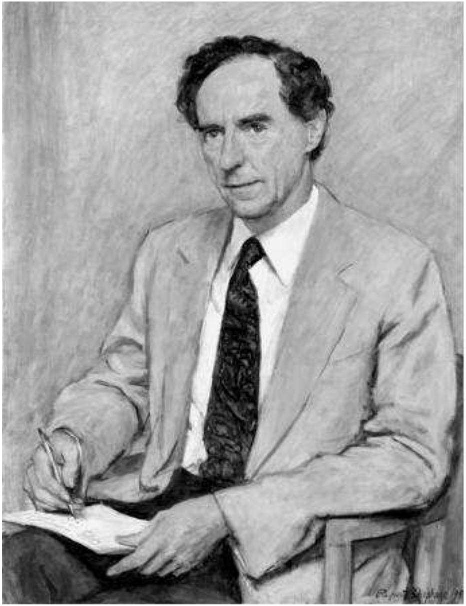

G. K. Batchelor 8 March 1920 – 30 April 2000

A perspective of GKB's Micro-hydrodynamics

John Hinch

DAMTP, Cambridge

September 10, 2010

Before: his turbulence research 1945–1961

- Understand, explain and exploit Kolmogorov

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- 1967 start of second wave of research (no more turbulence)

  in low-Reynolds-number suspensions of particles

  producing 8 of his top 10 most cited papers

GKB's Micro-hydrodynamics in perspective

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- Before
- Batchelor's research
- What followed

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  - To start:
    (i) bulk stress
    (ii) μ at c^2
- What followed

A start

Stress system in a suspension of force-free particles JFM 1970

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Suspension of small particles in a viscous fluid

- low Reynolds number flow about particles
- each particle sees a linear flow (local rheology)

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with viscous solvent stress and extra non-zero only inside particles

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Ensemble average (from turbulence research)

$$\langle \sigma_{ij} \rangle = -\langle p \rangle \delta_{ij} + 2\mu \langle e_{ij} \rangle + \langle \sigma_{ij}^{+} \rangle$$

Switch to volume average. Use divergence theorem for force-free particles

$$\langle \sigma_{ij}^{+} \rangle = n \int_A \sigma_{ik} n_k x_j - \mu(u_i n_j + u_j n_i)\, dA$$

with n number of particles per unit volume and A surface of typical particle.

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Much cited result - still 30 pa

L&L 1959

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Slender-body theory for particles of arbitrary cross-section in Stokes flow JFM 1970

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Evaluated force and couple

Second result

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- (Very) dilute: $\ell \ll h$
- Semi-dilute: $b \ll h \ll \ell$
- (Nematic phase transition to aligned rods: $h = \sqrt{b\ell}$)
- Concentrated: $h \sim b$

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Use bulk-stress paper for formula for stress

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Much larger than the viscosity of the solvent μ,
which is shear viscosity of suspension

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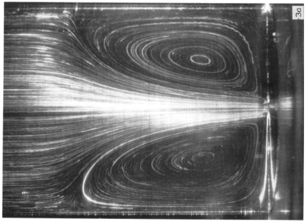

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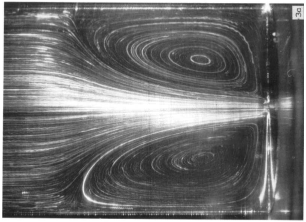

- Needed in polymer rheology

Renormalization of hydrodynamic interactions

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Far field form from reflections

$$\begin{aligned}
\Delta V(r)/V_0 \quad = \quad & \tfrac{a}{r} + \tfrac{a^3}{r^3} && \text{1st reflection} \\
& + \tfrac{a^4}{r^4} + \tfrac{a^6}{r^6} + \dots && \text{2nd reflection} \\
& + \tfrac{a^7}{r^7} + \tfrac{a^9}{r^9} + \dots && \text{3rd reflection} \\
& + \dots
\end{aligned}$$

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Naive pairwise addition of disturbances within large domain $r \leq R$, with n spheres per unit volume

$$\langle \Delta V \rangle = \int_{r=2a}^{R} V_0 \left(\frac{a}{r} + \ldots \right) n \, dV$$

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$$\langle \Delta V \rangle = \int_{r=2a}^{R} V_0 \left(\frac{a}{r} + \dots \right) n \, dV = O\left(V_0 c \frac{R^2}{a^2} \right)$$

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The divergence problem:

- Does mean settling velocity depend on size of domain?
- Or is it an intrinsic property independent of domain?
  i.e. is pairwise addition naive?

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Batchelor's renormalization:

$$\Delta V = \left(1 + \tfrac{a^2}{6}\nabla^2\right) u(x)\big|_{\text{test sphere}} + \text{higher reflections}$$

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$$\langle \text{higher reflections} \rangle = -1.55\, V_0 c$$

Hence

$$\langle V \rangle = V_0(1 - 6.55c)$$

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Assumes uniform separation of pairs for result

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For sedimentation: given force, find average velocity
For porous media: given velocity, find average force

$$\langle F\rangle = F_0\left(1+\tfrac{3}{\sqrt{2}}c^{1/2}+\tfrac{3}{2}c\right)$$

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- Erroneous applications subtracting wrong infinity
- Alternative *averaged equation* approach
  recognising divergences as change of ρ and μ
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- Much more from Batchelor, e.g.

Brownian diffusion of particle with hydrodynamic interactions JFM 1976

$$D = \frac{1-6.55c}{6\pi\mu a}\left[\frac{c}{1-c}\frac{\partial\mu}{\partial c} = kT(1+8c+30c^2)\right]$$

$O(c^2)$ correction to Einstein viscosity

The determination of the bulk stress in a suspension of spherical particles to order c^2
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Note strain-hardening and shear-thinning rheology

A refinement - polydispersity

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Sedimentation in a dilute polydisperse system of interacting spheres JFM 1982, Parts I, II, Corrigendum

$$\langle V_i \rangle = V_{i\,0}\,(1 + S_{ij} c_j)$$

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Also
Diffusion in a dilute polydisperse system of interacting spheres
JFM 1983

What followed beyond Batchelor's c^2

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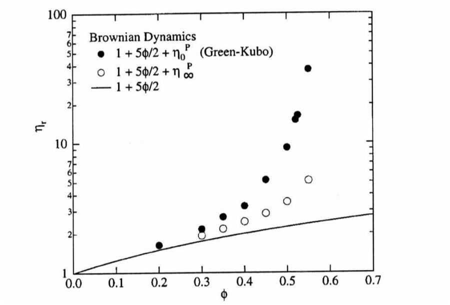
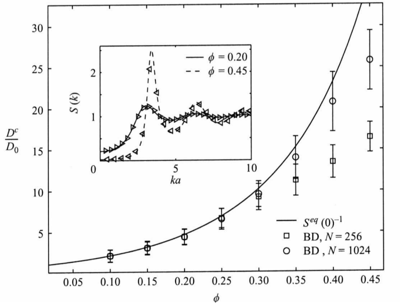
Brady's Stokesian Dynamics (1988) numerical simulations at moderate concentrations of hard spheres





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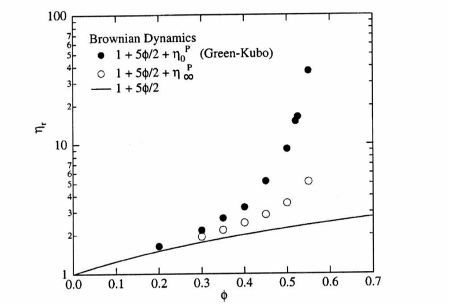

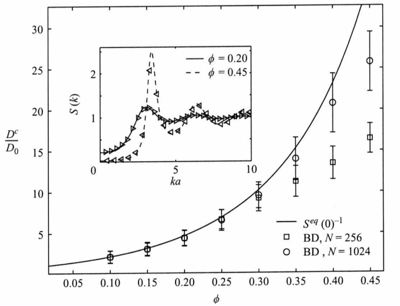

Boundary integral methods for emulsions (1996)

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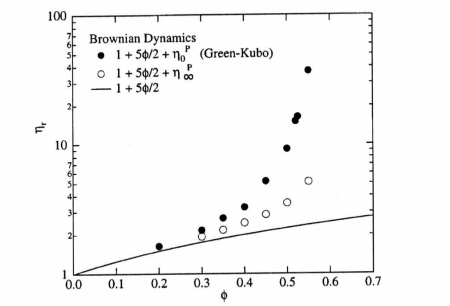
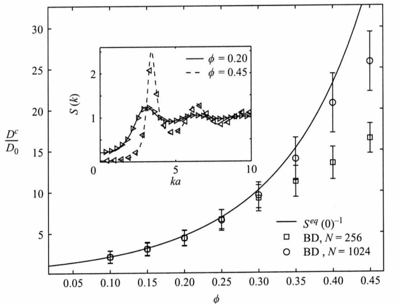




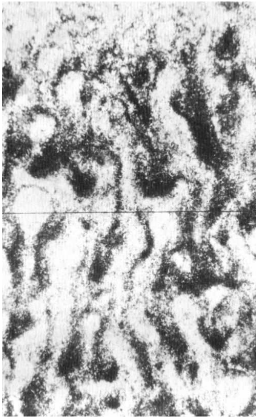
Boundary integral methods for emulsions (1996)
Ladd's Lattice Boltzmann simulations (1996)

More sedimentation

More sedimentation

Structure formation in bidisperse sedimentation JFM 1986

with van Rensburg



Light and heavy particles of same size, both at $c = 0.2$

Fast separation

What followed in sedimentation

- Inclined settling (Boycott effect) Acrivos 1979

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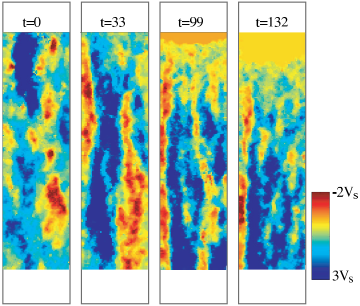

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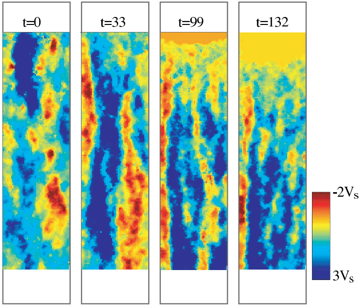
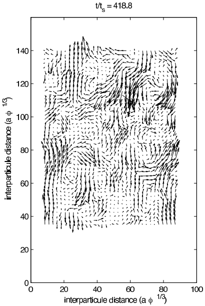
- Fluctuations depend on the size of box Guazzelli 2001

$$\langle V'^2 \rangle = O\left(V_0^2 c \frac{R}{a}\right)$$





Suspensions: what happened in parallel

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Batchelor's fluidized beds

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But dense regions have higher drag so rise!

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Break-up of a falling drop containing disperse particles JFM 1997 with Nitsche

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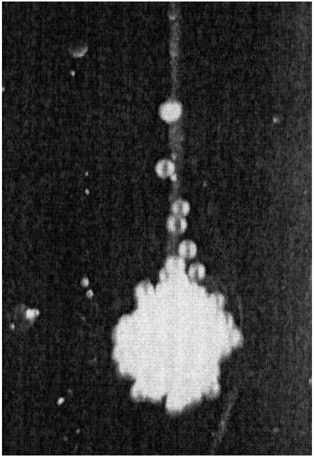

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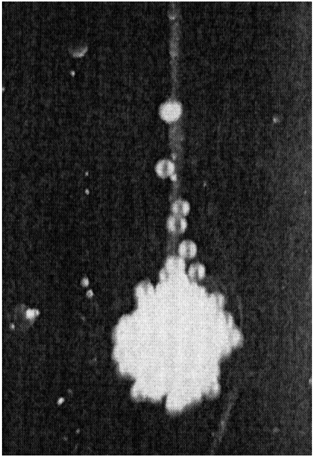

But a little later – video by Metzger, Nicolas & Guazzelli 2007

Batchelor's micro-hydrodynamics