The velocity of 'large' viscous drops falling on a coated vertical fiber

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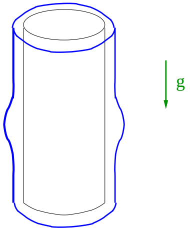
Revisiting: Kalliadasis & Chang (1994) *Drop formation during coating of vertical fibres* JFM **261**.

Thin coating on a vertical fibre

$$h_t + \left(h^3 \left(G + (h + h_{xx})_x\right)\right)_x = 0$$

where $G = \rho g a^3/\sigma h_0$.



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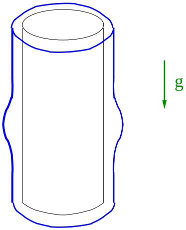

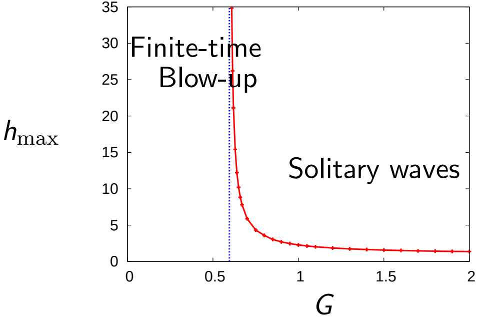

Large fast solitary waves

Kalliadasis & Chang (1994)

Main drop, constant capillary pressure

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Solutions

$$h \sim \tfrac{1}{2} P_{\pm} \xi^2 + Q\xi + R_{\pm} \quad \text{as} \quad \xi \to \pm\infty$$

where $Q = 0$, $P_{\pm} = 0.64$, $R_{+} = 2.90$ and $R_{-} = -0.85$.

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At leading order, matching curvatures

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But not yet found c as function of $G - G_0$.

Asymptotic expansion for main drop

$$h \sim c^{2/3}H_0 + c^{1/3}0 + c^0 H_2 + c^{-1/3}H_3 + c^{-2/3}H_4 + c^{-1}\ln c H_5 + c^{-1}H_6 \dots$$

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Amusing detail

$$(H_3 + H_3'')' = \frac{1}{H_0^2} = \frac{1}{P^2(1 - \cos x)^2}$$

with solution

$$H_3 = \frac{\sin x}{3P^2(1 - \cos x)}.$$

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Key correction from

$$h_2''' = \frac{3-2h_0}{h_0^4}h_2 - h_0' \sim -P\xi + \frac{0}{\xi^2} + O\left(\frac{1}{\xi^4}\right)$$

with solution

$$h_2 = -\tfrac{P}{24}\xi^4 + \tfrac{1}{2}a_\pm\xi^2 + 0\xi + c_\pm + O(\xi^{-1})$$

Matching 1

Inner=

$$
\begin{array}{rlllllll}
 & h_0 & h_2 & h_3 & h_4 & h_5 & \\
c^{2/3}\Big[& \frac{P}{2}x^2 & -\frac{P}{24}x^4 & & +\frac{P}{6!}x^6 & & +\dots\Big] \\
+c^0\Big[& R_+ & +\frac{a_2}{2}x^2 & -\frac{G_0}{6}x^3 & -\frac{a_2}{24}x^4 & +\frac{G_0}{5!}x^5 & +\dots\Big] \\
+c^{-1/3}\Big[& -\frac{2}{3P^2x} & +b_2x & +\frac{a_3}{2}x^2 & +K_3x^3 & +O(x^4) & +\dots\Big] \\
+c^{-2/3}\Big[& & +c_{2+} & +b_3x & +\frac{a_4}{2}x^2 & +\frac{G_2}{3!}x^3 & +\dots\Big] \\
+c^{-1}\ln c\Big[& & & +\frac{4G_0}{9P^3} & & O(x^2) & +\dots\Big] \\
+c^{-1}\Big[& \frac{K_1}{x^3} & +\frac{K_2}{x} & +\frac{4G_0}{3P^3}\ln x + c_{3+} & +b_4x & & +\dots\Big]
\end{array}
$$

Matching 2

Outer =

$$c^{2/3} \quad \left[\frac{P}{2}x^2 - \frac{P}{24}x^4 + \frac{P}{6!}x^6 + \dots\right]$$

$$+c^0 \quad \left[-G_0 \times 0 + A_2 + C_2 - \frac{C_2}{2}x^2 - \frac{G_0}{6}x^3 + \frac{C_2}{4!}x^4 + \frac{g_0}{5!}x^5 + \dots\right]$$

$$+c^{-1/3} \quad \left[-\frac{2}{3P^2x} + (A_3 + C_3) + (\frac{1}{18P^2} + B_3)x - \frac{C_3}{2}x^2 + (\frac{1}{1080P^2} - \frac{B_3}{3!})x^3 \dots\right.$$

$$+c^{-2/3} \quad \left[-G_2 \times 0 + A_4 + C_4 + B_4x - \frac{C_4}{2}x^2 - \frac{G_2}{3!}x^3 + \dots\right]$$

$$+c^{-1}\log c \quad \left[A_{4+} + C_{4+} - \frac{C_{4+}}{2}x^2 + \dots\right]$$

$$+c^{-1} \quad \left[\frac{2(1+2R_+)}{15P^3x^3} + \frac{4(1+2A_2-3C_2)}{15P^3x} + \frac{4G_0}{3P^3}\log x + A_5 + C_5 - G_3 \times 0 \dots\right]$$

Matching 3

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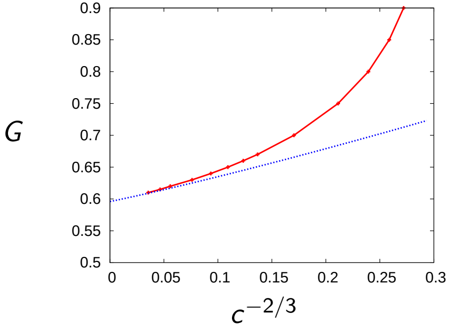

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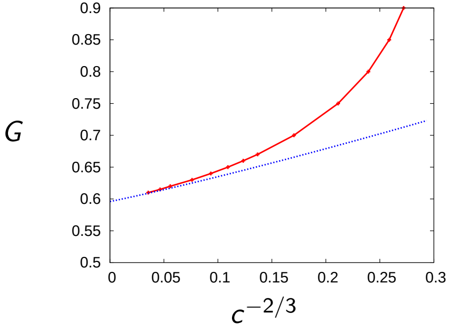

And for a shear-thinning liquid?