Explaining the flow of elastic liquids

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September 24, 2012

Complex fluids

- What & where? tooth paste, soup, ketchup, synthetic fibres, plastic bags, anti-splat ink-jet printing, oil well drilling muds
- Why & when? micron microstructure: nano reacts in 10^{-9}s, time $\propto$ volume, so micron in 1s
- Which today? not shear-thinning (easy physics of thinning and easy effect on flow)
- Now have:
  - reproducible experiments - standard well-characterised fluids
  - numerics consistent - 5 benchmark problems
  - Time to ask: what is underlying reason for effects

More than: Viscous + Elastic

- Viscous:
  Bernoulli, lift, added mass, waves, boundary layers, stability, turbulence
- Elastic:
  structures, FE, waves, crack, composites
- Visco-elastic is more
  Not halfway between Viscous & Elastic – strange flows to explain

Outline

- Observations to explain
- How well does Oldroyd-B do?
  – half correct
- The FENE modification
  – anisotropy & stress boundary layers
- Conclusions – the reasons why

Flows to explain

- Contraction flow

  large upstream vortices, large pressure drop
- Flow past a sphere

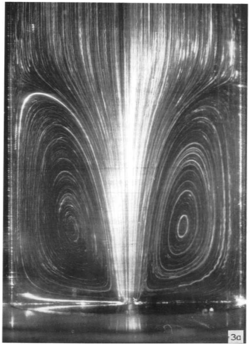
  long wake, increased drag
- M1 project on extensional viscosity

  large stresses but confusion for value of viscosity
- Capillary squeezing of a liquid filament

  very slow to break

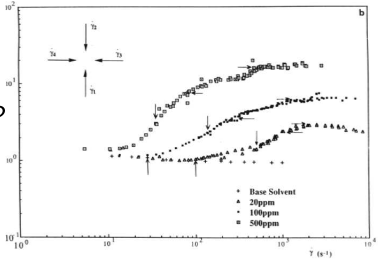
Contraction flow

large upstream vortex



large pressure drops

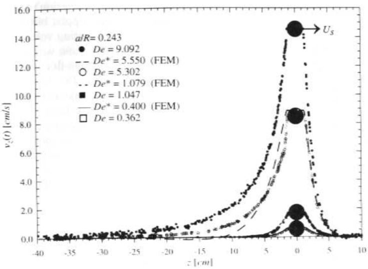
ΔP



Cartalos & Piau 1992 JNNFM 92

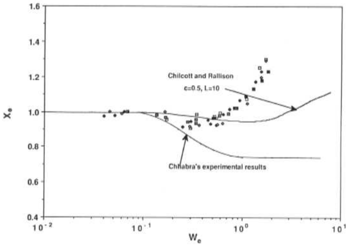
Flow past a sphere

long wake



Arigo, Rajagopalan, Shapley & McKinley 1995 JNNFM

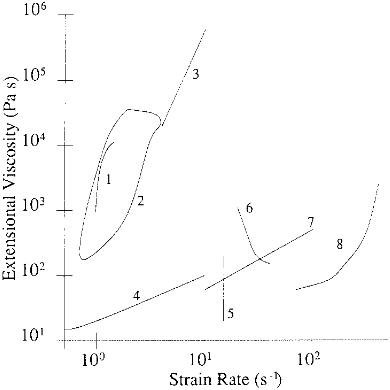
increased drag



Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

also negative wakes!

M1 project



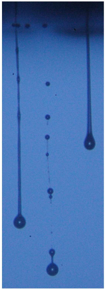
Keiller 1992 JNNFM

no simple extensional viscosity

Capillary squeezing of a liquid filament

Capillary squeezing of a liquid filament

Effect of polymer of Drop-on-Demand printer



Too much polymer for jet to break up into drops.

Flows to explain

- Contraction flow

  large upstream vortices, large pressure drop
- Flow past a sphere

  long wake, increased drag
- M1 project on extensional viscosity

  large stresses but confusion for value of viscosity
- Capillary squeezing of a liquid filament

  very slow to break

Governing equations

Mass $\nabla \cdot \mathbf{u} = 0$

Momentum $\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot \sigma$

Constitutive $\sigma(\nabla \mathbf{u})$ Not known

Start with simplest

Oldroyd-B model fluid simplest viscous + elastic

$$\sigma \quad = \quad -p\mathbf{I} \quad +2\mu_0\mathbf{E} \quad +G\mathbf{A}$$

stress — viscous — elastic

μ_0 viscosity — G elastic modulus

with $\mathbf{A}$ microstructure.

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A} \quad -\frac{1}{\tau}(\mathbf{A}-\mathbf{I})$$

deform with flow — relaxes

τ relaxation time

Deforming with the fluid

Fluid line element $\delta\ell$ deforms as

$$\frac{d\delta\ell}{dt} = \delta\ell \cdot \nabla\mathbf{u}$$

Hence the second order tensor (stress)

$$\mathbf{A} = \delta\ell\,\delta\ell$$

will deform as

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla\mathbf{u} + \nabla\mathbf{u}^T \cdot \mathbf{A}$$

Deborah/Weissenberg number

Fluid relaxation time τ gives nondimensional group

$$De = \frac{U\tau}{L} = \frac{\text{fluid time } \tau}{\text{flow time } L/U}$$

$De \ll 1$: fluid relaxed $\Longrightarrow$ liquid like

$De \gg 1$: little relaxed $\Longrightarrow$ solid like

Oldroyd-B model fluid simplest viscous + elastic

$$\sigma = -p\mathbf{I} \quad +2\mu_0\mathbf{E} \quad +G\mathbf{A}$$

stress — viscous (μ_0 viscosity) — elastic (G elastic modulus)

with $\mathbf{A}$ microstructure.

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A} \quad -\frac{1}{\tau}(\mathbf{A}-\mathbf{I})$$

deform with flow — relaxes (τ relaxation time)

Does this model work/fail?

Investigating Oldroyd-B

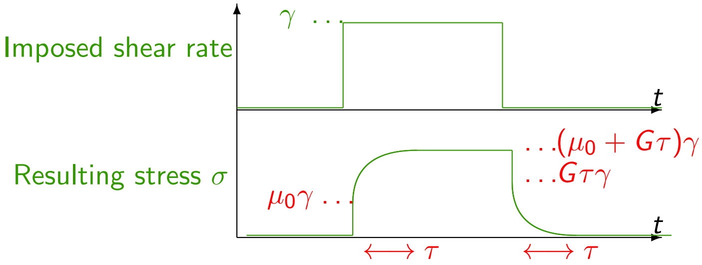
1. Steady & weak $\frac{D}{Dt}, \nabla \mathbf{u} \ll 1/\tau$
2. Unsteady & weak $\nabla \mathbf{u} \ll 1/\tau$

   – linear viscoelasticity
3. Slightly nonlinear $\nabla \mathbf{u} \lesssim 1/\tau$

   – 2nd order fluid
4. Very Fast $\nabla \mathbf{u} \gg 1/\tau$
5. Strongly elastic $2\mu_0 E \ll GA$

But will suppress detailed maths and numerics.

Linear visco-elasticity – common to all fluid models

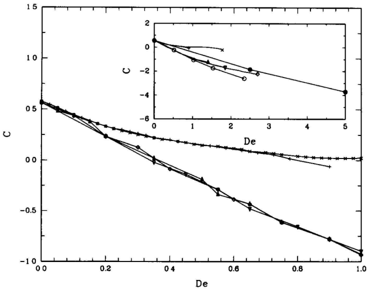


- Early viscosity μ_0
- Steady state viscosity $\mu_0 + G\tau$
- Takes τ to build up to steady state;
  steady deformation = shear rate $\gamma \times$ memory time τ

Stress relaxation is a special property of non-Newtonian fluids, which is not in elastic solids nor viscous liquids

Numerical Oldroyd-B

$\frac{\Delta P}{\text{Stokes}}$



$U\tau/L$

Debbaut, Marchal & Crochet 1988 JNNFM

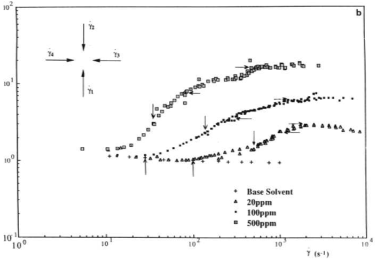
Coates, Armstrong & Brown 1992 JNNFM

Δp scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

But lower drop by early-time viscosity μ_0 if flow fast

Experiments

$\frac{\Delta P}{\text{Stokes}}$



$U\tau/L$

Cartalos & Piau 1992 JNNFM 92

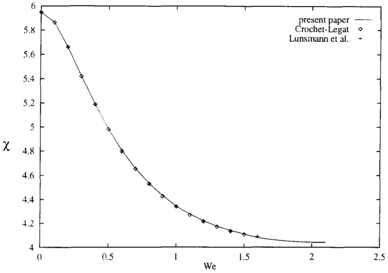
Experiments have a tiny decrease in pressure drop!

Oldroyd-B has no big increase is Δp, and no big upstream vortices

Flow past a sphere   Lagrangian unsteady

Numerical Oldroyd-B

$\frac{\text{Drag}}{\text{Stokes}}$



$U\tau/L$

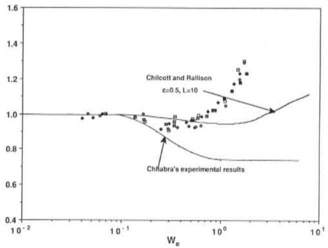
Yurun & Crochet 1995 JNNFM

Drag scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

Lower drag by early-time viscosity μ_0 if flow fast

Experiments

$\frac{\text{Drag}}{\text{Stokes}}$



$U\tau/L$

Tirtaatmadja, Uhlherr & Sridhar

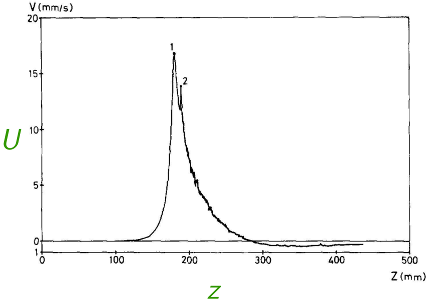
1990 JNNFM

Experiments have a tiny decrease in drag!

Oldroyd-B has no big increase in drag, and no big wake

. . . and negative wakes

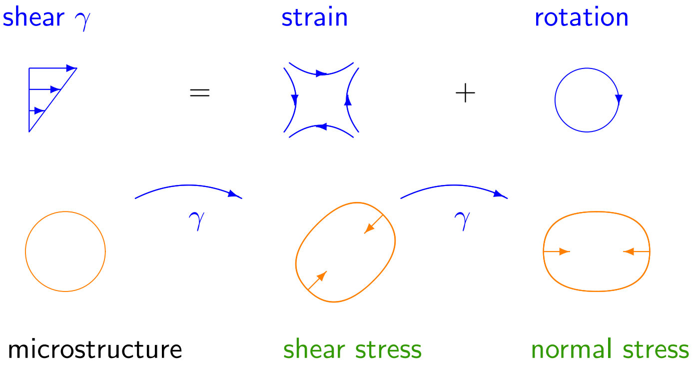
Experiment



Bisgaard 1983 JNNFM

Driven by unrelaxed elastic stress in wake.

Tension in streamlines – slightly nonlinear effect



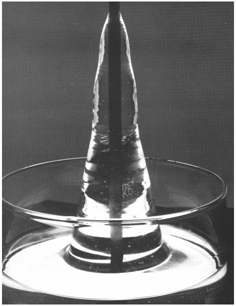
Shear stress $= G \times (\text{rate} = \gamma) \times (\text{memory time} = \tau)$

Normal stress (tension in streamlines) $=$ shear stress $\times \gamma\tau$.

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability

Rod climbing

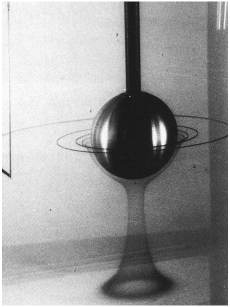


Bird, Armstrong & Hassager

1987, Vol 1 (2nd ed) pg 62

Tension in streamlines $\longrightarrow$ hoop stress
$\longrightarrow$ squeeze fluid in & up.

Secondary flow



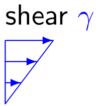
Bird, Armstrong & Hassager

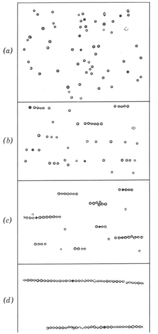
1987, Vol 1 (2nd ed) pg 70

Tension in streamlines $\longrightarrow$ hoop stress
$\longrightarrow$ squeeze fluid in.

Non-Newtonian effects opposite sign to inertial

Migration into chains





Bird, Armstrong & Hassager

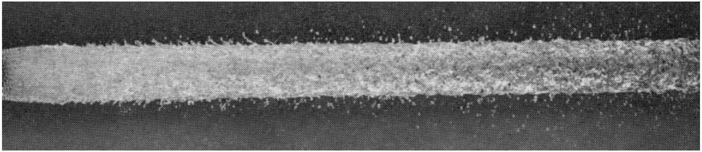
1987, Vol 1 (2nd ed) pg 87

Tension in streamlines $\longrightarrow$ hoop stress
$\longrightarrow$ squeeze particles together

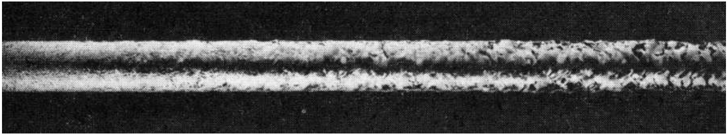
Also migration to centre of a tube,
and alignment with gravity of sedimenting rods.

Stabilisation of jets

Newtonian Jet



Non-Newtonian Jet (200ppm PEO)



Hoyt & Taylor 1977 JFM

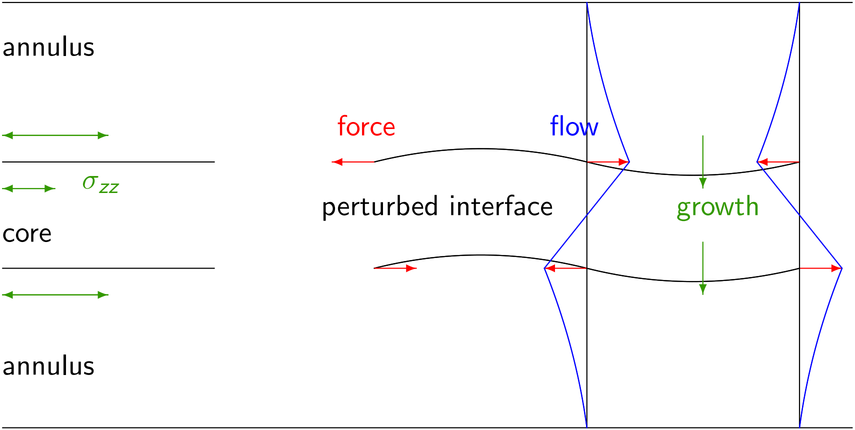
Tension in streamlines in surface shear layer

For fire hoses, and reduce explosive mist

Co-extrusion instability

If core less elastic, then jump in tension in streamlines

Jump OK is interface unperturbed



Hinch, Harris & Rallison 1992 JNNFM

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability

Tension in streamlines – when very fast

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \mathbf{A} - \frac{1}{\tau}(\mathbf{A} - \mathbf{I})$$

Fast: no time to relax: deforms where speeds up (steady flow)

$\mathbf{A} = g(\psi)\mathbf{u}\mathbf{u}$ tensioned streamlines again

g from matching to slower (relaxing) region

Momentum $\nabla \cdot \sigma = 0$, purely elastic $\sigma = G\mathbf{A}$

$$0 = -\nabla p + Gg^{1/2}\mathbf{u} \cdot \nabla g^{1/2}\mathbf{u}$$

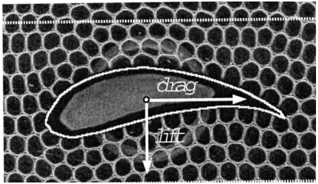
Euler equation!

. . . very fast

$$0 = -\nabla p + G g^{1/2} \mathbf{u} \cdot \nabla g^{1/2} \mathbf{u}$$

Anti-Bernoulli

$$p - \frac{1}{2} G g u^2 = \text{const}$$

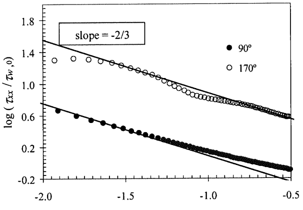


Dollet, Aubouy & Graner 2005 PRL

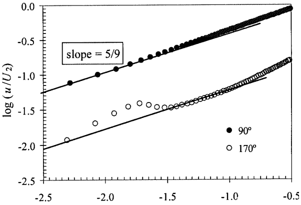
Potential flows $g^{1/2}\mathbf{u} = \nabla\phi$

Flow around sharp 270° corner: Hinch 1995 JNNFM

$$\phi = r^{2/3}\cos\tfrac{2}{3}\theta, \qquad \sigma \propto r^{-2/3} \qquad \psi = r^{14/9}\sin^{7/3}\tfrac{2}{3}\theta$$



(b)



(a)

Alves, Oliviera & Pinho 2003 JNNFM

Capillary squeezing – controlled by relaxation

Mass $\quad \dot{a} = -\frac{1}{2} E a$

Momentum $\quad \frac{\chi}{a} = 3\mu_0 E + G(A_{zz} - A_{rr})$

Microstructure $\quad \dot{A}_{zz} = 2 E A_{zz} - \frac{1}{\tau}(A_{zz} - 1)$

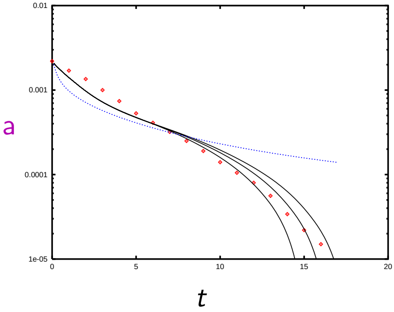
Solution $\quad a(t) = a(0) e^{-t/3\tau}$

Need slow $E = 1/3\tau$ to stop A_{zz} relaxing from $\chi / G a$

. . . capillary squeezing

Oldroyd-B $\quad a(t) = a(0)e^{-t/3\tau} \quad$ does not break

Experiments S1 fluid



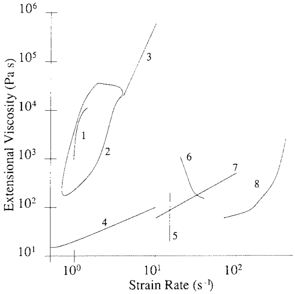
Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

Correct time scale, but filament eventually breaks in experiments

M1 project

no simple extensional viscosity



1. Open syphon
2. Spin line
3. Contraction
4. Opposing Jet
5. Falling drop
6. Falling bob
7. Contraction
8. Contraction

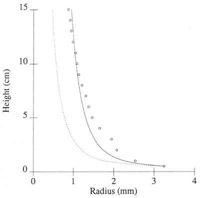
Keiller 1992 JNNFM

really elastic responses

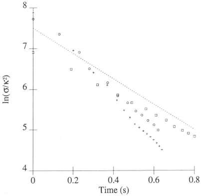
Fit data with Oldroyd-B: $\mu_0 = 5$, $G = 3.5$, $\tau = 0.3$ from shear

Keiller 1992 JNNFM

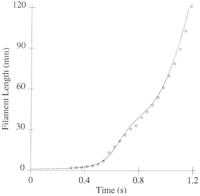
1. Open syphon   Binding 1990



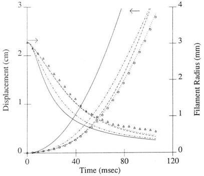
2. Spin line   Oliver 1992



5. Falling drop   Jones 1990



6. Falling bob   Matta 1990



Oldroyd B: Successes & Failures

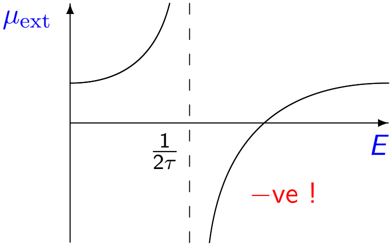
Simplest viscosity μ_0 + elasticity G + relaxation τ

- M1 Project
- Tension in streamlines
- Contraction:
  - Δp small decrease,
  - no big increase, no large vortices
- Sphere:
  - Force small decrease,
  - no increase, no long wake
- Capillary squeezing:
  - long time-scale,
  - no break

Also difficult numerically at $\frac{U\tau}{L} > 1$

Disaster in Oldroyd-B

Steady extensional flow



Microstructure deforms without limit if $E > \frac{1}{2\tau}$: $\quad A = e^{(2E - \frac{1}{\tau})t}$

Need to limit deformation of microstructure

FENE modification

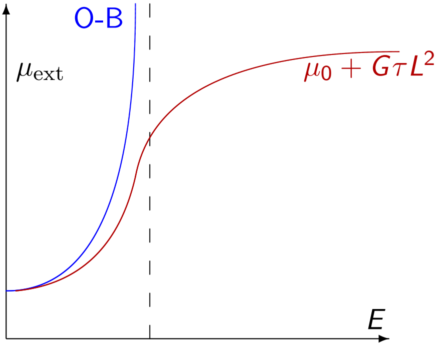
Finite Extension Nonlinear Elasticity

$$\frac{DA}{Dt} = A \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot A - \frac{f}{\tau}(A - \mathbf{I})$$

$$\sigma = -p\mathbf{I} + 2\mu_0 E + G f A$$

$$f = \frac{L^2}{L^2 - \mathrm{trace}\, A} \qquad \text{keeps} \quad A < L^2$$

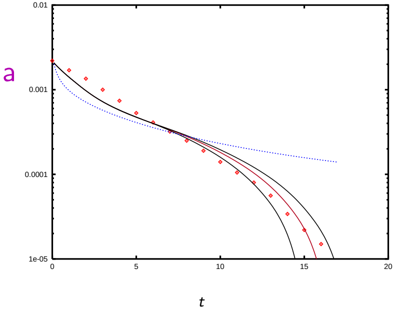
...FENE



Large extensional viscosity $\mu_0 + G\tau L^2$, but small shear viscosity μ_0

FENE capillary squeezing

Filament breaks in with FENE $L = 20$



Exp: Liang & Mackley 1994 JNNFM

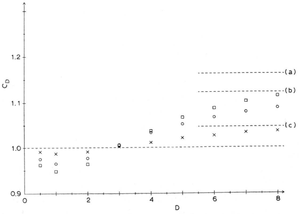
Thy: Entov & Hinch 1997 JNNFM

FENE flow past a sphere

Experiments M1

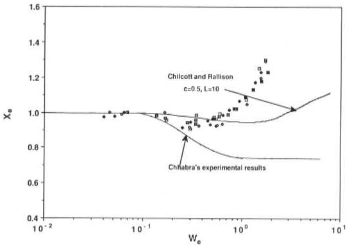
FENE

F



De

Chilcott & Rallison 1988 JNNFM

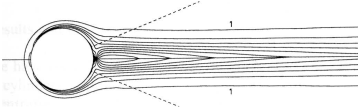


De

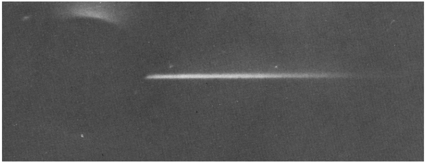
Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

FENE gives drag increase

FENE drag increase from long wake of high stress



Chilcott & Rallison 1988 JNNFM

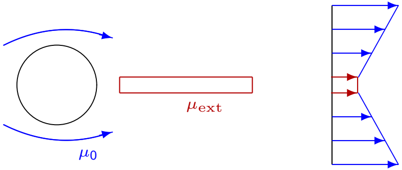


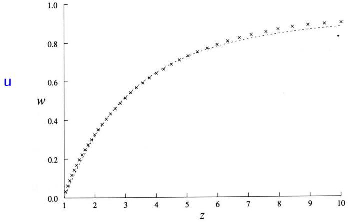
Cressely & Hocquart 1980 Opt Act

"Birefringent strand"

. . . birefringent strands

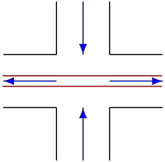
Boundary layers of high stress: μ_{ext} in wake, μ_0 elsewhere.

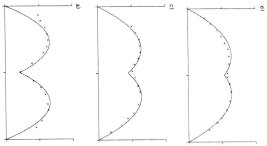




Harlen, Rallison & Chilcott 1990 JNNFM

Can apply to all flows with stagnation points, e.g.





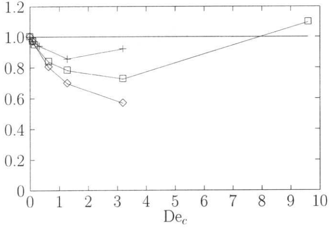
Harlen, Rallison & Chilcott 1990 JNNFM

Also cusps at rear stagnation point of bubbles.

FENE contraction flow

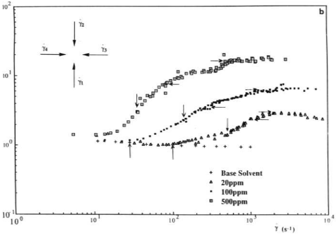
FENE $L = 5$

Δp



Szabo, Rallison & Hinch 1997 JNNFM

Experiments



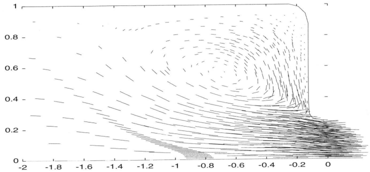
Cartalos & Piau 1992 JNNFM

FENE gives increase in pressure drop

...FENE contraction flow

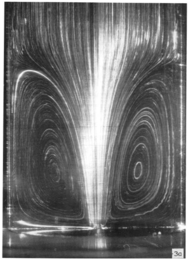
Increase in pressure drop from long upstream vortex

FENE $L = 5$



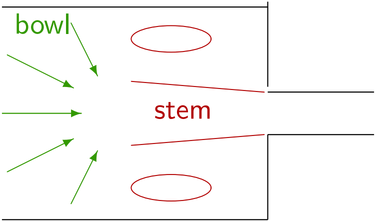
Szabo, Rallison & Hinch 1997 JNNFM

Experiments



Cartalos & Piau 1992 JNNFM

...a champagne-glass model



Bowl: point sink flow, full stretch if $De > L^{3/2}$.

Stem: balance $\mu_{\text{ext}} \dfrac{\partial^2 u}{\partial r^2} = \mu_{\text{shear}} \dfrac{1}{r^2} \dfrac{\partial^2 u}{\partial \theta^2}$

if small cone angle $\Delta\theta = \sqrt{\frac{\mu_{\text{shear}}}{\mu_{\text{ext}}}}$

Flow anisotropy from material anisotropy: $\mu_{\text{ext}} \gg \mu_{\text{shear}}$ TDR

Conclusions for FENE modification

- Contraction: Δp increases, large upstream vortex
- Sphere: drag increase, long wake
- Capillary squeezing: filament breaks
- Numerically safe

Failures of Oldroyd-B corrected.

But sometimes need small L to fit experiments.

Understanding flow of elastic liquids?

In Oldroyd B

- Tension in streamlines
- Stress relaxation – transients see $\mu_0 < \mu_{\text{steady}}$
- Flows controlled by relaxation – E to stop relaxation, very slow

In FENE – deformation of microstructure limited

- μ_{ext} large – increase Δp & drag
- $\mu_{\text{ext}} \gg \mu_{\text{shear}}$ – flow anisotropy

independent of details of model?

More than viscous + elastic