Lopsided coatings of a visco-elastic fluid on a vertical fibre

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Motivation

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- François Boulogne observed in his Paris PhD thesis that the coating of an elastic liquid was never axisymmetric, but was always thicker on one side.
- Flow in thin coating is mainly simple shear and quasi-steady
  - Hence rheology is a viscosity plus normal stresses.
  - First normal stress difference = tension in streamlines → enhanced effective surface tension.
  - Second normal stress difference = tension in vortex lines → new instability.
- Will use lubrication theory for thin film.

Governing equation

Extra non-Newtonian stress for a second-order fluid

$$\sigma^{NN} = -2\alpha \overset{\triangledown}{E} + \beta E^2,$$

α tension in the streamlines, $\beta < 0$ tension in the vortex lines.

Lubrication theory, suitably non-dimensionalised:

$$\frac{\partial h}{\partial t} + G\frac{\partial h^3}{\partial z} + \nabla h^3 \nabla(h + \nabla^2 h) + A\frac{\partial^2}{\partial z^2}h^5 + B\frac{\partial^2}{\partial \theta^2}h^5 = 0,$$

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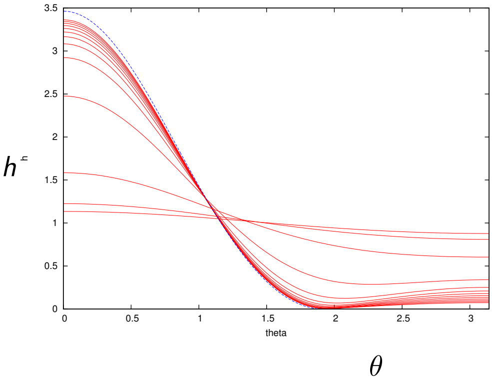
Now study development of lop-sided flow with $h(\theta, t)$,
no z-variations.

$$h_t + \left(h^3(h_{\theta\theta} + h + Bh^2)_\theta\right)_\theta = 0$$

Time evolution

$h(\theta, t)$ at $t = 2^n$ $n = -2, \ldots, 11$, for $B = 0.5$.

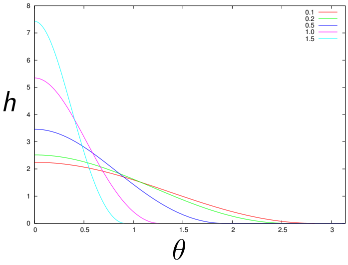


Dotted blue is a steady state which wets only $0 \leq \theta \leq 1.9071$

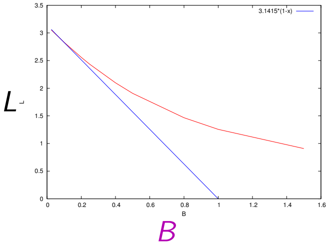
(Interesting intermediate times: drift of an off-centred cylinder.)

Steady states

Steady states for various *B*

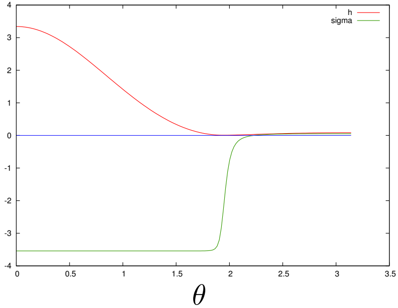


Length of steady state



Structure at late times

The shape and the pressure (stress $\sigma_{\theta\theta}$) at $t = 10^3$ for $B = 0.5$

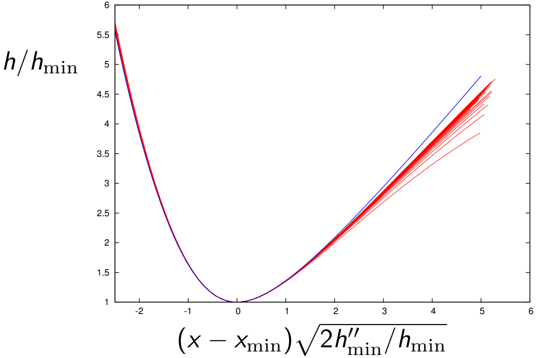


There two constant pressure regions.

Higher pressure region to the right drains into the lower pressure region to the left through a small neck.

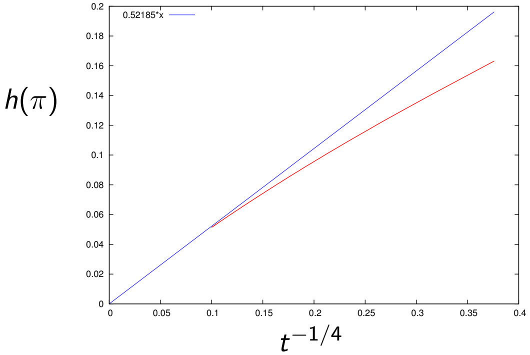
The neck between the two constant pressure regions

Universal shape of the neck between the two constant pressure regions, for $t = 50\,(50)\,10^3$ and for $B = 0.5$.



Blue shape from Bretherton's equation.

Draining of small region as $t^{-1/4}$



$$h(\pi) = \frac{1+\cos L}{t^{1/4}} \left(\frac{K((\pi - L)\cos L + \sin L}{4Q \sin^5 L} \right)^{1/4}$$

with Bretherton $Q = 1.20936$ and for $B = 0.5$ pressure in steady state $K = 3.7297$ and length of steady state $L = 1.9171$.