Levitation and locomotion on an air-table of plates with herringbone grooves.

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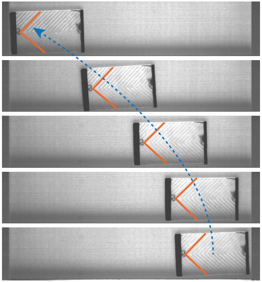
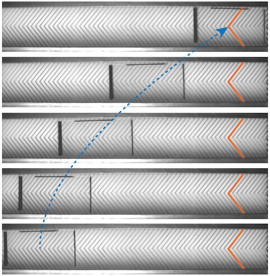
with Dan Soto & David Quéré in Paris, and
Maximilian Schür, Steffen Hardt & Tobias Baier in Darmstadt

Experiments on an air table

Grooves on the floating body, on the table

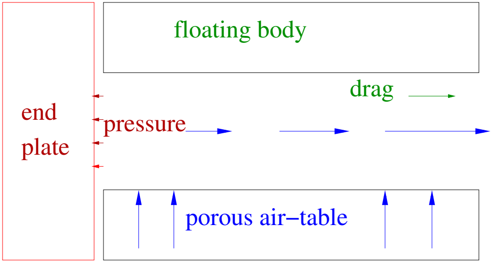




Accelerates to the left, to the right.

Why different direction?

Left or right?



End plate attached to base (grooved table)

$\longrightarrow$ body dragged by flow to right

End plate attached to top (grooved body)

$\longrightarrow$ pressure pushes to left

Pressure also levitates floating body.

Study 2D flow down groove

Boundary layer equations with $p = p(x)$:

$$u_x + v_y = 0$$
$$Re(u_t + uu_x + vu_y) = -K p_x + u_{yy}$$

BC on porous plate $y = 0$: $\quad u = 0, \quad v = 1 - p$

$$K = \frac{\Delta p \text{ across porous plate}}{\Delta p \text{ down groove}}$$

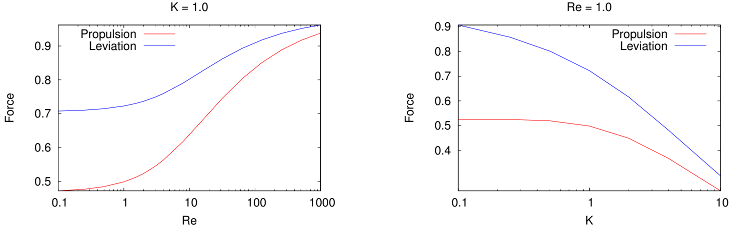
Forces:

$$\text{Propulsion } F_H = p(0) + \int_0^1 u_y\big|_{y=1} \, dx, \quad \text{Levitation } F_V = \int_0^1 p \, dx$$

NB: different non-dimensionalisation, F_H smaller by area ratio H/L

Numerical solution

Method: integrate $v_y = -u_x$ from $y = 1$ to $y = 0$ to find $v(x, y = 0)$,
then porous plate BC for $p(x) = 1 - v(x, 0)$ for momentum equation.



NB: two forces (in different non-dimensionalisations) within a factor of two
NB: small change with Re, but decrease at large K (short groove)

Asymptotics

- Short groove, $K \gg 1$, pressure drop mostly across porous plate
- $Re \ll 1$
- $Re \gg 1$
- Long groove, $K \ll 1$, $p = 1$ in groove except very near outlet

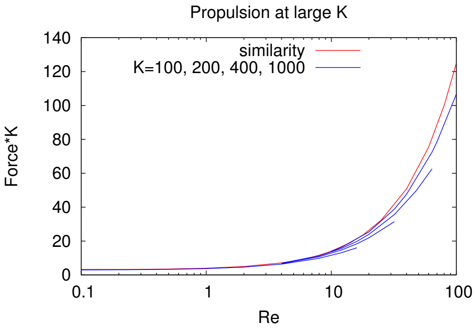
Short groove, $K \gg 1$

Most pressure drop across porous plate, so $p \sim 0$ in groove, so $v(x, y = 0) = 1$, hence similarity solution

$$u(x,y) = -xg'(y), \quad v(x,y) = g(y), \quad p = \frac{B}{K}(1 - x^2)$$

Momentum equation then

$$Re(g'^2 - gg'') = 2B - g'''$$



need $Re \ll K$

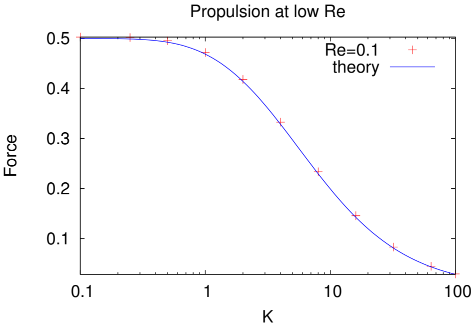
Low Reynolds number

Lubrication theory

$$q = \int_0^1 u(x,y)\,dy = -\frac{K}{12} p_x, \qquad q_x = v(x,0) = 1 - p$$

so

$$F_H = \tfrac{1}{2}\left(1 - \operatorname{sech}\sqrt{12/K}\right), \quad F_V = 1 - \sqrt{K/12}\,\tanh\sqrt{12/K}$$



High Reynolds number

Inviscid separable solution

$$u = -f(x)g'(y), \quad v = f'(x)g(y), \quad \text{with } g(y) = \cos\tfrac{\pi}{2}y$$

Bernoulli integral

$$\beta^2 f^2 = p_0 - p, \quad \text{with } \beta^2 = \pi^2 Re/8K$$

Porous plate $f' = 1 - p$, so

$$f(x) = \frac{\sqrt{1-p_0}}{\beta}\tan\left(\beta x\sqrt{1-p_0}\right)$$

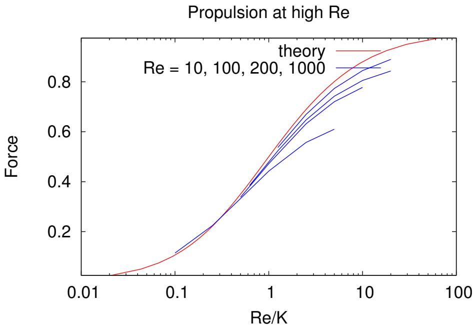
$p_0 = p(0)$ is determined by $p(1) = 0$:

$$\tan^2\left(\beta\sqrt{1-p_0}\right) = \frac{p_0}{1-p_0}$$

High Reynolds number, inviscid

Finally forces

$$F_H = -p_0, \quad F_V = 1 - \sqrt{p_0}/\beta$$

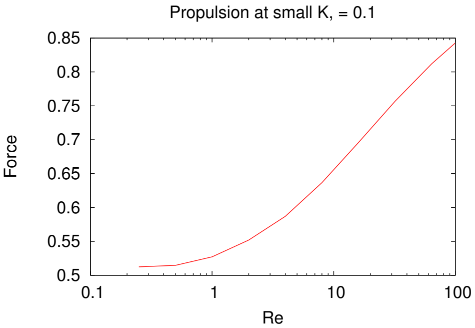


need $Re \gg K$

Need to add boundary layer on top

Long groove, $K \ll 1$

Pressure drop and flow only near exit of groove, so tough numerics.
Most of the groove is at $p = 1$, the pressure under the air-table.
Hence pressure part of Propulsion and Levitation ~ 1.



But frictional drag halves the Propulsion at low Re.

Double limits

	Propulsion F_H	Levitation F_V
$K, Re \ll 1$	0.5	$1 - \sqrt{K/12}$
$K \ll 1 \ll Re$	1	$1 - \sqrt{8K/\pi^2 Re}$
$Re \ll 1 \ll K$	$3/K$	$4/K$
$1 \ll Re \ll K$	$\pi^2 Re/8K$	$\pi^2 Re/12K$
$1 \ll K \ll Re$	$1 - 2K/Re$	$1 - \sqrt{8K/\pi^2 Re}$

At $Re \gg 1$ and $K \ll 1$, uniform levitation pressure = propulsion pressure, i.e. $F_H = F_V$

At $Re \ll 1$, frictional drag reduces F_H by 50%.
At $K \gg 1$, pressure is parabolic, reduces F_V by 2/3.

Hence $\quad F_H/F_V = 0.5$ to 1.5 (different non-dimensionalisations)

Experiments

Reinstating the different dimensional factors, and resolving force along direction that body moves, predict

$$\text{Propulsion/weight} = (0.5 \text{ to } 1.5)h \cos\alpha/\ell$$