Oldroyd B, and not A?

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Oldroyd B, and not A?

Thanks to the JNNFM Editors for commissioning the project to research the origins and later development of Oldroyd-A and -B. Quite an historical mystery tour!

Origin of Oldroyd A/B: **the 1950 paper**.

But what came before, and what triggered it?

Why is Oldroyd-B used so frequently used, and Oldroyd-A so rarely?

B – referred to in title/abstract of 100 papers per year in last 10 years

A – 6 references in 70 years

What is the underlying difference between A and B?

How to choose?

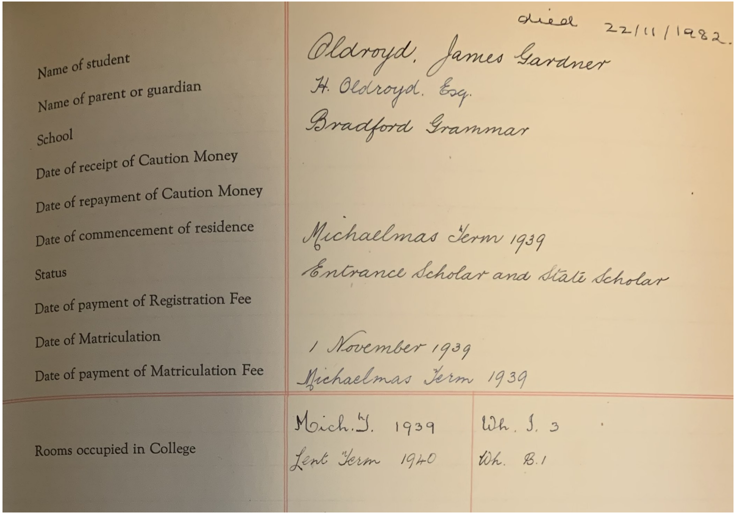
James Gardner Oldroyd

some biographical details

- 1921 Born Bradford, Bradford Grammar School
- 1939 Undergraduate at Trinity College, Cambridge, Mayhew prize 1942
- 1942 Rocket research, Ministry of Supply, Aberporth
- 1945 Courtaulds Fundamental Laboratory, Maidenhead
- 1947 Prize Fellow of Trinity
- 1949 Ph.D.
- 1950 paper
- 1953 Swansea University, Chair Applied Maths
- 1958 Sc.D.
- 1964 Adams Prize
- 1965 Liverpool University
- 1980 Gold Medal of BSR
- 1982 Died

James Gardner Oldroyd at Trinity

from members' record . . .

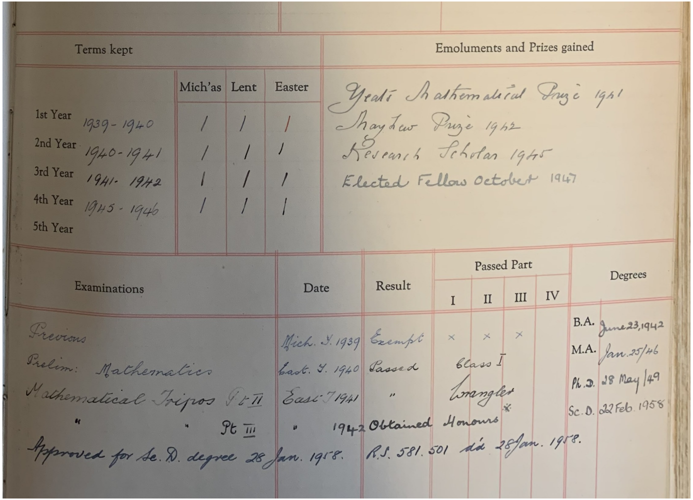


died 22/11/1982.

Name of student	Oldroyd, James Gardner	
Name of parent or guardian	H. Oldroyd. Esq.	
School	Bradford Grammar	
Date of receipt of Caution Money		
Date of repayment of Caution Money		
Date of commencement of residence	Michaelmas Term 1939	
Status	Entrance Scholar and State Scholar	
Date of payment of Registration Fee		
Date of Matriculation	1 November 1939	
Date of payment of Matriculation Fee	Michaelmas Term 1939	
Rooms occupied in College	Mich. T. 1939	Wh. I. 3
	Lent Term 1940	Wh. B.1

James Gardner Oldroyd at Trinity

. . . from members' record



Terms kept		Mich'as	Lent	Easter
1st Year	1939 - 1940	/	/	/
2nd Year	1940 - 1941	/	/	/
3rd Year	1941 - 1942	/	/	/
4th Year	1945 - 1946	/	/	/
5th Year				

Emoluments and Prizes gained

Yeats Mathematical Prize 1941
Mayhew Prize 1942
Research Scholar 1945
Elected Fellow October 1947

Examinations	Date	Result	Passed Part I	II	III	IV
Previous	Mich. T. 1939	Exempt	×	×	×	
Prelim: Mathematics	East. T. 1940	Passed	Class I			
Mathematical Tripos Pt II	East. T 1941	"	Wrangler*			
" " Pt III	"	1942 Obtained Honours				

Approved for Sc. D. degree 28 Jan. 1958. R.S. 581.501 dd 28 Jan. 1958.

Degrees	
B.A.	June 23, 1942
M.A.	Jan. 25/46
Ph.D.	28 May /49
Sc.D.	22 Feb. 1958

In Trinity October 1939/42,

Research Scholar 1945/46.

Elected Fellow October 1947,

also elected that year
George Batchelor, Fritz Ursell,
Thomas Gold.

Trinity Fellowship thesis 1947 – Contents

CONTENTS

Part I: 6 papers on flow of “Bingham solids”,
published in Proc. Camb. Phil. Soc. in 1947 and 1948.

Part II lead to the ground-breaking 1950 paper.

Trinity Fellowship thesis 1947 – Preface

PREFACE

The mathematical theories of the mechanics of two special types of continuous material, the elastic solid which obeys Hooke's law and the liquid which obeys Newton's viscosity law, have been developed extensively in the past and have become classical. There is pressing need for a theory of the mechanics of continuous materials which behave in other ways when stressed. In the absence of a theory to coordinate and interpret experimental results, the accepted measures of deformation and flow properties of many commercially important materials are often entirely empirical. In order to build up a comprehensive theory of rheological phenomena, it is necessary to make a detailed study of carefully chosen prototype materials, and at the same time a general study of the mechanics of continua. The theoretical work described in the present dissertation comprises a contribution to each of the two parallel approaches to a unified theory.

A great deal of interest is now being expressed in rheology, chiefly by experimentalists but also by some theoreticians. So far as can be gathered from published work to date, the approaches to the subject of other theoreticians differ from that presented here. Detailed references to the work of other writers are listed at the ends of the introduction and the individual chapters.

After only two years of research in a subject which offers such a wide field, the ideas put forward in this dissertation are necessarily in an initial stage of development. The dissertation is therefore to be regarded as an interim, rather than a final, report of my work. Generally speaking, Part I is intended to be complete in itself, further developments depending on overcoming the technical difficulties of solving certain differential equations. On the other hand, Part II is quite obviously incomplete, and only Chapters VIII and IX are to be regarded as in their final form.

I am greatly indebted to Mr. W. R. Dean and to Mr. A. H. Wilson for most helpful suggestions and criticisms throughout the course of this work.

J.G.O.

August, 1947.

4, Strande View Walk,
Cookham, Berkshire.

Thanks Cambridge teachers:

W.R. Dean of Dean flows in curved pipes,

A.H. Wilson, who had become Research Director of Courtaulds in 1945.

Dated August 1947, after only two years of research.

The 1950 paper

On the formulation of rheological equations of state

BY J. G. OLDROYD, *Courtaulds Limited, Research Laboratory, Maidenhead, Berks.*

(*Communicated by A. H. Wilson, F.R.S.—Received* 26 *July* 1949—
Revised 4 *November* 1949)

Motivated by the Jefferys model of Fröhlich & Sack (1946),

$$\sigma_{ij} + \tau_1 \dot{\sigma}_{ij} = 2\mu^*(E_{ij} + \tau_2 \dot{E}_{ij}).$$

But Oldroyd remarked: need to differentiate the full tensor $\boldsymbol{\sigma}$ not just the values of the components σ_{ij}:

$$\dot{\boldsymbol{\sigma}} = \dot{\sigma}_{ij}\mathrm{e}^i\mathrm{e}^j + \sigma_{ij}\dot{\mathrm{e}}^i\mathrm{e}^j + \sigma_{ij}\mathrm{e}^i\dot{\mathrm{e}}^j,$$

i.e. also differentiate basis vectors e^i, and *moving with the material*.

Differentiate moving with the Material

– co-translating, co-rotating, and. . .

Three modes of movement.

1. Advected (Lagrangian)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathsf{u} \cdot \boldsymbol{\nabla}.$$

2. Rotating (Zaremba 1903, Jaumann 1911)

$$\overset{\circ}{\mathsf{A}} = \frac{D\mathsf{A}}{Dt} - \mathsf{A} \cdot \Omega + \Omega \cdot \mathsf{A},$$

where Ω is the vorticity tensor

$$\Omega = \tfrac{1}{2}\left(\boldsymbol{\nabla}\mathsf{u} - \boldsymbol{\nabla}\mathsf{u}^{T}\right).$$

Differentiate moving with the Material

– co-translating, co-rotating, and co-deforming

3. Deforming (Hencky 1925 no details) – really?, why? Oldroyd gave no reason.

Differentiate moving with the Material

– co-translating, co-rotating, and co-deforming

3. Deforming (Hencky 1925 no details) – really?, why? Oldroyd gave no reason.

Complicated. Two varieties, when coordinates deform to be non-orthogonal (skew):

- co-variant components A_{ij},
  – use the lower-convected time-derivative,

$$\overset{\triangle}{\mathsf{A}} = \frac{D\mathsf{A}}{Dt} + \mathsf{A} \cdot (\nabla \mathsf{u})^T + \nabla \mathsf{u} \cdot \mathsf{A}.$$

- contra-variant components A^{ij},
  – use the upper-convected time-derivative,

$$\overset{\triangledown}{\mathsf{A}} = \frac{D\mathsf{A}}{Dt} - \mathsf{A} \cdot \nabla \mathsf{u} - (\nabla \mathsf{u})^T \cdot \mathsf{A}.$$

1947 thesis: "need to establish the co-variant or contra-variant nature of any physical quantity before differentiating it". But Oldroyd failed to do so, ever.

Oldroyd A & B arrive

In 1947 Oldroyd gave the first proper invariant generalisation of the Jefferys model of Fröhlich & Sack (1946),

$$\sigma_{ij} + \tau_1 \dot{\sigma}_{ij} = 2\mu^* (E_{ij} + \tau_2 \dot{E}_{ij}),$$

$$\text{Oldroyd A:} \qquad \boldsymbol{\sigma} + \tau_1 \overset{\triangle}{\boldsymbol{\sigma}} = 2\mu^* (\mathsf{E} + \tau_2 \overset{\triangle}{\mathsf{E}}),$$

$$\text{Oldroyd B:} \qquad \boldsymbol{\sigma} + \tau_1 \overset{\triangledown}{\boldsymbol{\sigma}} = 2\mu^* (\mathsf{E} + \tau_2 \overset{\triangledown}{\mathsf{E}}).$$

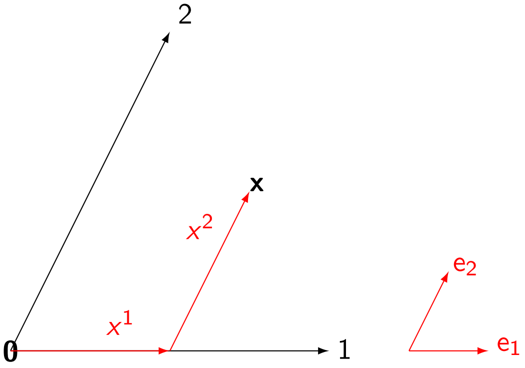
To choose, need to examine structural models or experiments.

Oldroyd-B gives rod climbing (Weissenberg 1947),
Oldroyd-A anti-climbing.

Hence Oldroyd-B wins, but underlying reason unclear.

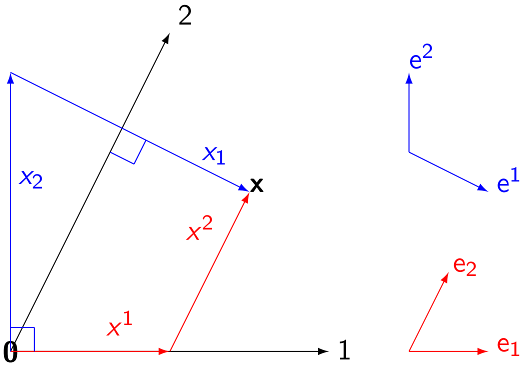
Non-orthogonal (skew) coordinates 1

Measure *contra-variant* components, x^i, of vector x parallel to basis vectors e_i.



Non-orthogonal coordinates 2

Measure *co-variant* components, x_i, of vector $\mathbf{x}$ perpendicular to basis vectors $\mathbf{e_i}$, i.e. parallel to reciprocal basis vectors $\mathbf{e}^i = \mathbf{e_j} \times \mathbf{e_k}$.



Used for $\nabla\phi$ and surface areas.

Non-orthogonal coordinates 3

Time-derivatives of basis vectors,

$$\frac{d\mathbf{e}_i}{dt} = \mathbf{e}_i \cdot \nabla \mathbf{u},$$

gives upper-convected time-derivative for contra-variant components.

While time-derivatives of the reciprocal basis vectors, $\mathbf{e}^i = \mathbf{e}_j \times \mathbf{e}_k$,

$$\frac{d\mathbf{e}^i}{dt} = \mathbf{e}^i \cdot (\nabla \mathbf{u})^T, \qquad \text{student exercise!},$$

gives lower-convected time-derivative for co-variant components.

Mis-education: a vector IS NOT (x, y, z), nor x_i, but is $\mathbf{x} = x^i \mathbf{e}_i$.

Related papers 1

The elastic and viscous properties of emulsions and suspensions

BY J. G. OLDROYD
Courtaulds Limited, Maidenhead, Berks

(*Communicated by A. H. Wilson, F.R.S.—Received 29 January* 1953)

1953 paper: an emulsion, a dilute suspension of liquid drops,
Fröhlich & Sack (1946) was elastic particles.

$$\sigma_{ij} + \tau_1 \dot{\sigma}_{ij} = 2\mu^* (E_{ij} + \tau_2 \dot{E}_{ij}),$$

$$\tau_1 = \frac{a\mu}{\gamma} A\left(3 + 2\lambda + 8A\phi\right), \qquad \tau_2 = \frac{a\mu}{\gamma} A\left(3 + 2\lambda - 12A\phi\right),$$

$$\mu^* = \mu\left(1 + \frac{2 + 5\lambda}{2(1+\lambda)}\phi\right), \qquad \text{where} \qquad A = \frac{16 + 19\lambda}{40(1+\lambda)}.$$

Same Jefferys model, different coefficients.
No nonlinear terms/physics to select A or B.

Related papers 2

THE MOTION OF AN ELASTICO-VISCOUS LIQUID CONTAINED BETWEEN COAXIAL CYLINDERS. I

By J. G. OLDROYD

(*Courtaulds Limited, Research Laboratory, Maidenhead, Berks.*)

[Received 20 December 1950]

1951 paper: 5-constant model,

$$\boldsymbol{\sigma} + \tau_1 \overset{\triangle}{\boldsymbol{\sigma}} - 2\kappa_1(\mathsf{E}\cdot\boldsymbol{\sigma} + \boldsymbol{\sigma}\cdot\mathsf{E}) = 2\mu_0(\mathsf{E} + \tau_2 \overset{\triangle}{\mathsf{E}}) - 8\mu_0\kappa_2 \mathsf{E}\cdot\mathsf{E}.$$

Normal stress differences.
Shear-thinning, but shear viscosity constant for Oldroyd-A & Oldroyd-B.

No physics given to justify the added terms.

Non-Newtonian effects in steady motion of some idealized elastico-viscous liquids

By J. G. Oldroyd

Department of Applied Mathematics, University College of Swansea

(*Communicated by Sir Geoffrey Taylor, F.R.S.—Received 7 February* 1958)

1958 paper: 8-constant model

$$\boldsymbol{\sigma} + \tau_1 \overset{\circ}{\boldsymbol{\sigma}} - \tau_3(\mathsf{E}\cdot\boldsymbol{\sigma} + \boldsymbol{\sigma}\cdot\mathsf{E}) + \tau_5(\boldsymbol{\sigma} : \mathsf{I})\mathsf{E} + \tau_6(\boldsymbol{\sigma} : \mathsf{E})\mathsf{I}$$
$$= 2\mu_0 \left(\mathsf{E} + \tau_2 \overset{\circ}{\mathsf{E}} - 2\tau_4 \mathsf{E}\cdot\mathsf{E} + \tau_7(\mathsf{E} : \mathsf{E})\mathsf{I}\right).$$

Added all possible terms either bilinear in stress and strain-rate or quadratic in strain-rate.
These nonlinear terms are *insufficient* to describe observed extensional behaviour.

No physics given to justify the added terms.

Related papers 4

Some steady flows of the general elastico-viscous liquid

By J. G. Oldroyd

Department of Applied Mathematics, University College of Swansea

(Communicated by G. K. Batchelor, F.R.S.—Received 27 May 1964)

1965 paper: using symmetries, calculates kinematics of the flow, in order to demonstrate that one can work in coordinates deforming with the flow.

But has no constitutive equation.

Related papers 5

Journal of Non-Newtonian Fluid Mechanics, 14 (1984) 9–46
Elsevier Science Publishers B.V., Amsterdam – Printed in The Netherlands

AN APPROACH TO NON-NEWTONIAN FLUID MECHANICS

J.G. OLDROYD

Part of an essay submitted in competition for the Adams Prize, December 1964

1984 paper: two chapters selected by Ken Walters from Oldroyd's 1964 Adams Prize essay.

A careful and extended account of 1950 paper.

But still no physics to select between Oldroyd-A and Oldroyd-B, and no awareness of the extensional viscosity crisis.

Microstructure

Time for some physics, suggested as necessary by Oldroyd, but never supplied by him.

Microstructure 1 – rotation

Jeffery (1922) found that rigid ellipsoids rotated with *all of the vorticity*,

but *only a fraction of strain-rate*, a fraction $\frac{r^2-1}{r^2+1}$.

Microstructure 1 – rotation

Jeffery (1922) found that rigid ellipsoids rotated with *all of the vorticity*, but *only a fraction of strain-rate*, a fraction $\frac{r^2-1}{r^2+1}$.

Hence non-affine slip, introduced by Johnson & Segalman (1977), but earlier by Gordon & Schowlater (1972). Both gave no argument for why.

$$\overset{\blacksquare}{\mathsf{A}} = \overset{\circ}{\mathsf{A}} - \frac{r^2-1}{r^2+1}(\mathsf{E}\cdot\mathsf{A} + \mathsf{A}\cdot\mathsf{E}) \equiv \frac{r^2}{r^2+1}\overset{\triangledown}{\mathsf{A}} + \frac{1}{r^2+1}\overset{\vartriangle}{\mathsf{A}}.$$

NB fibres $r \gg 1$ upper-convective, discs $r \ll 1$ lower-convective.

Non-affine slip gives shear-thinning, Burgers (1938), Peterlin (1938), and a second normal stress difference.

Note 1922 and 1938 papers all before the 1950 paper.

Microstructure 2 – deformation

- Suspension of elastic spheres
  - Linear by Fröhlich & Sack (1946),
  - Nonlinear by Cerf (1951), Roscoe (1967),
  - finding shear-thinning, both normal stresses, critical extension rate beyond which no steady state.
- Suspension of liquid drops
  - Linear Oldroyd (1953),
  - Nonlinear by Barthès-Biesel & Acrivos (1973), Rallison (1984).
- Both suspensions have rotation with full vorticity and deformation with only a fraction of strain-rate (non-affine slip).

Microstructure 3 - wrong view

Wrong: Jefferys equation of Fröhlich & Sack (1946),:

$$\sigma + \tau_1 \dot{\sigma} = 2\mu^*(E + \tau_2 \dot{E}),$$

Correction 1. Expose microstructural state – deformation of spheres D, which satisfies evolution equation

$$\mathsf{D} + \tau \dot{\mathsf{D}} = \tfrac{5}{3}\tau \mathsf{E}. \qquad (*)$$

The stress is related to deformation

$$\boldsymbol{\sigma} = 2\mu(1 - \tfrac{5}{3}\phi)\mathsf{E} + \tfrac{10}{3}\phi G \mathsf{D}.$$

Microstructure 3 - wrong view

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$$\sigma + \tau_1 \dot{\sigma} = 2\mu^* (E + \tau_2 \dot{E}),$$

Correction 1. Expose microstructural state – deformation of spheres D, which satisfies evolution equation

$$\mathsf{D} + \tau \dot{\mathsf{D}} = \tfrac{5}{3} \tau \mathsf{E}. \qquad (*)$$

The stress is related to deformation

$$\boldsymbol{\sigma} = 2\mu (1 - \tfrac{5}{3}\phi) \mathsf{E} + \tfrac{10}{3} \phi G \mathsf{D}.$$

But Oldroyd said stress comes from deforming with the material, i.e. not created by a source term as in the RHS of (*).

Microstructure 3 - corrected view

Correction 2. Included the undeformed state, by defining

$$\mathsf{A} = \mathsf{I} + \mathsf{D}.$$

Then can generate stress by deforming A with the material, with no source term

$$(\mathsf{A} - \mathsf{I}) + \tau\left(\overset{\circ}{\mathsf{A}} - \tfrac{5}{6}\left(\mathsf{E}\cdot\mathsf{A} + \mathsf{A}\cdot\mathsf{E}\right)\right) = 0,$$

i.e. non-affine slip of $5/6$.

This is all in Fröhlich & Sack (1946) if one were to dig into the physics.

Similarly dilute emulsion: Oldroyd's 1953 paper has all the info for non-affine slip of $5/(2(2\lambda + 3))$.

Popularity of Oldroyd-B

Enduring use of Oldroyd-B:

- Only 3 parameters to fit data,
- Fair fit to Boger fluids,
- Hence sensible choice for numerical simulations,
- Elastic-dumbbell model is governed by Oldroyd-B.

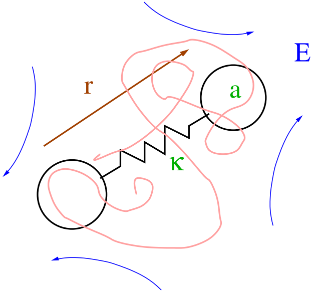
Oldroyd-B constant shear viscosity, first NSD, zero second NSD (approx Boger).

Problem: infinite extensional viscosity at finite strain-rate. Oldroyd unaware? (in print)

Look at micro model to cure.

Elastic-dumbbell model 1

Kuhn & Kuhn 1945, Werner & student Hans, no relation



Hydrodynamic drag on beads
$6\pi\mu a(\mathbf{r}\cdot\nabla\mathbf{u}-\dot{\mathbf{r}})$,
with $a = r_0 = b(N/6)^{1/2}$.

Spring force $-\kappa\mathbf{r}$ between beads,
with $\kappa = 3kT/Nb^2$.

$$\dot{\mathbf{r}} = \mathbf{r}\cdot\boldsymbol{\nabla}\mathbf{u} - \frac{\kappa}{3\pi\mu a}\mathbf{r}.$$

Add Brownian motion:

$$\frac{D}{Dt}\langle\mathbf{rr}\rangle = \langle\mathbf{rr}\rangle\cdot\boldsymbol{\nabla}\mathbf{u} + \boldsymbol{\nabla}\mathbf{u}^T\cdot\langle\mathbf{rr}\rangle - \frac{1}{\tau}\left(\langle\mathbf{rr}\rangle - \frac{r_0^2}{3}\mathbf{I}\right),$$

with relaxation time $\tau = 3\pi\mu a/2\kappa$.

Elastic-dumbbell model 2

$$\frac{D}{Dt}\langle \mathsf{rr}\rangle = \langle \mathsf{rr}\rangle \cdot \boldsymbol{\nabla}\mathsf{u} + \boldsymbol{\nabla}\mathsf{u}^T \cdot \langle \mathsf{rr}\rangle - \frac{1}{\tau}\left(\langle \mathsf{rr}\rangle - \frac{r_0^2}{3}\mathsf{I}\right),$$

Note upper-convected time-derivative

And bulk stress, with number density of polymers n,

$$\boldsymbol{\sigma} = -p\mathsf{I} + \mu(\boldsymbol{\nabla}\mathsf{u} + \boldsymbol{\nabla}\mathsf{u}^T) + n\kappa\langle \mathsf{rr}\rangle.$$

This pair of equations govern an Oldroyd-B fluid.

Elastic-dumbbell model 3

Why Oldroyd-B?

Follows from

$$\dot{\mathsf{r}} = \mathsf{r} \cdot \nabla \mathsf{u} - \frac{\kappa}{3\pi\mu a}\mathsf{r},$$

which has r deforming like a fibre, a material line-element.

Why?

Because drag force on a bead depends only on velocity at its centre, a single point.

But distributed hydrodynamics gives reduce efficiency of strain-rate,
so non-affine slip (ejh 1974).

Refinements

1. Linear chain of beads-and-springs: Rouse 1953, and hydrodynamic interactions Zimm 1956. Gives spectrum. But in strong flows only lowest modes matters.

2. In extensional flow, the viscosity blows up at a finite strain-rate $E = 1/2\tau$. Corrected with *Finite-Extensible-Nonlinear-Elastic* (FENE) spring force

$$-\frac{\kappa r}{1 - r^2/N^2 b^2}.$$

Harold R. Warner (1972), Bob Bird's student.

Refinements

2b. Using Peterlin's the pre-averaging gives FENE-P

$$\boldsymbol{\sigma} = -p\mathsf{I} + \mu(\nabla \mathsf{u} + \nabla \mathsf{u}^T) + G f \mathsf{A},$$

$$\overset{\triangledown}{\mathsf{A}} = -\frac{1}{\tau}\left(f\mathsf{A} - \mathsf{I}\right), \quad \text{with} \quad f = \frac{1}{1 - \mathrm{Tr}(\mathsf{A})/L^2},$$

where $\mathsf{A} = 3\langle \mathsf{rr} \rangle / r_0^2$ and $L = \sqrt{3}Nb/r_0$.

2c. Chilcott and Rallison (1988) suggested FENE-CR, with a constant shear viscosity,

$$\overset{\triangledown}{\mathsf{A}} = -\frac{f}{\tau}\left(\mathsf{A} - \mathsf{I}\right).$$

3. Also bead friction increased with deformation τ to $\tau\sqrt{\mathrm{Tr}(\mathsf{A})}$.
De Gennes 1974, ejh 1974.

Refinements

4. PTT Phan Thien and Tanner (1977), includes non-affine slip $\overset{\square}{\mathsf{A}}$

$$\overset{\square}{\mathsf{A}} = -\frac{1 + \alpha\,\mathrm{Tr}(\mathsf{A})}{\tau}\,(\mathsf{A} - \mathsf{I})\,.$$

4b. Later Phan Thien (1978) proposed an exponential decrease in relaxation time

$$\overset{\square}{\mathsf{A}} = -\frac{1}{\tau}\exp(\alpha\,\mathrm{Tr}(\mathsf{A}))\cdot(\mathsf{A} - \mathsf{I})\,.$$

5. For different reasons, Giesekus (1982) suggested including a quadratic term to account for the effects of anisotropic drag,

$$\overset{\triangledown}{\mathsf{A}} = -\frac{1}{\tau}(\mathsf{I} + \alpha\mathsf{A})\cdot(\mathsf{A} - \mathsf{I})\,.$$

6. White-Metzner (1963)

$$\boldsymbol{\sigma} + \tau(\dot\gamma)\,\overset{\triangledown}{\boldsymbol{\sigma}} = 2\mu(\dot\gamma)\mathsf{E}, \quad \text{where} \quad \dot\gamma = \sqrt{\tfrac{1}{2}\mathsf{E} : \mathsf{E}}.$$

Use experimental $\mu(\dot\gamma)$, and $N_1(\dot\gamma)$
by shear-dependent relaxation time $\tau(\dot\gamma) = N_1(\dot\gamma)/2\mu(\dot\gamma)\dot\gamma^2$.

Refinements

7. Pom-pom, McLeish and Larson (1998), simplified version

$$\text{Orientation:} \quad \overset{\triangledown}{\mathsf{A}} = -\frac{1}{\tau_b}(\mathsf{A} - \mathsf{I}) \quad \text{and then} \quad \mathsf{S} = \mathsf{A}/\operatorname{Tr}(\mathsf{A}).$$

$$\text{Stretch:} \quad \frac{D\lambda}{Dt} = \lambda(\boldsymbol{\nabla}\mathsf{u} : \mathsf{S}) - \frac{1}{\tau_s}(\lambda - 1) \quad \text{for} \quad \lambda < q,$$

$$\text{Stress:} \quad \boldsymbol{\sigma} = -p\mathsf{I} + 3G\lambda^2\mathsf{S}.$$

8. Rollie-poly, Likhtman and Graham (2003)

$$\overset{\triangledown}{\mathsf{A}} = -\frac{1}{\tau_d}(\mathsf{A} - \mathsf{I}) - \frac{2(1 - \sqrt{3/\operatorname{Tr}(\mathsf{A})})}{\tau_R}\left[\mathsf{A} + \beta\left(\frac{\operatorname{Tr}(\mathsf{A})}{3}\right)^{\delta}(\mathsf{A} - \mathsf{I})\right],$$

Conclusions

1. The hugely important 1950 paper introduced proper time-derivatives, essential ingredients for any viscoelastic constitutive equation; and also introduced the Oldroyd-B equation, much used today. ✓✓

Conclusions

1. The hugely important 1950 paper introduced proper time-derivatives, essential ingredients for any viscoelastic constitutive equation; and also introduced the Oldroyd-B equation, much used today. ✓✓

2. In the 1950 paper and in all subsequent papers, Oldroyd seems never to have understood how the physics makes the selection between B and A, or how the physics should be expressed (Jefferys model is not correct way), although he made a pressing case for exactly that.
That has distorted our subject, to expect full co-deformation, whereas reality always has non-affine slip. **X**

Conclusions

3. Further, Oldroyd never seems to have understood the extensional viscosity crisis, which is easily fixed by a FENE modification, when one understands the physics. That has distorted our subject with failed computations due to the entirely avoidable high Weissenberg-number non-problem. **X**

Conclusions

3. Further, Oldroyd never seems to have understood the extensional viscosity crisis, which is easily fixed by a FENE modification, when one understands the physics. That has distorted our subject with failed computations due to the entirely avoidable high Weissenberg-number non-problem. **X**

Hence: our subject would be nowhere without 1, but it would have been so much better if we had not had 2 and 3.

Thanks again to the editors for setting the fascinating historical mystery tour.