Levitation and locomotion on an air-table of plates with herringbone grooves.

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with Dan Soto & David Quéré in Paris, and
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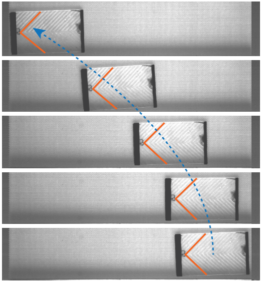
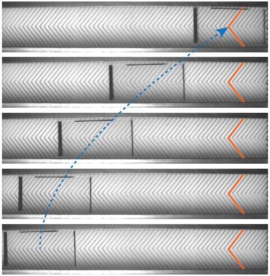
Experiments on an air table

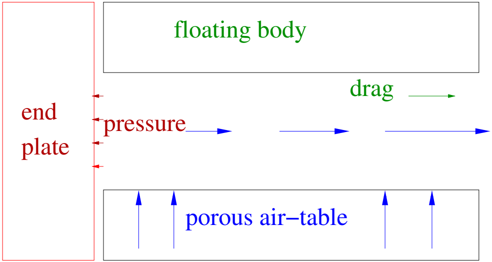
Grooves on the floating body, on the table

Accelerates to the left, to the right.

Why different direction?

Left or right?



End plate attached to base (grooved table)

$\longrightarrow$ body dragged by flow to right

End plate attached to top (grooved body)

$\longrightarrow$ pressure pushes to left

Pressure also levitates floating body.

Some approximations

Grooves: width $\gg$ height $\quad\rightarrow$ 2D flow, ignore spanwise

Grooves: length $\gg$ height $\quad\rightarrow$ boundary layer equations
(lubrication theory with inertia)

Two more approximations later.

Study 2D flow down groove

Boundary layer equations with $p = p(x)$:

$$u_x + v_y = 0$$
$$\rho(u_t + uu_x + vu_y) = -p_x + \mu u_{yy}$$

BC on porous plate $y = 0$: $\quad u = 0$ and

$$v = \frac{k}{\mu}(P_2 - p) \qquad \text{with plate permeability} \quad k = \frac{\pi r^4}{8\ell d^2}$$

for holes of radius r, length ℓ, separation d

Nondimensionalise

Boundary layer equations with $p = p(x)$:

$$u_x + v_y = 0$$

$$Re(u_t + uu_x + vu_y) = -Kp_x + u_{yy}$$

BC on porous plate $y = 0$: $\quad u = 0, \quad v = 1 - p$

$$K = \frac{\Delta p \text{ across porous plate}}{\Delta p \text{ down groove}} = \frac{8H^3 \ell d^2}{\pi L^2 r^4}$$

Forces:

Propulsion $F_H = p(0) + \int_0^1 u_y\big|_{y=1}\, dx$, $\quad$ Levitation $F_V = \int_0^1 p\, dx$

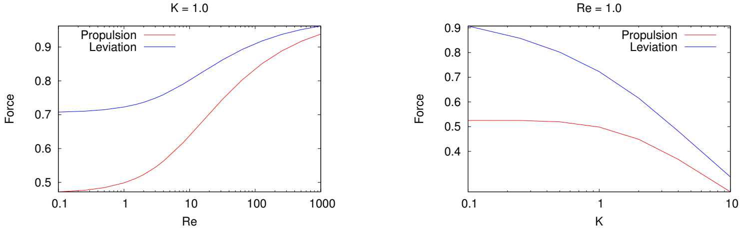
NB: different non-dimensionalisation, F_H smaller by area ratio H/L

Numerical solution

Integrate $v_y = -u_x$ from $y = 1$ to $y = 0 \quad \to \quad v(x, y = 0)$

Porous plate BC $p(x) = 1 - v(x, 0)$ into momentum equation.

Time step momentum to steady state.



NB: two forces (in different non-dimensionalisations) within a factor of two

NB: small change with *Re*, but decrease at large *K* (short groove)

Asymptotics

- Short groove, $K \gg 1$,

  pressure drop mostly across porous plate, so $v(x, 0) \approx 1$

- $Re \ll 1$
- $Re \gg 1$
- Long groove, $K \ll 1$,

  so $p = 1$ in groove except very near outlet

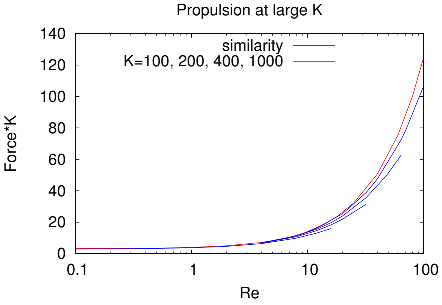
Short groove, $K \gg 1$, $\quad Re$ arbitrary

Most pressure drop across porous plate, so $p \sim 0$ in groove, so $v(x, y = 0) = 1$, hence similarity solution

$$u(x,y) = -xg'(y), \quad v(x,y) = g(y), \quad p = \frac{B}{K}(1 - x^2)$$

Momentum equation then

$$Re(g'^2 - gg'') = 2B - g'''$$



need $K \gg Re$

also $F_V = \frac{2}{3}p(0)$

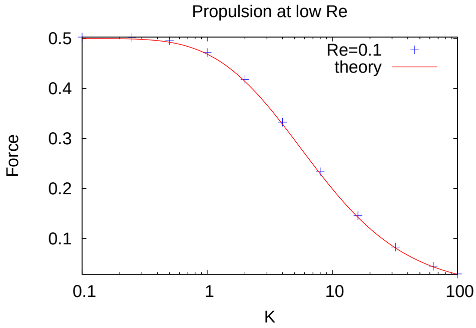
Low Reynolds number, K arbitrary

Lubrication theory

$$q = \int_0^1 u(x,y)\,dy = -\frac{K}{12}p_x, \qquad q_x = v(x,0) = 1 - p$$

so

$$F_H = \tfrac{1}{2}\left(1 - \operatorname{sech}\sqrt{12/K}\right), \quad F_V = 1 - \sqrt{K/12}\tanh\sqrt{12/K}$$



At $Re \ll 1$:
Drag on top =
Drag on bottom =
-$\frac{1}{2}$ Pr force on end

High Reynolds number, K arbitrary

Separable solution with streamfunction

$$\psi(x,y) = -f(x)g(y).$$

Inviscid, vorticity constant along streamlines. Try

$$\omega(\psi) = k^2\psi, \qquad \omega = -\psi_{yy}.$$

Hence

$$g(y) = \cos\tfrac{\pi}{2}y.$$

$$u = -f(x)g'(y), \quad v = f'(x)g(y)$$

Bernoulli integral

$$\beta^2 f^2 = p_0 - p, \quad \text{with } \beta^2 = \pi^2 Re/8K, \quad p_0 = p(0)$$

High Reynolds number, inviscid

Bernoulli integral

$$\beta^2 f^2 = p_0 - p.$$

Porous plate $f' = 1 - p$, so

$$f(x) = \frac{\sqrt{1 - p_0}}{\beta} \tan\left(\beta x \sqrt{1 - p_0}\right)$$

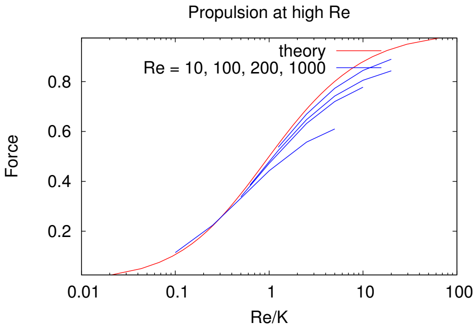
$p_0 = p(0)$ is determined by $p(1) = 0$:

$$\tan^2\left(\beta \sqrt{1 - p_0}\right) = \frac{p_0}{1 - p_0}$$

High Reynolds number, inviscid

Finally forces

$$F_H = -p_0, \quad F_V = 1 - \sqrt{p_0}/\beta$$



Need boundary layer corrections

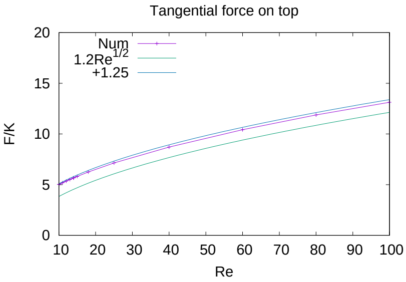
Boundary layer corrections

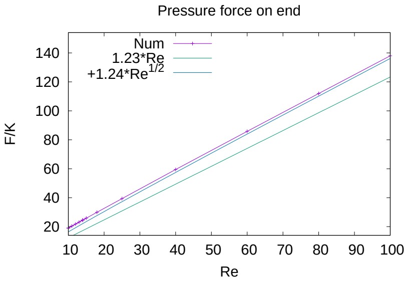
For short groves, there is a $Re^{1/2}$ similarity boundary layer

The boundary layer exerts a drag $1.2133Re^{1/2}$ on the top plate,

and has a displacement thickness $1.0366Re^{-1/2}$,

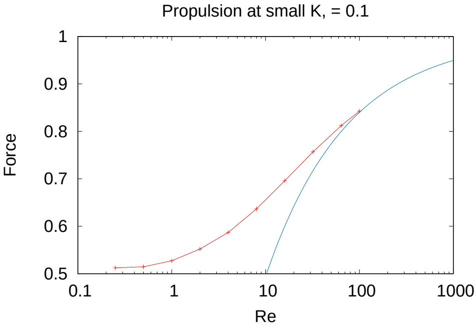
which leads to an enhanced pressure on the left end





Long groove, $K \ll 1$, $\quad Re$ arbitrary

Most of the groove is at $p = 1$, the pressure under the air-table.
Hence pressure part of Propulsion and Levitation ~ 1.
Pressure drop and flow only near exit of groove, so tough numerics.



But frictional drag halves the Propulsion at low *Re*.

Double limits

	Propulsion F_H	Levitation F_V
$K, Re \ll 1$	0.5	$1 - \sqrt{K/12}$
$K \ll 1 \ll Re$	1	$1 - \sqrt{8K/\pi^2 Re}$
$Re \ll 1 \ll K$	$3/K$	$4/K$
$1 \ll Re \ll K$	$\pi^2 Re/8K$	$\pi^2 Re/12K$
$1 \ll K \ll Re$	$1 - 2K/Re$	$1 - \sqrt{8K/\pi^2 Re}$

At $Re \gg 1$ and $K \ll 1$, uniform levitation pressure = propulsion pressure, i.e. $F_H = F_V$

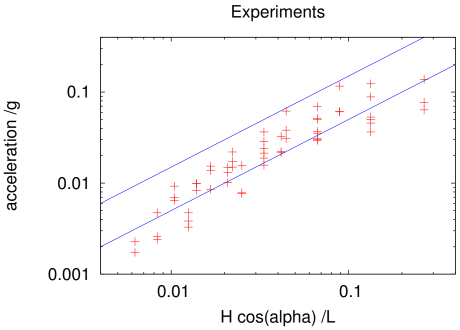
At $Re \ll 1$, frictional drag reduces F_H by 50%.
At $K \gg 1$, pressure is parabolic, reduces F_V by $2/3$.

Hence $\quad F_H/F_V = 0.5$ to 1.5 (different non-dimensionalisations)

Experiments

Reinstating the different dimensional factors, and resolving force along direction that body moves, predict

$$\text{Propulsion/weight} = (0.5 \text{ to } 1.5) h \cos\alpha/\ell$$



Grooves 5 heights & 4 lengths, Plates 3 thickness,
($K = 0.15 \to 2800$, $Re = 1 \to 200$) $a = 0.02 \to 1.4\,\mathrm{m/s^2}$