BRIEF COMMUNICATIONS

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Intrinsic convection in a settling suspension

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The intrinsic convection in a settling suspension is shown to have its origins in the buoyancy of the particle-depleted layer next to the side walls. The convection is calculated using a simple model in which the particles are represented by an excluded volume and by point forces at their centres. A boundary layer approximation is developed for vessels which are much wider than the particle diameter. It is shown that the direction of the convection depends critically on assumptions about the formation of the particle-depleted layer during the placement of the suspension into the vessel. © *1996 American Institute of Physics.* [S1070-6631(96)00505-3]

We consider the sedimentation of rigid spheres suspended in a viscous fluid in a vessel with vertical side walls. In addition to the sedimentation velocity of the heavy particles relative to the local fluid, there can also be a global convection of the suspension in which the fluid and the particle move together. This convection has been studied by Geigenmüller and Mazur,[1] and is called the ''intrinsic convection.'' They have shown that in a dilute suspension the magnitude of this convection is $O(V_0 c)$ where V_0 is the sedimentation velocity of an isolated sphere and c is the volume fraction of the particles in the suspension. The intrinsic convection is therefore comparable with the correction to the settling velocity due to a small non-zero concentration c. It is important to note that this intrinsic convection occurs in vessels with vertical walls and so is different from the Boycott effect, which requires inclined walls to generate its faster convection currents.

In the Geigenmüller and Mazur model, the particles are rigid spheres and their influence on the fluid is approximated by a uniform distribution of point forces distributed over the surface of the particles. Particle-particle and wall-particle interactions are thereby neglected. We adopt an even simpler model in which the particles are replaced by point forces acting at their centers. These point forces are uniformly distributed in the container so that particles can overlap with one another. We retain however the constraint that the particles cannot overlap with the wall.

The governing equations for the bulk motion of the suspension, i.e. of the particles and fluid moving together, are thus taken to be

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$0 = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho(\mathbf{x})\mathbf{g}. \tag{2}$$

The point particle approximation leaves the viscosity of the suspension equal to the solvent value μ. The approximation also gives a simple expression for the density $\rho(\mathbf{x})$, equal to the solvent value ρ_f within the particle-depleted layer next to the wall of thickness a the radius of the particles, and equal to the suspension value $\rho_s = \rho_f(1-c) + \rho_p c$ where ρ_p is the density of the particles in the bulk of the suspension. The above equations are subject to the no slip boundary condition,

$$\mathbf{u} = 0, \tag{3}$$

on the walls of the vessel.

We consider a tall vessel with vertical side walls. Making the side walls vertical suppresses the rapid convection of the Boycott effect. Away from the bottom and the top, the flow will be vertical and the same over each horizontal section. From the incompressibility (1) and the condition of no net flux through the bottom of the vessel in (3), we deduce that there is no net flow across any horizontal section,

$$\int_{z=\mathrm{const}} w\,dx\,dy = 0. \tag{4}$$

Applying this condition determines the vertical pressure gradient which is constant in a tall vessel.

It is a simple matter to solve (2) subject to (3) and (4) in closed form for the space between two vertical plane walls. In Fig. 1 we compare our results with those of Geigenmüller

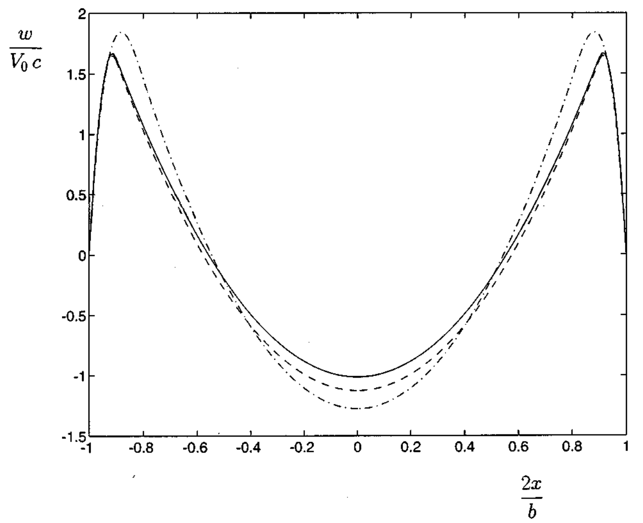

FIG. 1. The vertical motion of the intrinsic convection in a homogeneous dilute suspension, normalised by the settling speed of an isolated sphere V_0 and the volume fraction c. The dash–dotted curve is the result of Geigenmüller and Mazur, the continuous curve is the solution of our simplified model with the weight of the spheres exerted at their centers, and the dashed curve is the result of our boundary analysis.

and Mazur for that case of a channel of width of 10 particle diameters. We see the buoyant particle-depleted layer next to the wall drives an upward velocity with a maximum approximately $2V_0c$ at one particle radius from the wall. The circulation is completed by a downward return flux in the center with a maximum approximately $-V_0c$. There is little difference between the results of Geigenmüller and Mazur which have the weight of the particles distributed over their surface and our simpler model with the weight concentrated at the center. Similar remarks apply to the case of a vessel of a circular cylinder, which we do not display here.

The intrinsic convection has not to our knowledge been observed experimentally. Most measurements of the sedimentation velocity track the front between the suspension and the clear fluid above. This method cannot see circulation inside the suspension. The predicted circulation would deflect the interface by a fraction of a particle diameter, $O(2a^2/9b)$ where b is the width of the vessel, thus well within the uncertainty of the position of the interface. There have been some observations of the velocity of individual particles in a sedimenting suspension. These find large variations in velocity between different particles due to hydrodynamic interactions. The average velocities reported so far involve many particles in different parts of the suspension, so averaging out any internal circulation.

When the particles are small compared with the width of the vessel, a boundary layer analysis is possible. To leading order the pressure is the hydrostatic pressure required to support the weight of the suspension in the majority of the vessel (the outer region of the boundary layer analysis),

$$\frac{dp}{dz} \sim -\rho_s g. \tag{5}$$

This pressure gradient is unchanged in the particle-depleted wall layer, because with no motion perpendicular to the wall the pressure gradient can have no component in that direction, and so the pressure is solely a function of height. Thus the governing momentum equation (2) reduces in the wall layer to

$$\mu \frac{d^2 w}{dx^2} = -(\rho_p - \rho_f) c g, \tag{6}$$

where x is measured normal to the wall. We solve this equation subject to the no slip condition $w=0$ on the wall and a no stress condition $\mu dw/dx=0$ at the edge of the wall region one particle radius a from the wall. This no stress condition applies at lowest order because the majority of the vessel will have velocities of the same magnitude but with much longer length scales than in the wall layer. Integrating (6), we obtain the velocity on the edge of the wall layer,

$$w_* = \frac{9}{4} V_0 c, \tag{7}$$

using the settling speed of an isolated sphere $V_0 = 2a^2(\rho_p - \rho_f)g/9\mu$.

In the majority of the vessel outside the particle-depleted layer, the suspension is dragged upwards by the effective slip velocity w_* on the edge of the wall layer. In order to comply with the no net flux condition (4) a small downward pressure gradient is required to drive a Poiseuille return flow superimposed on this uniform upward w_*. We note that the extra pressure gradient required is $O(\mu w_* / b^2)$ where b is the width of the vessel, and that this a small correction $O(a^2/b^2)$ relative to the principal hydrostatic balance.

While the effective slip velocity w_* is independent of the shape of the cross-section of the vertically sided vessel, the Poiseuille return flow does depend on the shape. For two plane walls, the Poiseuille flow is parabolic with a maximum downwards velocity at the centre $-\frac{1}{2} w_*$. We plot our boundary layer solution also in Fig. 1. It over predicts the maximum velocity in the wall region due to higher order corrections which modify the no stress condition on the edge of the layer.

The boundary layer analysis permits further extensions to vessels which are not tall, although they must still have vertical side walls. The argument which gave the slip velocity w_* would be applicable near to the top and the bottom of the suspension, so long as the vessel is wide compared with the particles and the suspension is uniformly concentrated in the interior. Of course within a couple of particle radii of the top and bottom the no stress condition would fail, but this is a negligibly small region. Thus the problem reduces to finding the three-dimensional return flow inside the non-tall vessel, which is to be superimposed on a uniform upward velocity w_*, the return flow ensuring that there is no net flux across any horizontal section. This remark could lead to a simpler approach than the one used by Geigenmüller and Mazur[2] to calculate the intrinsic convection taking into account the meniscus at the top of the suspension.

The boundary analysis emphasises the origin of the intrinsic convection is the buoyancy of the particle-depleted wall layer compared with the uniformly concentrated suspension outside. This raises the questions of how the suspension was placed in the vessel and what happened to the particles which were associated with the fluid which is destined for the wall layer. If the suspension is vigorously stirred, or if there is strong Brownian motion, then the particles will be uniformly distributed outside the wall layers. On the other hand, if vertical walls are carefully introduced into an initially well stirred suspension, and the particles have no Brownian motion, then the particles associated with the wall fluid will be displaced sideways just to the edge of the wall layer.

We can model the latter case of particles which were in the wall-layer fluid being displaced to the edge of the wall layer as a mass $(\rho_s-\rho_f)a=(\rho_p-\rho_f)ca$ per unit area on the edge of the wall layer. The weight of this mass requires a stress $\mu dw/dx=-(\rho_p-\rho_f)cag$ to be exerted from just inside the wall layer, as the outside is unable to exert any stress to leading order. Solving (6) using this condition instead of the earlier no stress boundary condition produces a velocity on the edge of the wall layer,

$$w_*=-\frac{9}{4}V_0c.$$

This velocity is the same magnitude as our original model, but is in the opposite direction. It is clear that the displaced mass at the edge of the wall layer is more successful at driving a downward flow than its deficit nearer to the wall is at driving an upward flow.

Finally we must remark the hydrodynamic interactions of the spherical particles with the wall have been taken into account only in a very approximate way, treating the particles as point forces. It is necessary to return to this in the future using instead the actual distribution of force over the surface of a particle which can be found from the solution of one particle interacting with the wall.

[1]U. Geigenmüller and P. Mazur, ''Sedimentation of homogeneous suspensions in finite vessels,'' J. Stat. Phys. **53**, 137 (1988).
[2]U. Geigenmüller and P. Mazur, ''Intrinsic convection near a meniscus,'' Physica A **171**, 475 (1991).