Explaining the flow of elastic liquids

John Hinch

CMS-DAMTP, University of Cambridge

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  - Time to ask: what is underlying reason for effects

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- Elastic:
  structures, FE, waves, crack, composites
- Visco-elastic is more
  Not halfway between Viscous & Elastic – strange flows to explain

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- Conclusions – the reasons why

Flows to explain

- Contraction flow

  large upstream vortices, large pressure drop
- Flow past a sphere

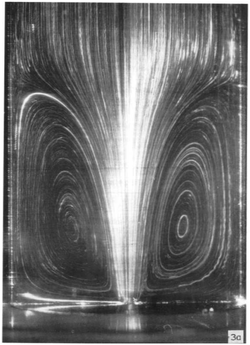
  long wake, increased drag
- M1 project on extensional viscosity

  large stresses but confusion for value of viscosity
- Capillary squeezing of a liquid filament

  very slow to break

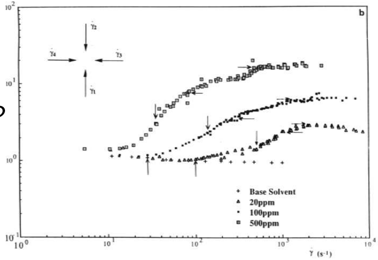
Contraction flow

large upstream vortex



large pressure drops

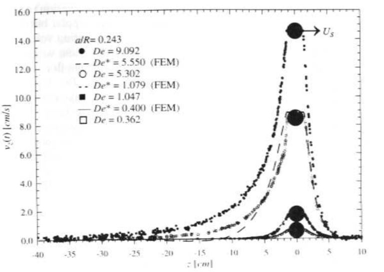
ΔP



Cartalos & Piau 1992 JNNFM 92

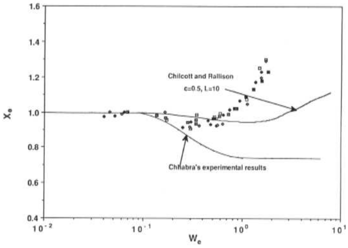
Flow past a sphere

long wake



Arigo, Rajagopalan, Shapley & McKinley 1995 JNNFM

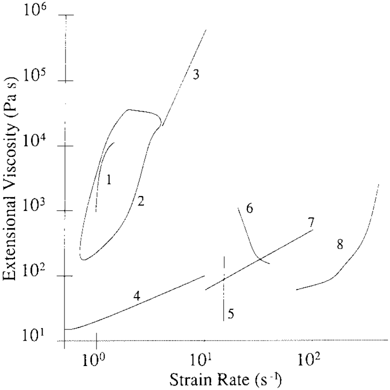
increased drag



Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

also negative wakes!

M1 project



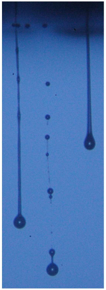
Keiller 1992 JNNFM

no simple extensional viscosity

Capillary squeezing of a liquid filament

Capillary squeezing of a liquid filament

Effect of polymer of Drop-on-Demand printer



Too much polymer for jet to break up into drops.

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Governing equations

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Start with simplest

Oldroyd-B model fluid   simplest viscous + elastic

$$\sigma \quad = \quad -p\mathbf{I} \quad +2\mu_0\mathbf{E} \quad +G\mathbf{A}$$

stress   viscous   elastic

μ_0 viscosity G elastic modulus

with $\mathbf{A}$ microstructure,

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with **A** microstructure,

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A} - \frac{1}{\tau}(\mathbf{A}-\mathbf{I})$$

deform with flow   relaxes

τ relaxation time

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Hence the second order tensor (stress)

$$\mathbf{A} = \delta\ell\,\delta\ell$$

will deform as

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla\mathbf{u} + \nabla\mathbf{u}^T \cdot \mathbf{A}$$

Deborah/Weissenberg number

Fluid relaxation time τ gives nondimensional group

$$De = \frac{U\tau}{L} = \frac{\text{fluid time } \tau}{\text{flow time } L/U}$$

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$De \ll 1$: very relaxed $\Longrightarrow$ liquid like

$De \gg 1$: little relaxed $\Longrightarrow$ solid like

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Does this simple model work/fail?

Investigating Oldroyd-B

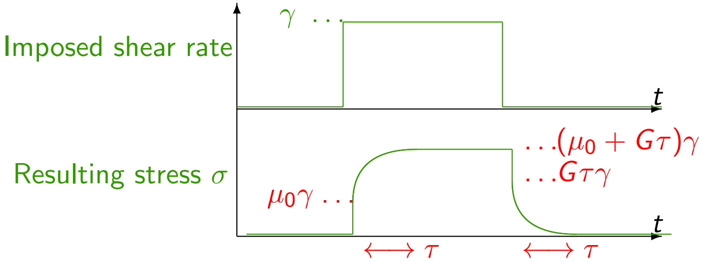
1. Steady & weak $\frac{D}{Dt}, \nabla \mathbf{u} \ll 1/\tau$
2. Unsteady & weak $\nabla \mathbf{u} \ll 1/\tau$

   – linear viscoelasticity
3. Slightly nonlinear $\nabla \mathbf{u} \lesssim 1/\tau$

   – 2nd order fluid
4. Very Fast $\nabla \mathbf{u} \gg 1/\tau$
5. Strongly elastic $2\mu_0 E \ll GA$

But will suppress detailed maths and numerics.

Linear visco-elasticity – common to all fluid models



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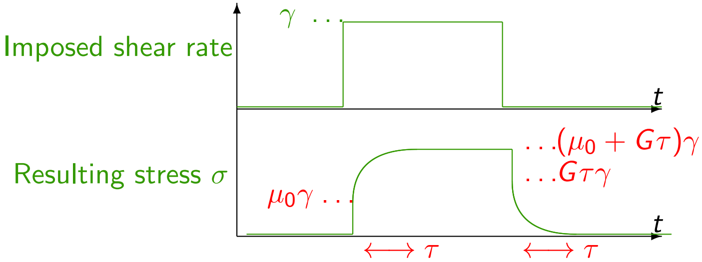

- Early viscosity μ_0

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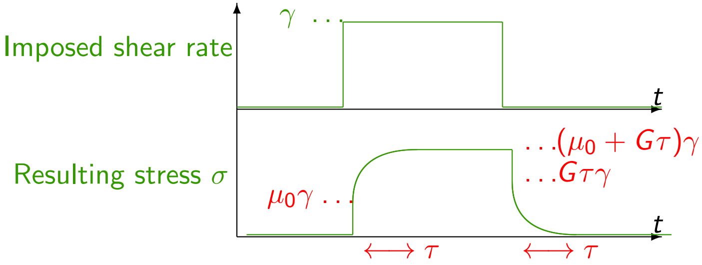

- Early viscosity μ_0
- Steady state viscosity $\mu_0 + G\tau$

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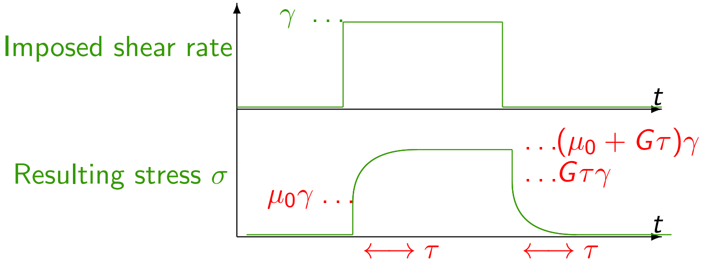

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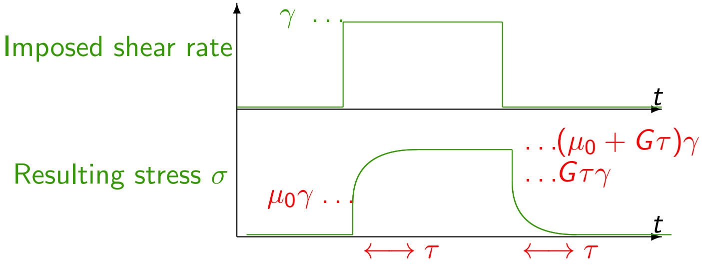

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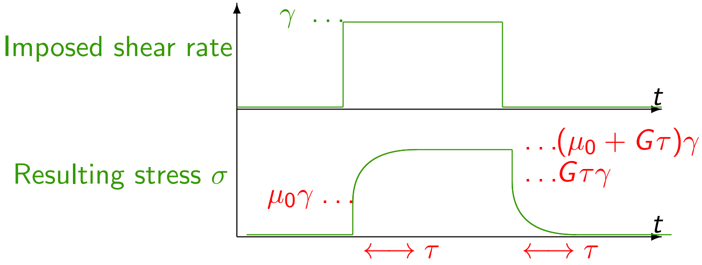

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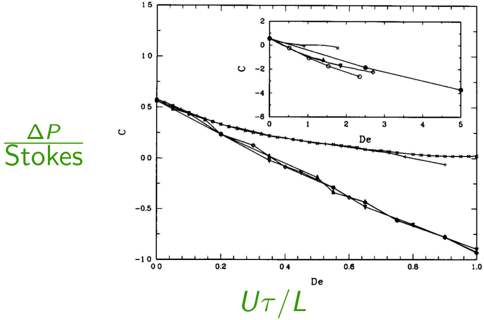
Stress relaxation is a special property of non-Newtonian fluids, which is not in elastic solids nor viscous liquids

Numerical Oldroyd-B



Debbaut, Marchal & Crochet 1988 JNNFM

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Δp scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

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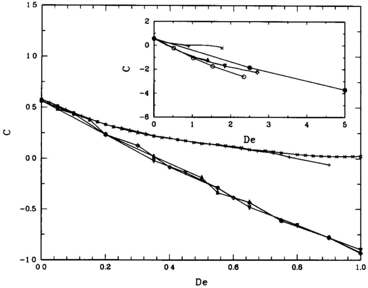

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. . . contraction flow

Experiments

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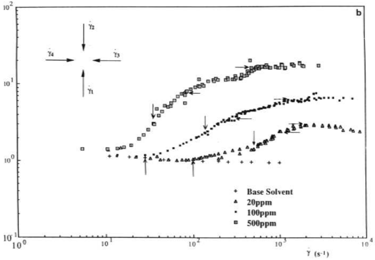

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Experiments have a tiny decrease in pressure drop!

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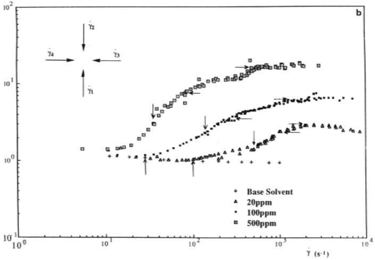

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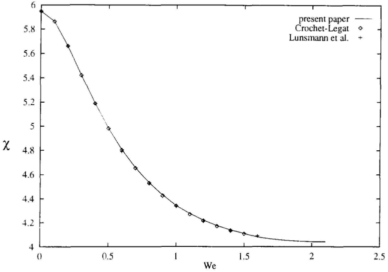
Experiments have a tiny decrease in pressure drop!

Oldroyd-B has no big increase is Δp, and no big upstream vortices

Numerical Oldroyd-B

Drag / Stokes



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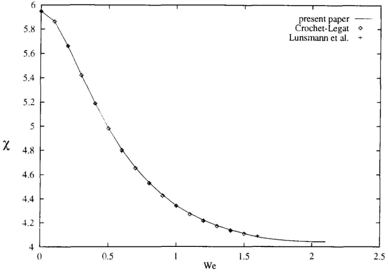

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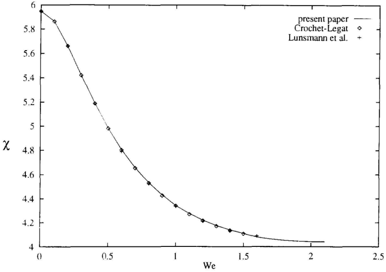

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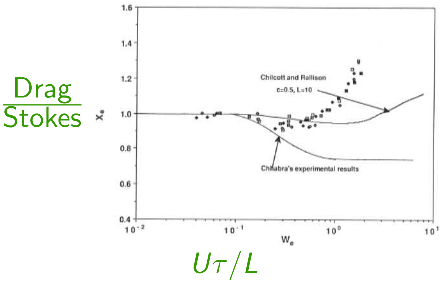
Mathematical question: Does a solution exist for $De > 2$?

. . . flow past a sphere

Experiments

Experiments



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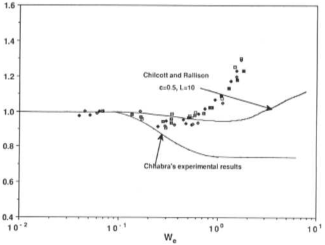

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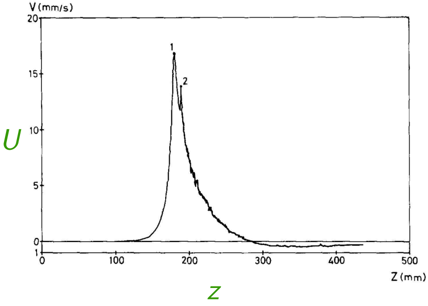
Experiments have a tiny decrease in drag!

Oldroyd-B has no big increase in drag, and no big wake

. . . and negative wakes

Experiment



Bisgaard 1983 JNNFM

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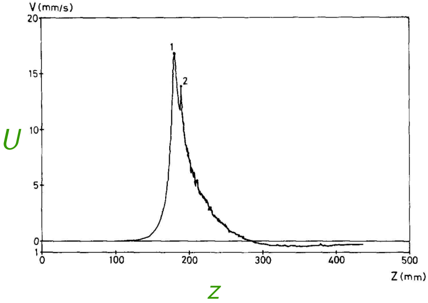
Experiment



Bisgaard 1983 JNNFM

Driven by unrelaxed elastic stress in wake.

Tension in streamlines – slightly nonlinear effect

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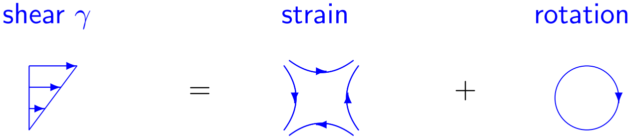

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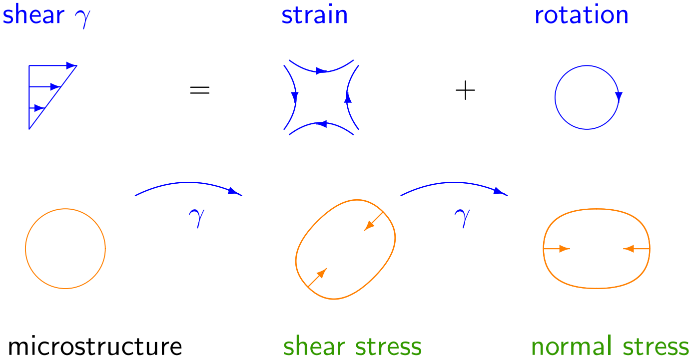

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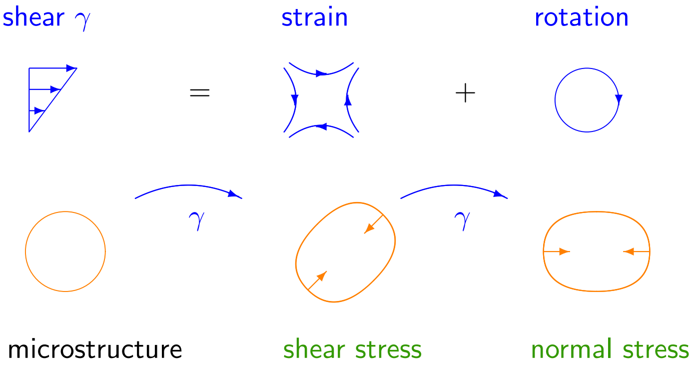
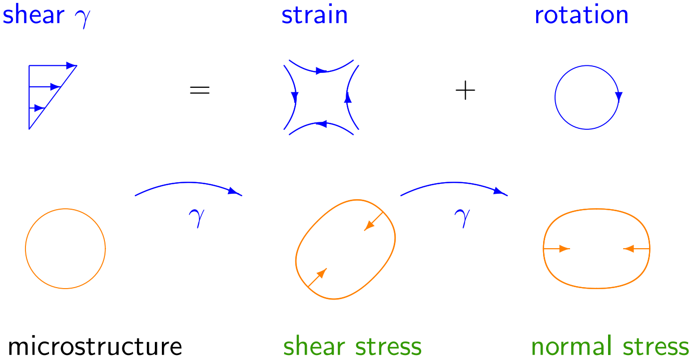


Shear stress $= G \times (\text{rate} = \gamma) \times (\text{memory time} = \tau)$

Tension in streamlines – slightly nonlinear effect



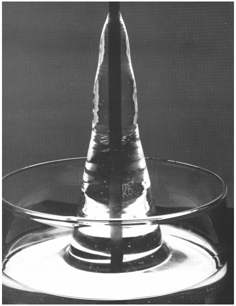
Shear stress $= G \times (\text{rate} = \gamma) \times (\text{memory time} = \tau)$

Normal stress (tension in streamlines) $=$ shear stress $\times \gamma\tau$.

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability

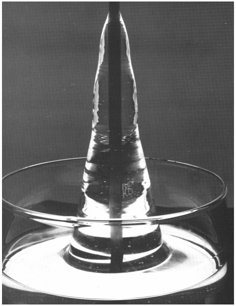
Rod climbing



Bird, Armstrong & Hassager

1987, Vol 1 (2nd ed) pg 62

Rod climbing



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Tension in streamlines

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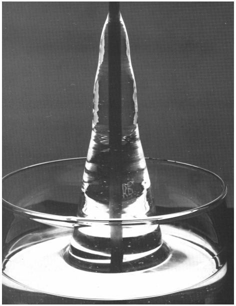

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Tension in streamlines $\longrightarrow$ hoop stress

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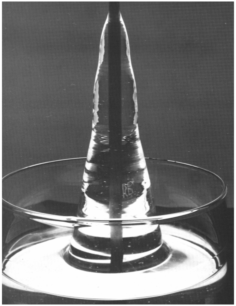

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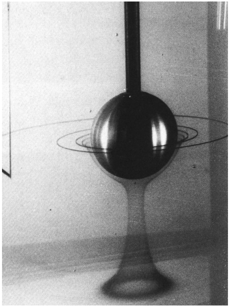
Tension in streamlines $\longrightarrow$ hoop stress
$\longrightarrow$ squeeze fluid in & up.

Secondary flow



Bird, Armstrong & Hassager

1987, Vol 1 (2nd ed) pg 70

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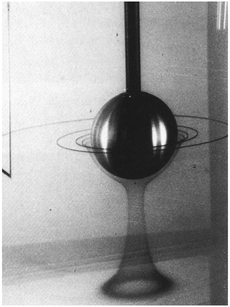

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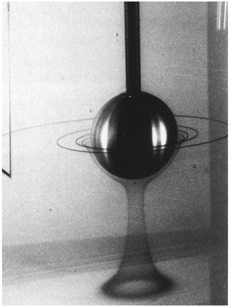

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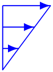
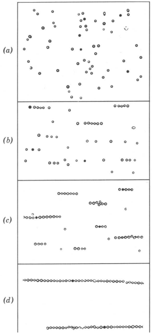
Tension in streamlines ⟶ hoop stress
⟶ squeeze fluid in.

Non-Newtonian effects opposite sign to inertial

Migration into chains

shear γ





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1987, Vol 1 (2nd ed) pg 87

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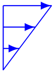

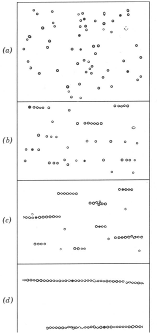

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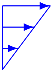
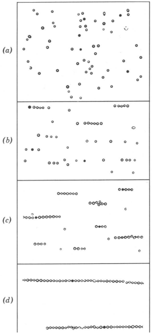
$\longrightarrow$ squeeze particles together

Migration into chains

shear γ





Bird, Armstrong & Hassager

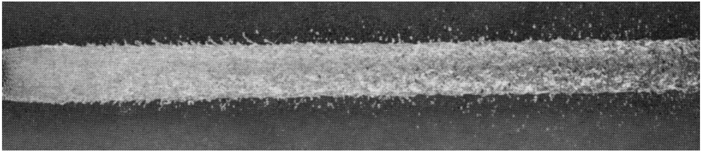
1987, Vol 1 (2nd ed) pg 87

Tension in streamlines $\longrightarrow$ hoop stress
$\longrightarrow$ squeeze particles together

Also migration to centre of a tube,
and alignment with gravity of sedimenting rods.

Stabilisation of jets

Newtonian Jet



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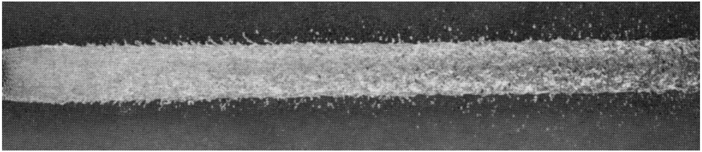
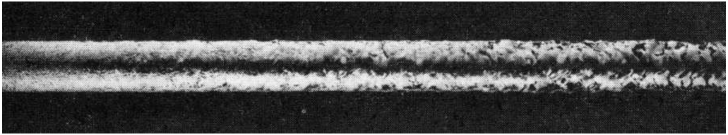


Non-Newtonian Jet (200ppm PEO)



Hoyt & Taylor 1977 JFM

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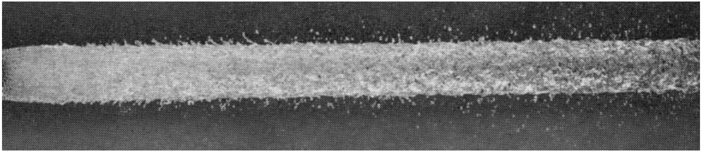

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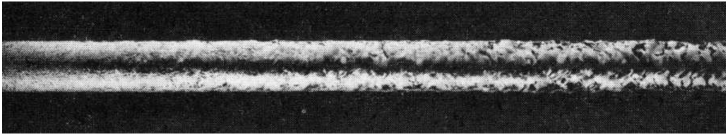

Hoyt & Taylor 1977 JFM

Tension in streamlines in surface shear layer

Stabilisation of jets

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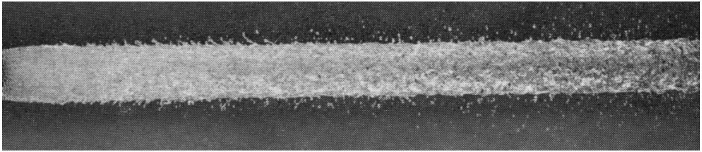

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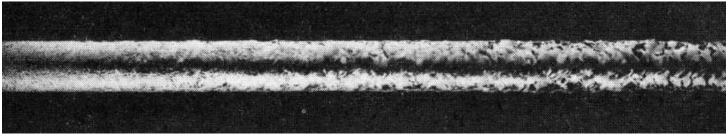

Hoyt & Taylor 1977 JFM

Tension in streamlines in surface shear layer

For fire hoses, and reduce explosive mist

Co-extrusion instability

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If core less elastic, then jump in tension in streamlines

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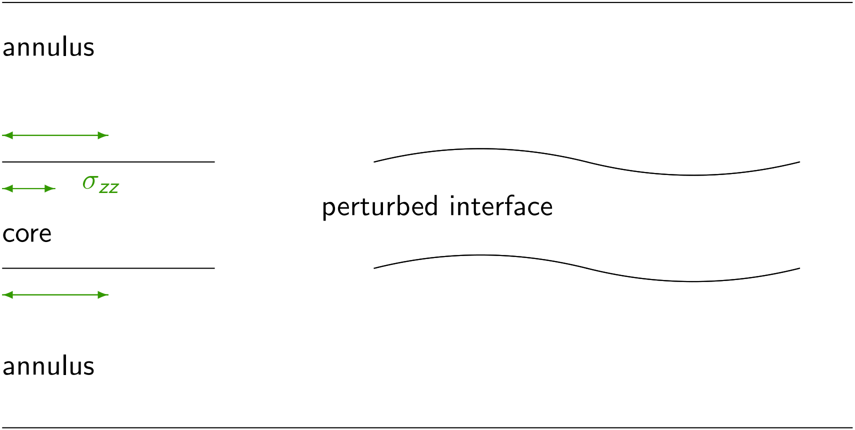

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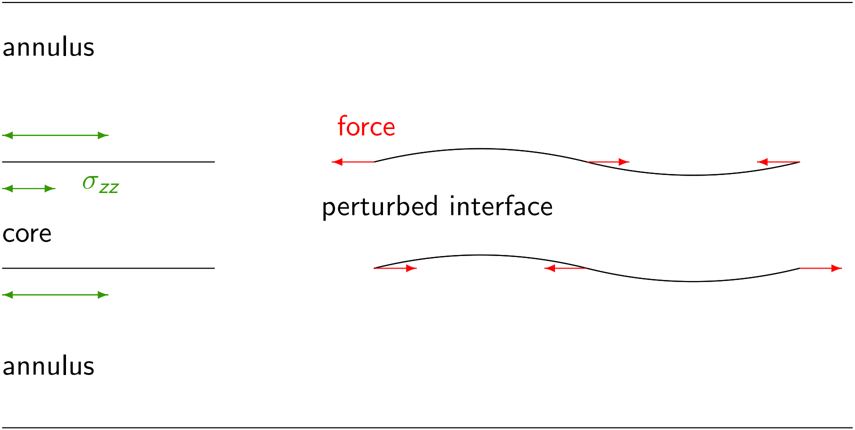

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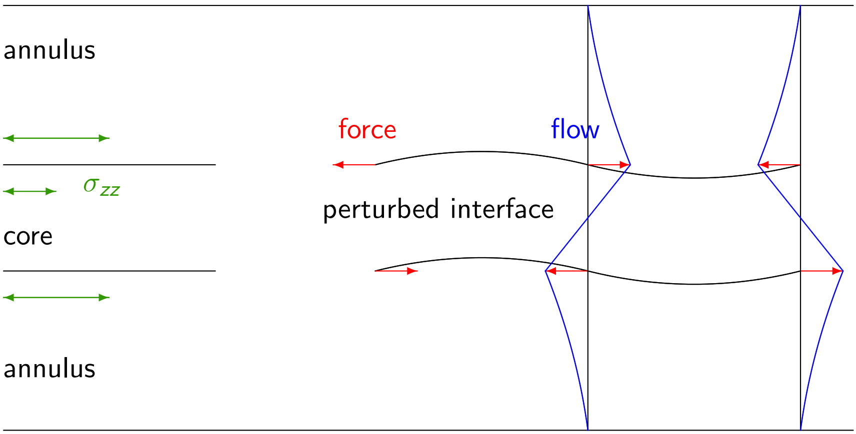

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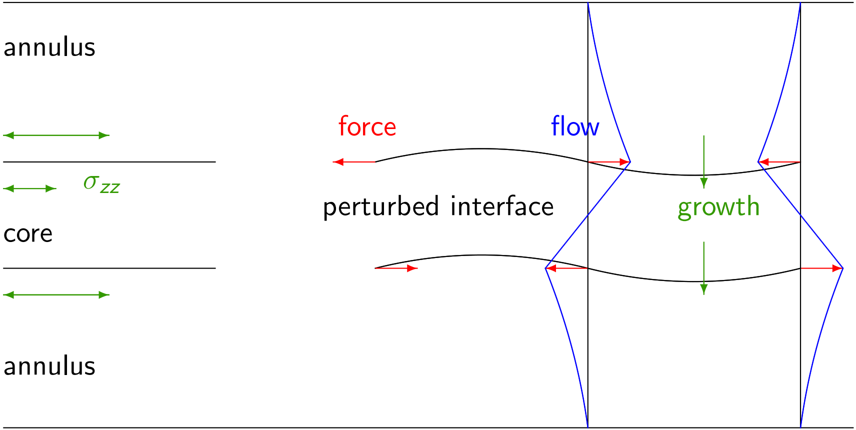

Hinch, Harris & Rallison 1992 JNNFM

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability

Tension in streamlines – when very fast

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla\mathbf{u} + \nabla\mathbf{u}^T \cdot \mathbf{A} - \frac{1}{\tau}(\mathbf{A} - \mathbf{I})$$

Fast:

Tension in streamlines – when very fast

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$$\mathbf{A} = g(\psi)\mathbf{uu}$$

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g from matching to slower (relaxing) region

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$$0 = -\nabla p + Gg^{1/2}\mathbf{u} \cdot \nabla g^{1/2}\mathbf{u}$$

Euler equation!

. . . very fast

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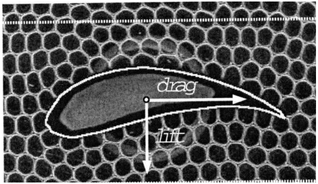
Anti-Bernoulli

$$p - \frac{1}{2} G g u^2 = \text{const}$$



Dollet, Aubouy & Graner 2005 PRL

. . . very fast

Potential flows $g^{1/2}\mathbf{u} = \nabla\phi$

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Flow around sharp 270° corner: Hinch 1995 JNNFM

$$\phi = r^{2/3}\cos\tfrac{2}{3}\theta,$$

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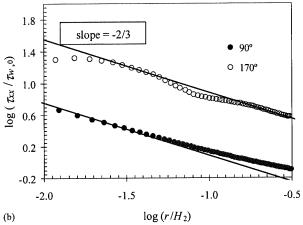

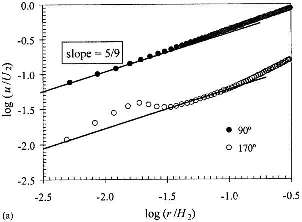

Alves, Oliviera & Pinho 2003 JNNFM

Capillary squeezing – controlled by relaxation

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Mass $\qquad \dot{a} = -\frac{1}{2} E a$

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Solution $\qquad a(t) = a(0) e^{-t/3\tau}$

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Microstructure	$\dot{A}_{zz} = 2 E A_{zz} - \frac{1}{\tau}(A_{zz} - 1)$

Solution $\quad a(t) = a(0) e^{-t/3\tau}$

Need slow $E = 1/3\tau$ to stop A_{zz} relaxing from $\chi / G a$

. . . capillary squeezing

Oldroyd-B $\quad a(t) = a(0)e^{-t/3\tau} \quad$ does not break

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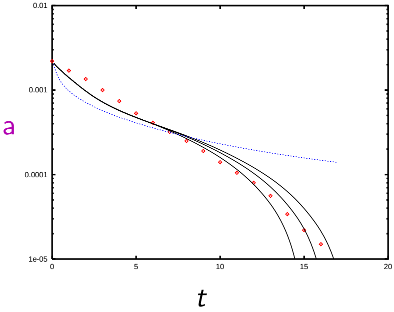
Experiments S1 fluid



Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

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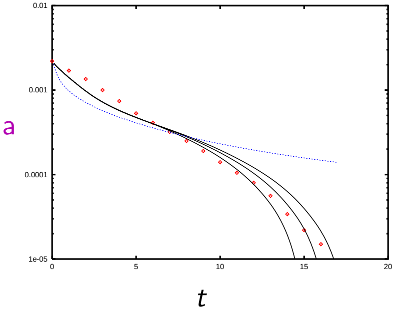

Exp: Liang & Mackley 1994 JNNFM

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Correct time scale,

. . . capillary squeezing

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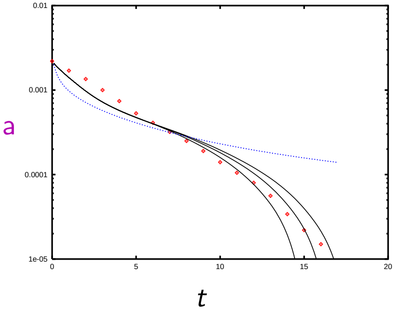


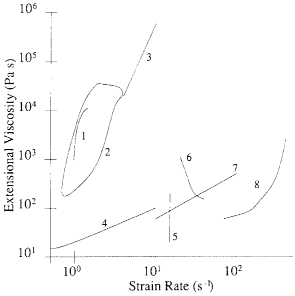
Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

Correct time scale, but filament eventually breaks in experiments

M1 project

no simple extensional viscosity



1. Open syphon
2. Spin line
3. Contraction
4. Opposing Jet
5. Falling drop
6. Falling bob
7. Contraction
8. Contraction

Keiller 1992 JNNFM

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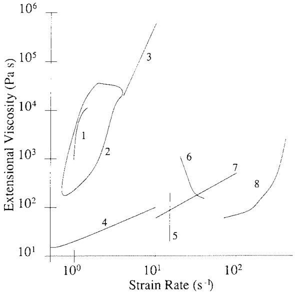
no simple extensional viscosity



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Keiller 1992 JNNFM

really elastic responses

Fit data with Oldroyd-B: $\mu_0 = 5$, $G = 3.5$, $\tau = 0.3$ from shear

Keiller 1992 JNNFM

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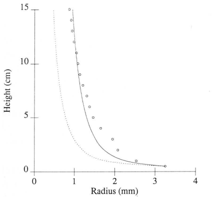

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Keiller 1992 JNNFM

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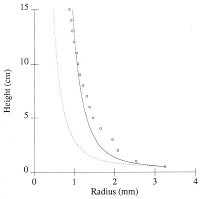

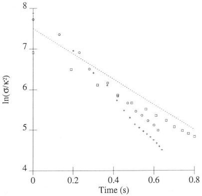

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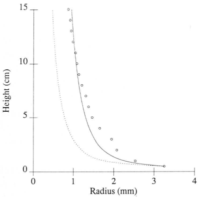
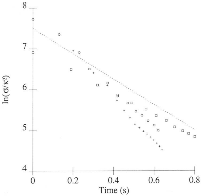


2. Spin line   Oliver 1992



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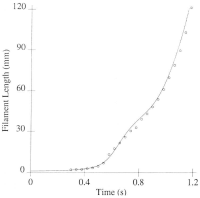

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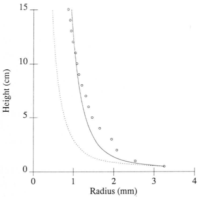

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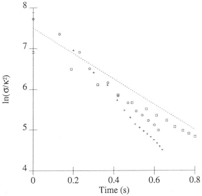

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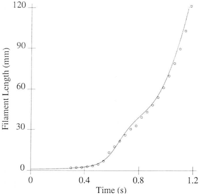
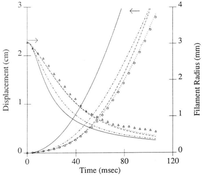


6. Falling bob   Matta 1990



Oldroyd B: Successes & Failures

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Simplest viscosity μ_0 + elasticity G + relaxation τ

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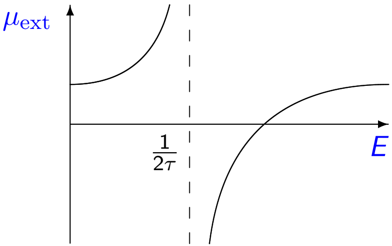
Also difficult numerically at $\frac{U\tau}{L} > 1$

Disaster in Oldroyd-B

Steady extensional flow:



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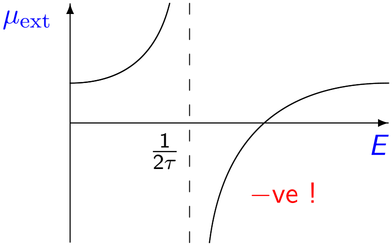

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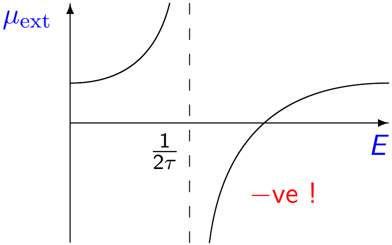

Microstructure deforms without limit if $E > \frac{1}{2\tau}$: $\quad A = e^{(2E - \frac{1}{\tau})t}$

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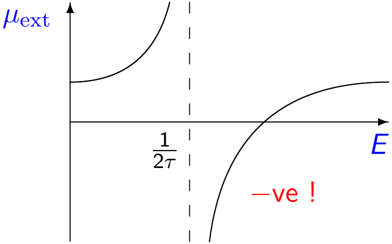

Microstructure deforms without limit if $E > \frac{1}{2\tau}$: $\quad A = e^{(2E - \frac{1}{\tau})t}$

Need to limit deformation of microstructure

FENE modification

Finite Extension Nonlinear Elasticity

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$$\frac{DA}{Dt} = A \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot A - \frac{f}{\tau}(A - \mathbf{I})$$

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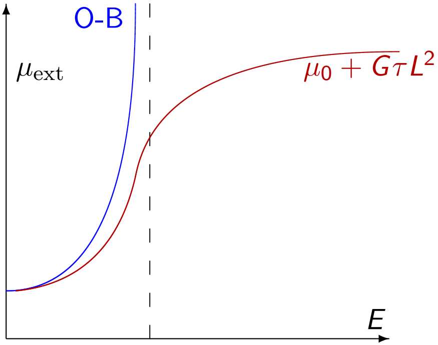
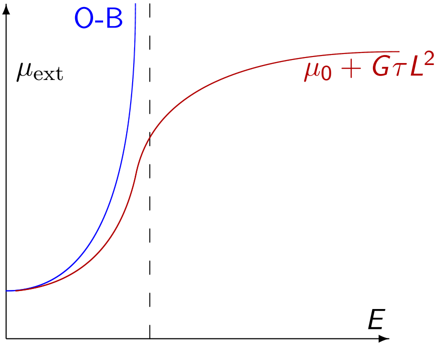
$$f = \frac{L^2}{L^2 - \mathrm{trace}\, A} \qquad \text{keeps} \quad A < L^2$$

...FENE



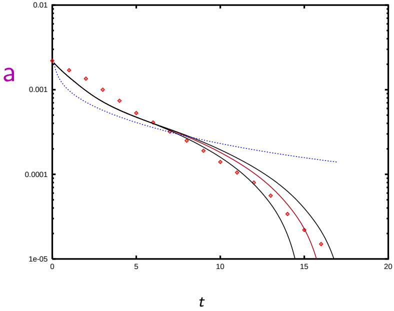
. . . FENE



Large extensional viscosity $\mu_0 + G\tau L^2$, but small shear viscosity μ_0

FENE capillary squeezing

Filament breaks in with FENE $L = 20$



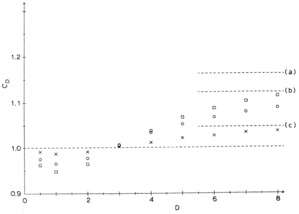
Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

FENE flow past a sphere

FENE

F



De

Chilcott & Rallison 1988 JNNFM

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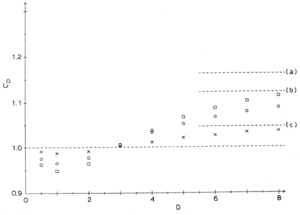
FENE

F



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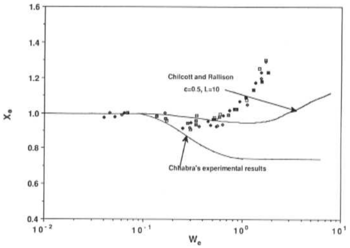
Chilcott & Rallison 1988 JNNFM

Experiments M1



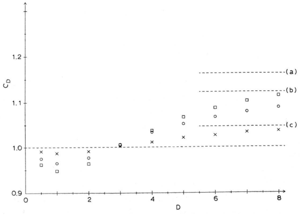
De

Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

FENE flow past a sphere

FENE

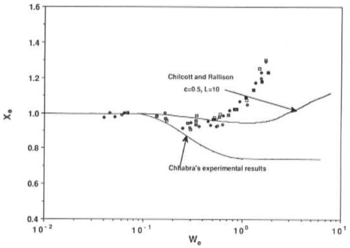
F



De

Chilcott & Rallison 1988 JNNFM

Experiments M1



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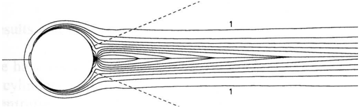
FENE gives drag increase

FENE drag increase from long wake of high stress



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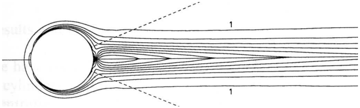

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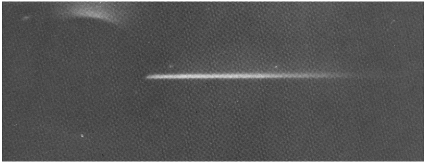

Cressely & Hocquart 1980 Opt Act

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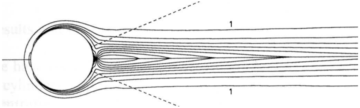

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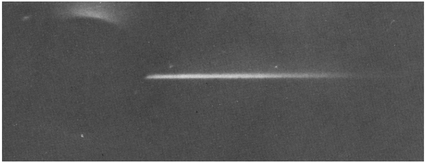

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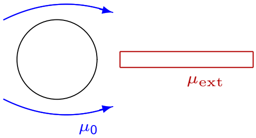
"Birefringent strand"

...birefringent strands

Boundary layers of high stress: μ_{ext} in wake, μ_0 elsewhere.



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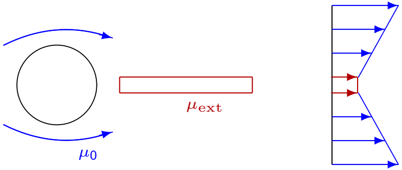

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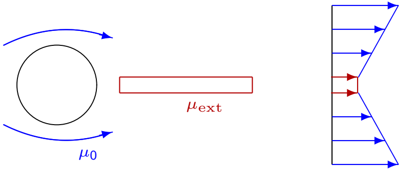

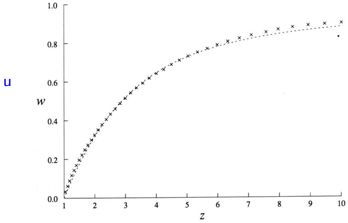

Harlen, Rallison & Chilcott 1990 JNNFM

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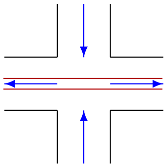

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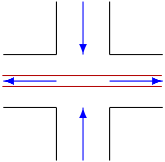

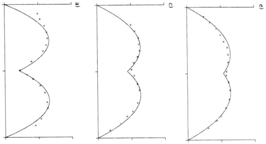

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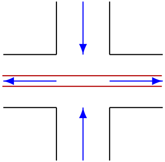

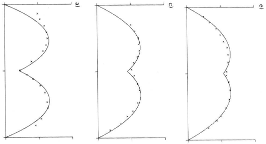

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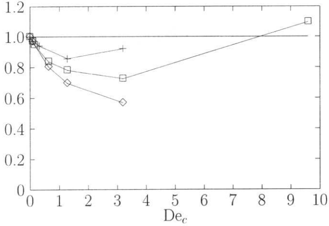
Also cusps at rear stagnation point of bubbles.

FENE contraction flow

FENE $L = 5$

Δp



Szabo, Rallison & Hinch 1997 JNNFM

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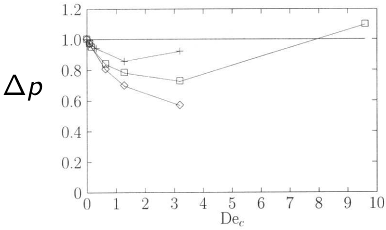

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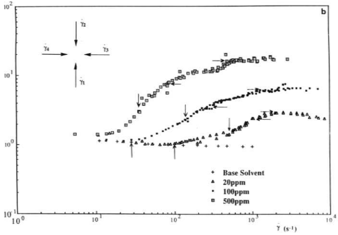
Experiments



Cartalos & Piau 1992 JNNFM

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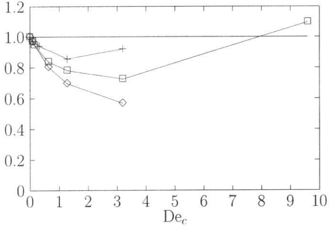

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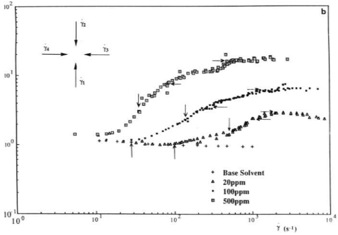

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FENE gives increase in pressure drop

Increase in pressure drop from long upstream vortex

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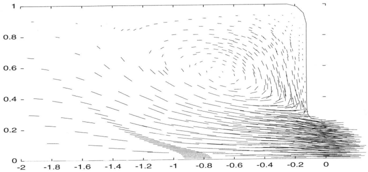

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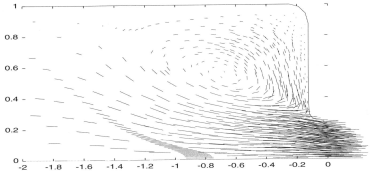

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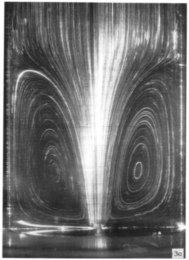

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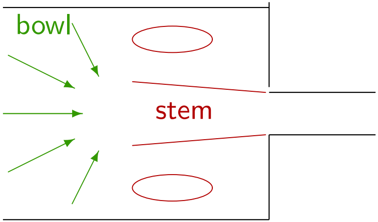

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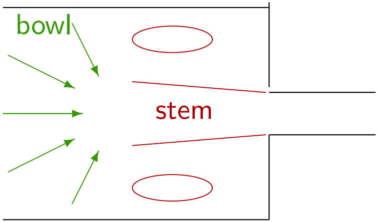

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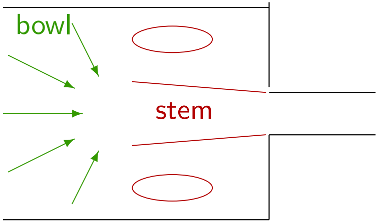

Bowl: point sink flow,

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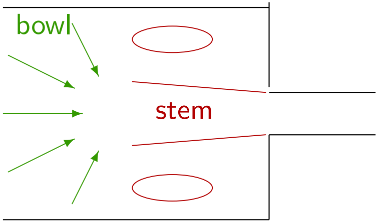

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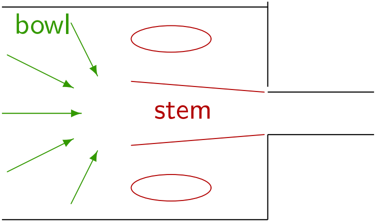

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Stem:

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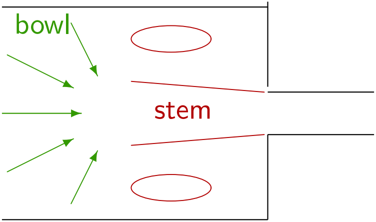

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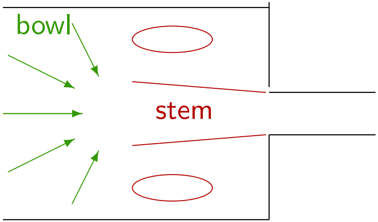

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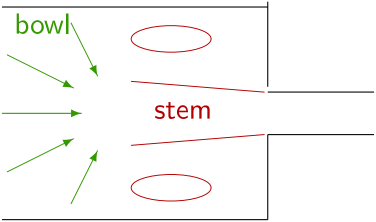

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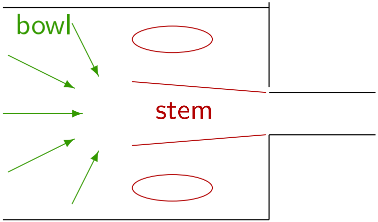

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But sometimes need small L to fit experiments.

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