Elastic liquids:
so common, yet so strange

John Hinch

CMS-DAMTP, University of Cambridge

April 13, 2012

Elastic liquids

Something between liquids and solids, but . . .

More complicated than simple viscous fluids,

More complicated than simple elastic solids.

Outline

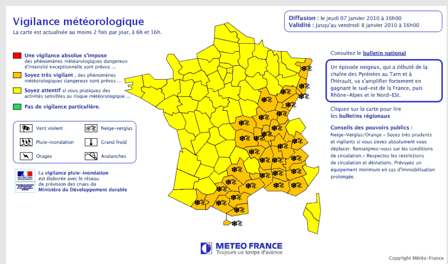
- Review of simple fluids and simple solids
- Complex fluids
- Tension in the streamlines
- Inhibition of stretching
- A little theory

Simple fluids

Studied for 100+ years

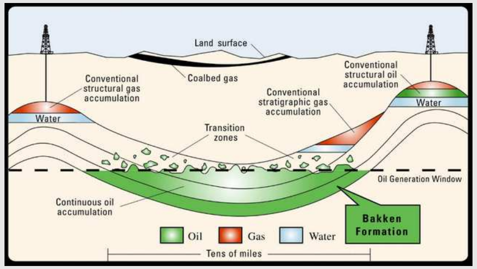
Well understood: library of behaviour; equations, techniques to solve, numerical approach; experimental techniques

Some examples (fluid = air)



More examples of simple fluids

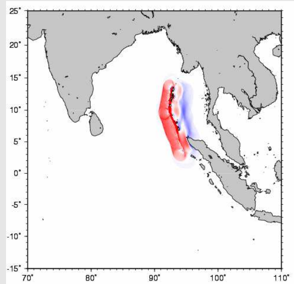
Historic subjects at IMFT: hydroelectricity, porous media



And today, combustion, bio-mechanics, environment

More examples of simple fluids

Propagation of waves: tsunami

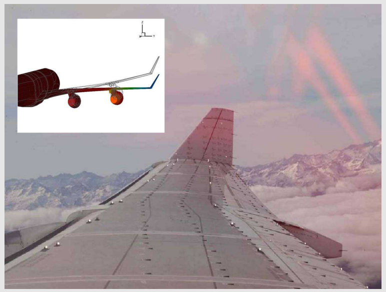


Simple solids

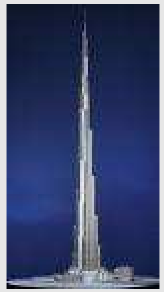
Studied for 100+ years

Well understood: library of behaviour, equations, techniques to solve, numerical approach, experimental techniques.

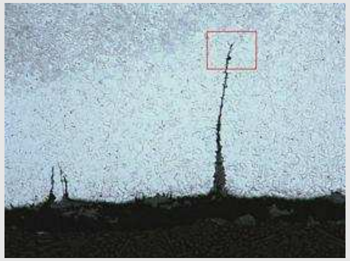
An important example: aeroelasticity

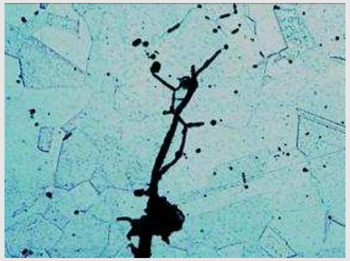


More examples of simple solids: structures (FE)



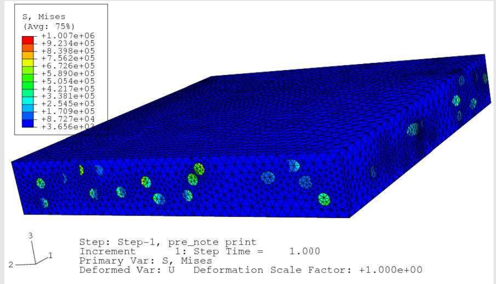
More examples of simple solids: fatigue





More examples of simple solids

Composite materials and Earthquakes



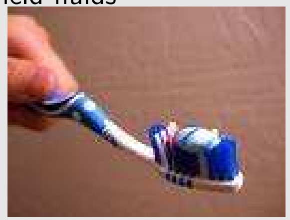
simple fluids and simple solids

Well understood: library of behaviour, equations, techniques to solve, numerical approach, experimental techniques.

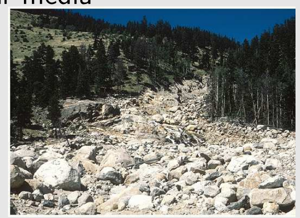
One can predict the values of forces and velocities. One can predict their instabilities.

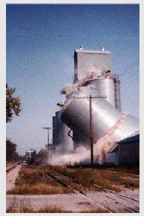
Complex fluids

- Elastic liquids – following subject
- Yield fluids



- Granular media





Elastic liquids

- Where & What

  Plastic products, food processing, biological fluids

- Why & When

  Microstructure of several microns.

  Relaxation time for a nanometre $= 10^{-9}$s.

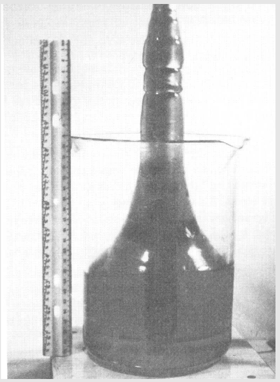
  Time $\propto$ volume. Hence 1s for a micron.

- Review without maths

Tension in the streamlines

- Rod climbing
- Secondary flows
- Migration to form chains of particles
- Migration to the centreline of a pipe
- Vertical alignment of sedimenting fibres
- Stabilisation of jets
- Instability of co-extrusions
- Negative lift force
- Source of tension in the streamlines

Climbing a rotation rod

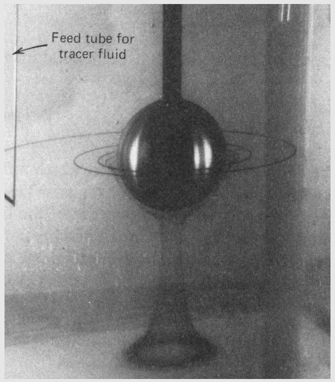


In the kitchen: Whisking egg whites

Bird, Armstrong & Hassager 1987, Vol 1 (2nd ed) pg 62

Tension in the streamlines $\longrightarrow$ "hoop-stress" (perpendicular force) $\longrightarrow$ squeezing liquid towards the centre, so climbs

Secondary flow

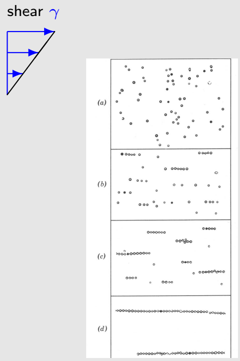


Bird, Armstrong & Hassager 1987, Vol 1 (2nd ed) pg 70

Tension in the streamlines $\longrightarrow$ "hoop-stress"

Opposite direction to effect of inertia

Particle migration to form chains



Bird, Armstrong & Hassager

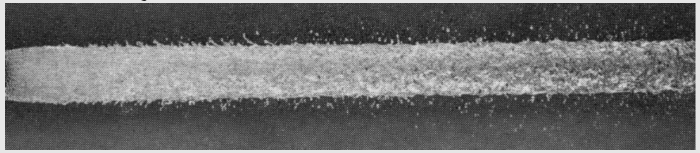
1987, Vol 1 (2nd ed) pg 87

Tension in the streamlines $\longrightarrow$ "hoop-stress"
$\longrightarrow$ brings particles together

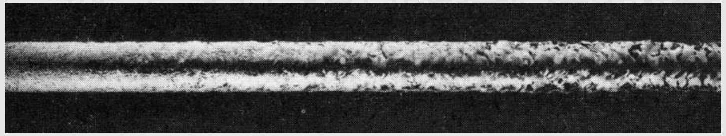
Also: migration to the centreline of a pipe, and
alignment of fibres with gravity

Stabilisation of jets

Newtonian jet



Non-Newtonian jet (200ppm PEO)



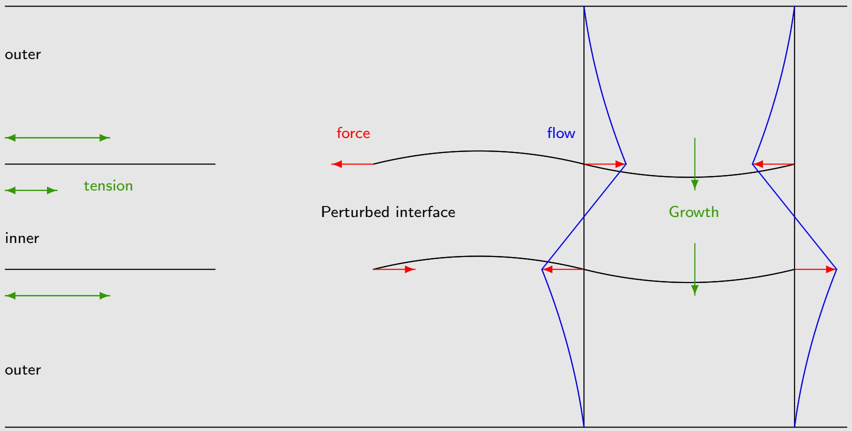
Hoyt & Taylor 1977 JFM

Tension in streamlines near the surface $\longrightarrow$ increase effective surface tension

For fire fighting,
and for stopping explosive aerosols of petrol

Instability in co-extrusion

Jump in tension of streamlines. Case of less elastic central liquid
No problem if interface is flat

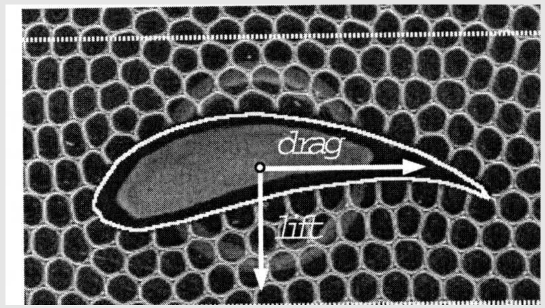


Hinch, Harris & Rallison 1992 JNNFM

Negative lift force

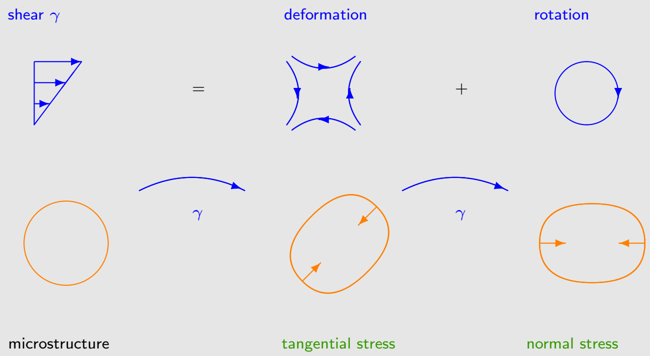
Anti-Bernoulli

$$p - \tfrac{1}{2} G g u^2 = \text{const}$$



Dollet, Aubouy & Graner 2005 PRL

Source of tension in streamlines



Tension in the streamlines

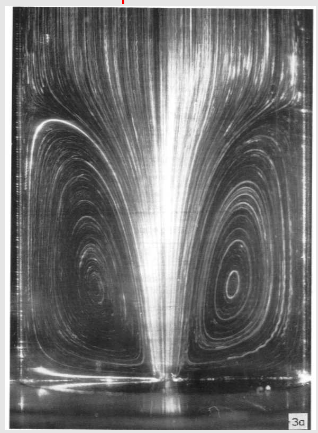
- Rod climbing
- Secondary flows
- Migration to form chains of particles
- Migration to the centreline of a pipe
- Vertical alignment of sedimenting fibres
- Stabilisation of jets
- Instability of co-extrusions
- Negative lift force
- Source of tension in the streamlines

Inhibition of stretching

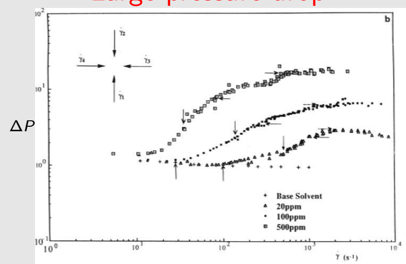
- Contraction
- Flow past a sphere
- M1 project
- Polymers in DoD ink-jet printing
- Effect on a capillary liquid bridge

Contraction from a large tube to a small tube

Large recirculating eddy upstream



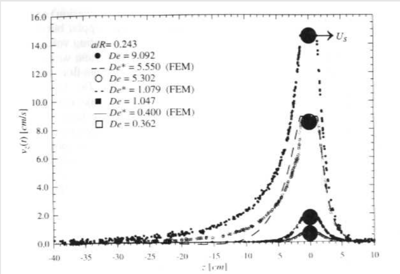
Large pressure drop



Cartalos & Piau 1992 JNNFM 92

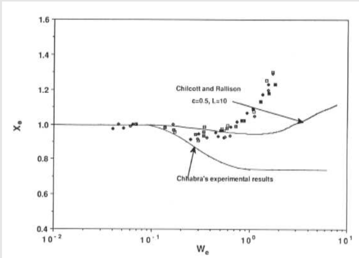
Flow past a sphere

Long wake



Arigo, Rajagopalan, Shapley & McKinley 1995 JNNFM

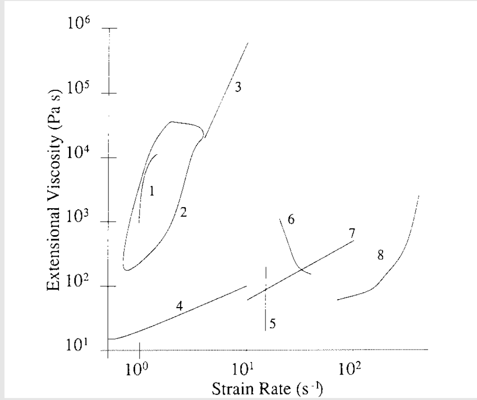
Increase in drag



Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

also negative wake

M1 project to measure the extensional viscosity



1. Open syphon
2. Spin line
3. Contraction
4. Opposing Jet
5. Falling drop
6. Falling bob
7. Contraction
8. Contraction

Keiller 1992 JNNFM

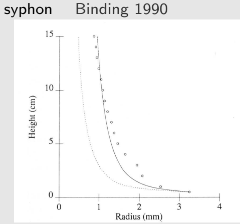
'The' extensional viscosity does not exist

. . . M1 project

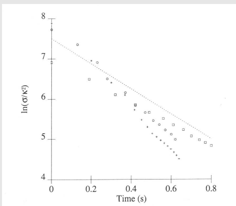
The Oldroyd-B model works well with: $\mu_0 = 5,\ G = 3.5,\ \tau = 0.3$

Keiller 1992 JNNFM

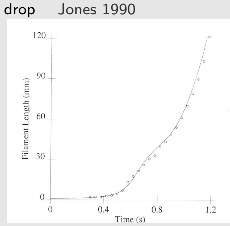
1. Open syphon Binding 1990



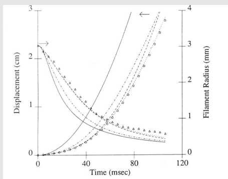
2. Spin line Oliver 1992



5. Falling drop Jones 1990



6. Falling bob Matta 1990

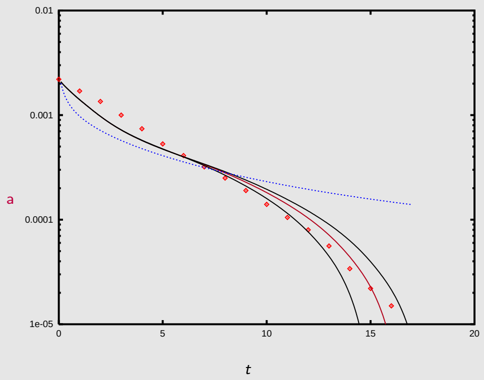


Polymer in a DoD ink-jet printer

– inhibition of stretching

Capillary squeezing of a liquid bridge

Example: for eating



Results for the model fluids Oldroyd-B and FENE

Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

Inhibition of stretching

- Flow through a contraction
- Flow past a sphere
- M1 project
- Polymers in a Drop-on-Demand ink-jet printer
- Effect on a capillary bridge

A little theory

- Oldroyd-B model fluid
- FENE modification
- FENE predictions for flow past a sphere
- . . . “birefringent strands”
- FENE predictions for flow through a contraction

Oldroyd-B model

Simplest combination of viscosity + elasticity

$$\underset{\text{stress}}{\sigma} = -p\mathbf{I} \underset{\text{viscous}}{+2\mu_0\mathbf{E}} \underset{\text{elastic}}{+G\mathbf{A}}$$

μ_0 viscosity G elastic modulus

with **A** microstructure

$$\frac{D\mathbf{A}}{Dt} = \underset{\text{deformation by the flow}}{\mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A}} \underset{\text{relaxation}}{-\frac{1}{\tau}(\mathbf{A}-\mathbf{I})}$$

τ relaxation time

FENE modification

Finite Extension Nonlinear Elasticity
– to avoid certain infinities

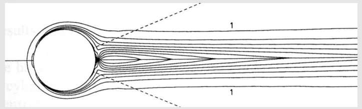
$$\frac{DA}{Dt} = A \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot A - \frac{f}{\tau}(A - \mathbf{I})$$

$$\sigma = -p\mathbf{I} + 2\mu_0 E + G f A$$

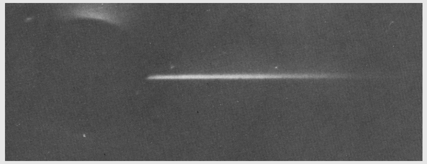
$$f = \frac{L^2}{L^2 - \mathrm{trace}\, A} \qquad \text{for} \quad A < L^2$$

FENE Prediction for flow past a sphere

long thin wake with high stresses



Chilcott & Rallison 1988 JNNFM

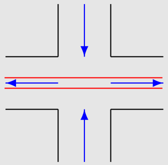


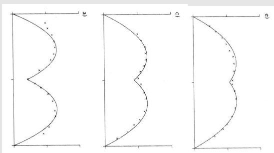
Cressely & Hocquart 1980 Opt Act

"Birefringent strand"

. . . theory of "birefringent strands"

Applicable to flows with a stagnation point





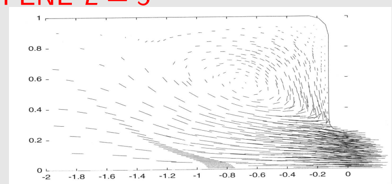
Harlen, Rallison & Chilcott 1990 JNNFM

Also cusps at the rear of shampoo bubbles

FENE predictions for flow through a contraction

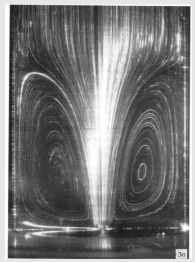
Increase in pressure drop + long upstream vortex

FENE $L = 5$



Szabo, Rallison & Hinch 1997 JNNFM

Experience



Cartalos & Piau 1992 JNNFM

A little theory

- Oldroyd-B model fluid
- FENE modification
- FENE predictions for flow past a sphere
- . . . "birefringent strands"
- FENE predictions for flow through a contraction

Outline

- Review of simple fluids and simple solids
- Complex fluids
- Tension in the streamlines
- Resistance to deformation
- A little theory

Elastic liquids

Studied for 20 years.

Well understood now?

- library of behaviour? – beginning
- equations? - some models
- techniques to solve them - beginning
- numerical approach? – Lagrangian finite elements
- experimental techniques? – standardised test liquids