}(\frac{\pi}{2}) = 0$, so

$$\Delta T = Z_{2T}(\tfrac{\pi}{2}) = \tfrac{\pi}{4}\left(\ln\sqrt{\ln V_0} + k - \tfrac{1}{2} + \ln 2\right).$$

Hence,

$$t_\infty \sim \frac{1}{\sqrt{\ln V_0}} \frac{\pi}{2} + \frac{1}{(\ln V_0)^{3/2}} \frac{\pi}{4}\left(\ln\sqrt{\ln V_0} + k - \tfrac{1}{2} + \ln 2\right).$$

This agrees with expression (7) obtained in Sect. 3.

5.6 Penetration depth

Evaluating the depth at the stopping time,

$$z(t_\infty) \sim \ln V_0 + Z_1(\tfrac{\pi}{2}) + \frac{1}{\ln V_0}\left(Z_{1T}(\tfrac{\pi}{2})\Delta T + Z_2(\tfrac{\pi}{2})\right).$$

Now $Z_1(\frac{\pi}{2}) = -\ln\sqrt{\ln V_0}$ and $Z_{1T}(\frac{\pi}{2}) = 0$ and $Z_2(\frac{\pi}{2})$ can be evaluated simply, giving

$$z(t_\infty) \sim \ln V_0 - \ln\sqrt{\ln V_0} + \frac{1}{\ln V_0}\left(\tfrac{1}{2}\left(\ln\sqrt{\ln V_0} + k + \tfrac{1}{2}\right)\right).$$

This agrees with expression (6) obtained in Sect. 3.

6 Discussion

The asymptotic analysis of §4 and §5 reveals how the particle stops. While the velocity is large, the inertial part of the resistive force dominates, so

$$\ddot{z} \sim -\dot{z}^2,$$

with the solution

$$\dot{z} \sim \frac{V_0}{1 + V_0 t} \qquad \text{and} \qquad z \sim \ln(1 + V_0 t).$$

The two terms in the resistive force eventually become comparable when

$$\left(\frac{V_0}{1 + V_0 t}\right)^2 \sim \ln(1 + V_0 t),$$

i.e. at a time

$$t \sim \frac{1}{\sqrt{\ln V_0}} \quad \text{when} \quad \dot{z} = O(\sqrt{\ln V_0}) \quad \text{and} \quad z \sim \ln V_0.$$

Thereafter the inertial term drops to zero as the particle stops while the dry-solid friction term stays asymptotically constant, producing a deceleration $-\ln V_0$. This deceleration will finally stop the particle moving at a speed $O(\sqrt{\ln V_0})$ in a time $\pi/(2\sqrt{\ln V_0})$, with the particle advancing only a $O(1)$ distance during this time. Hence the penetration depth increases logarithmically with impact velocity V_0, while the stopping time decreases as $\pi/(2\sqrt{\ln V_0})$.

This behaviour is special to the combination of the two terms in the resistive force: with just one of the two terms the outcome is different. If there were only the quadratic inertial term, then the particle would never stop while the penetration would increase indefinitely in time. If there were only the dry-solid friction term, the motion would be a simple harmonic motion. Thus at low impact speeds, the penetration depth would be $2k$ while the stopping time is π, and at high impact speeds the penetration depth increases linearly with V_0 and the stopping time tends down to the plateau value $\pi/2$. It is only with the combination of the two terms that the stopping time continues to decrease as the impact velocity increases, albeit very slowing as the inverse of the square root of the logarithm.

Translating the results of the asymptotic analysis into the original dimensional variables of the problem, the penetration depth is predicted at leading order to be

$$\frac{\delta}{D} \sim c_1 \frac{\rho_s}{\rho_b} \ln\left(\frac{\rho_b}{\rho_s}\frac{H}{D}\right),$$

while the stopping time is predicted to be

$$t_\infty \sqrt{\frac{g}{D}} \sim c_2 \left(\frac{\rho_s}{\rho_b}\right)^{1/2} \left(\ln \frac{\rho_b}{\rho_s}\frac{H}{D}\right)^{-1/2},$$

with two constants c_1 and c_2.

The existence of the logarithms explains the variability in the previous results for the exponents when fitting power laws to the experimental observations. Indeed if a power law is fitted to the the numerical results for the penetration depth in Fig. 1, then one finds different exponents if different ranges are used. Fitting over the range $1 < V_0 < 2$, one finds $\alpha = 0.75$ and $\beta = 0.25$. On the other hand, if one fits over the range $5 < V_0 < 20$, then one finds $\alpha = 0.83$ and $\beta = 0.17$. The same ranges fitted to the stopping times in Fig. 2 produce very similar exponents, so $t_\infty \propto (\rho_s/\rho_g)^{0.42}(D/H)^{0.08}$, close to the scaling suggested by [7]. The power laws for the penetration depth are different from those found by fitting experimental observations. It may be that the force law needs refinement.

Acknowledgments Milt was a friend who inspired us to exploit any small parameter and learn.

References

1. Ambroso MA, Santore CR, Abate AR, Durian DJ (2005) Penetration depth for shallow impact cratering. Phys Rev E 71:051305
2. de Bruyn JR, Walsh AM (2006) Penetration of spheres into loose granular media. Can J Phys 82:439–446
3. Ciamarra MP, Lara AH, Lee AT, Goldman DI, Vishik I, Swinney HL (2004) Dynamics of drag and force distributions for projectile impact in a granular medium. Phys Rev Lett 92:194301
4. Goldman DI, Umbanhowar P (2008) Scaling and dynamics of sphere and disk impact into granular media. Phys Rev E 77:021308
5. Katsuragi H, Durian DJ (2007) Unified force law for granular impact cratering. Nat Phys 3:420–443
6. Pacheco-Vázquez F, Caballero-Robledo GA, Solano-Altamirano JM, Altshuler E, Batista-Leyva AJ, Ruiz-Suárez JC (2011) Infinite penetration of a projectile into a granular medium. Phys Rev Lett 106:218001
7. Seguin A, Bertho Y, Gondret P, Crassous J (2009) Sphere penetration by impact in a granular medium: a collisional process. Europhys Lett 88:44002
8. Tsimring LS, Volfson D (2005) Modeling of impact cratering in granular media. In: Garca-Rojo R, Herrmann HJ, McNamara S (eds) Powders and grains 2005, vol 2. Balkema, Rotterdam, pp 1215–1223
9. Uehara JS, Ambroso MA, Ojha RP, Durian DJ (2003) Low-speed impact craters in loose granular media. Phys Rev Lett 90, 194301 Erratum: low-speed impact craters in loose granular media. Phys Rev Lett 91:149902