Jet break-up and polymers: a numerical study of beads-on-string

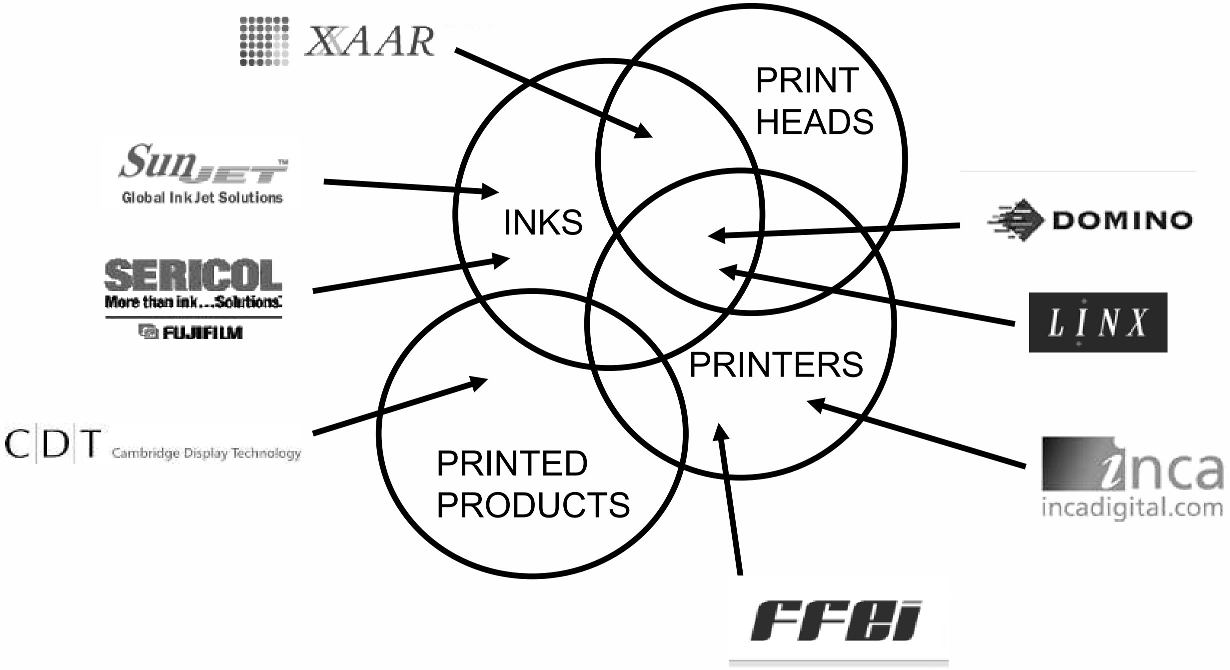
J. Etienne[1,3], J. Hinch[1] & J. Li[2]

[1] DAMTP & [2] BP Institute, Cambridge University

[3]SPECTRO, UJF & CNRS, Grenoble

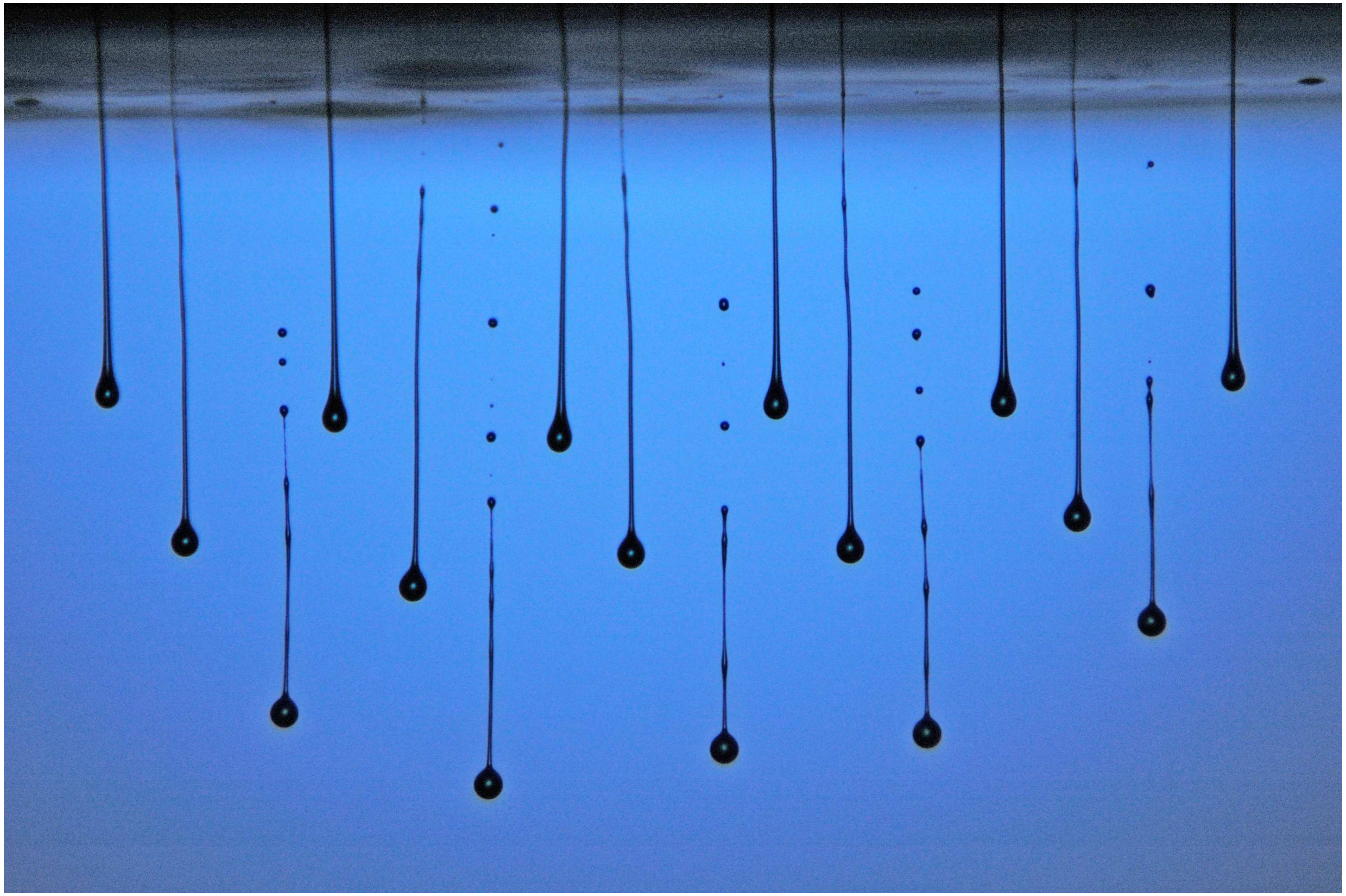
EPSRC collaboration

Companies



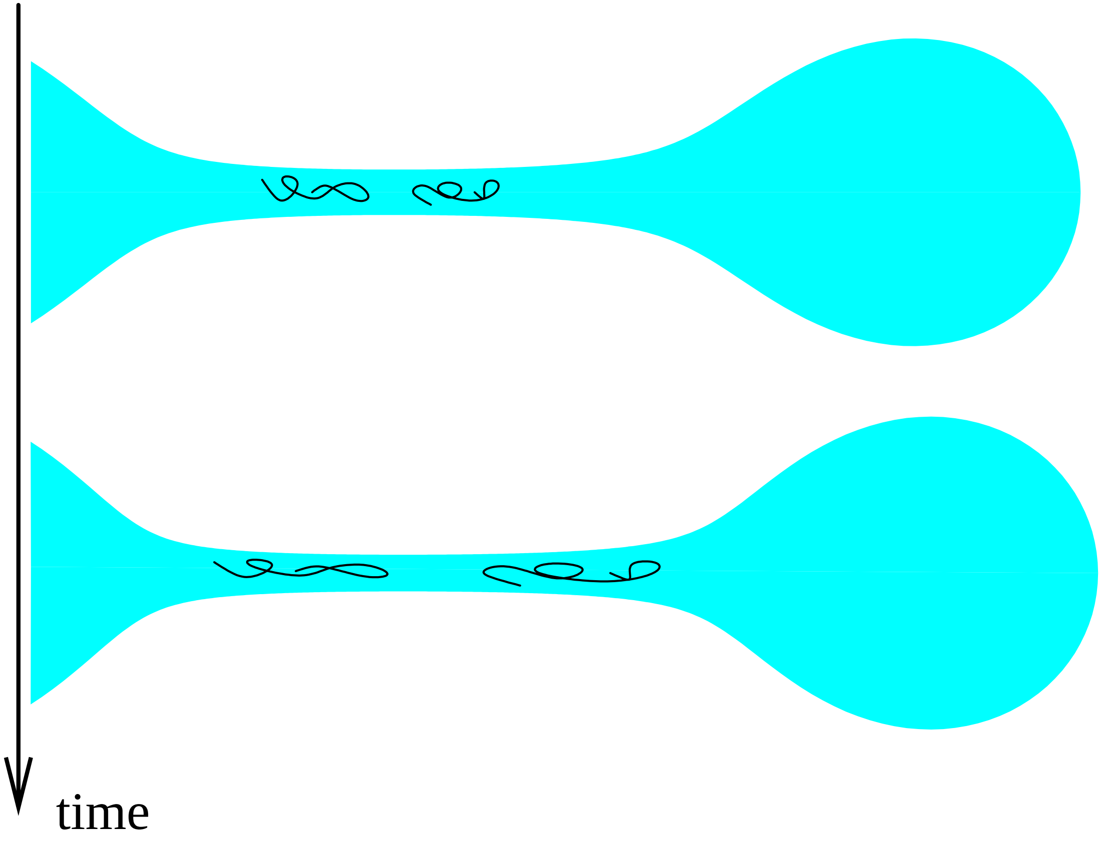
Universities: Cambridge, Durham, Leeds, Manchester, Wales

Drop-on-Demand printer



Effect of polymer

Difficult to break non-Newtonian jets



Polymers resist stretching – need rheology equation

Oldroyd-B model fluid **simplest viscous + elastic**

$$\sigma \quad = \quad -p\mathbf{I} \qquad +2\mu_0\mathbf{E} \qquad\qquad +G\mathbf{A}$$

stress viscous elastic

μ_0 viscosity G elastic modulus

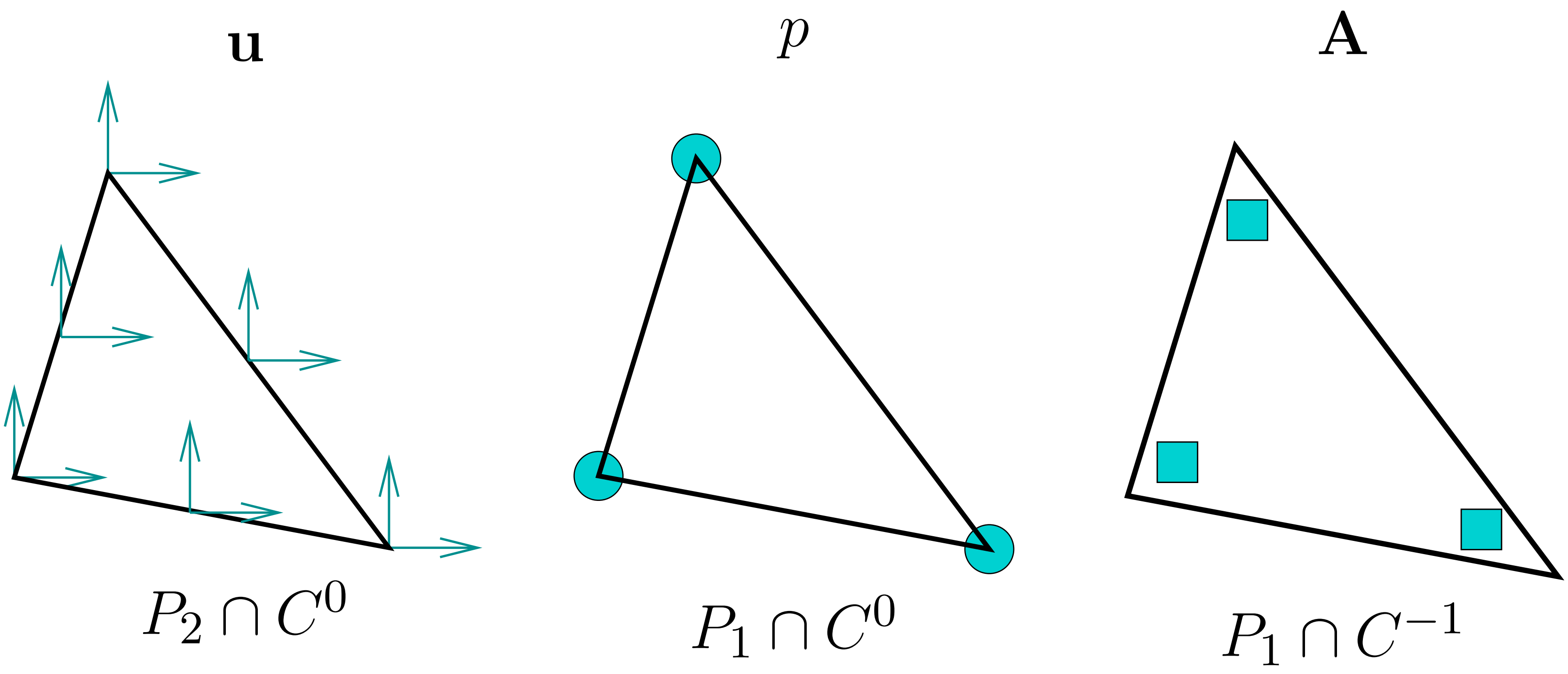
with $\mathbf{A}$ microstructure.

$$\frac{D\mathbf{A}}{Dt} = \quad \mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A} \qquad -\frac{1}{\tau}\left(\mathbf{A}-\mathbf{I}\right)$$

deform with fluid relaxes

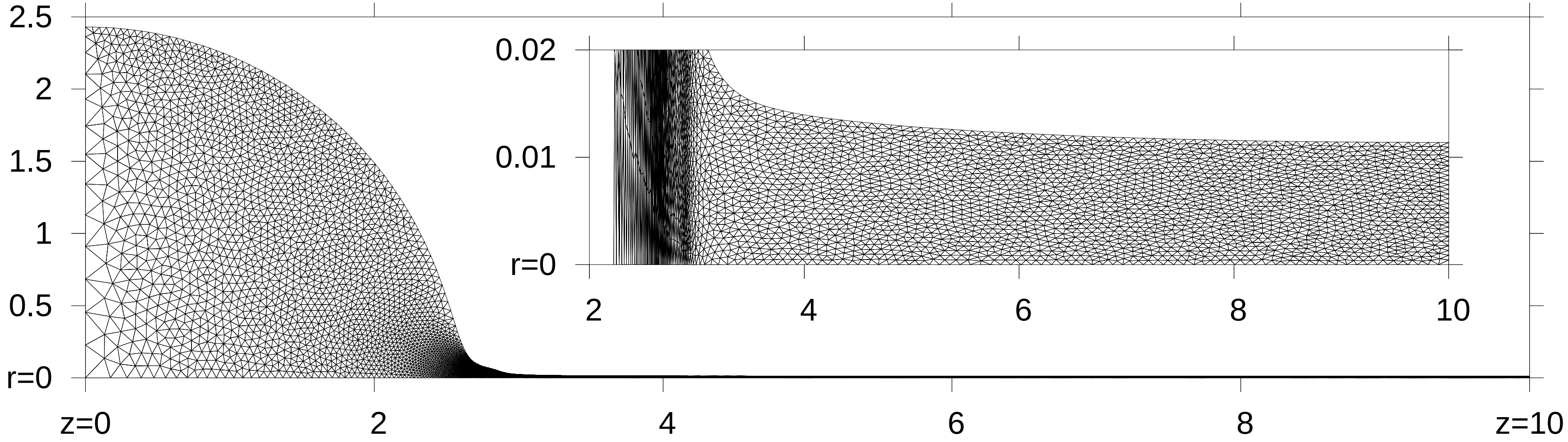
τ relaxation time

Numerical: Finite Elements



C^{++} code embedded in the free-software FEM environment `rheolef` (P. Saramito, N. Roquet, J. E.)

Auto-adaptive mesh

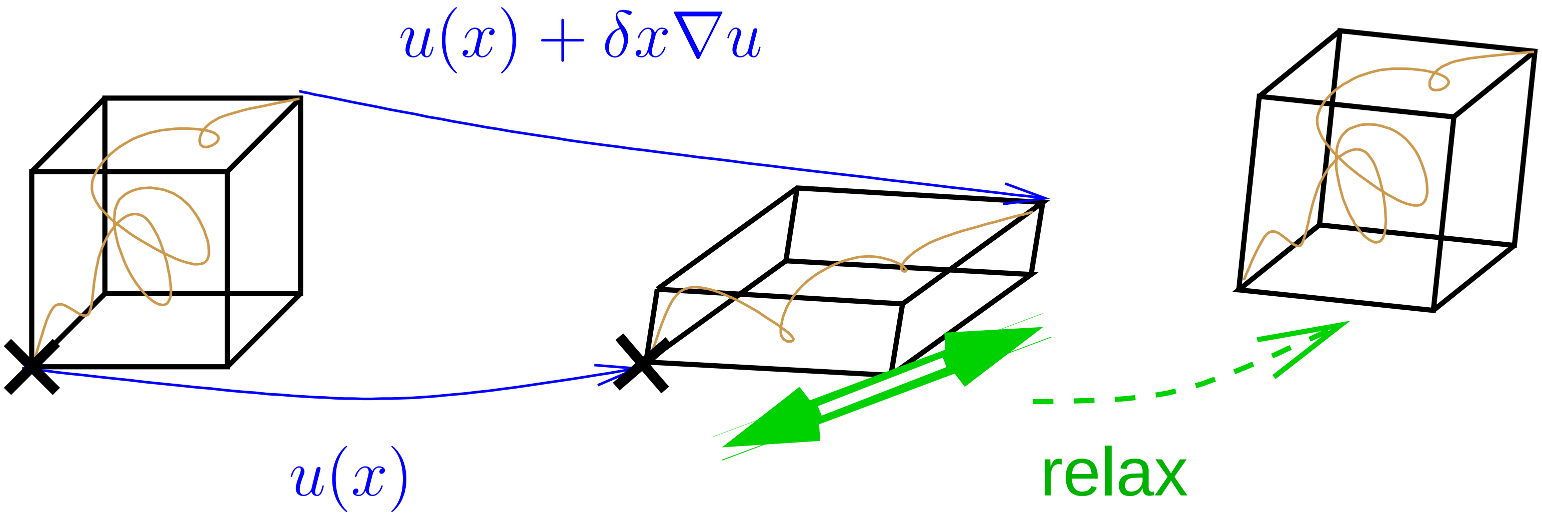


Generated by free-software `Bamg` (F.Hecht)

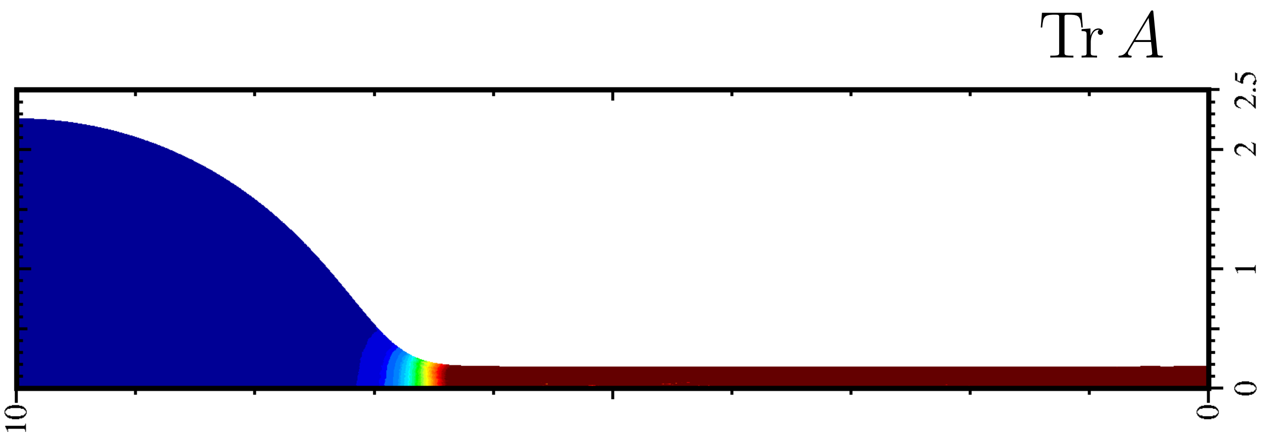
Lagrangian approach

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A} \quad -\frac{1}{\tau}(\mathbf{A}-\mathbf{I})$$

deform with fluid relaxes



Oldroyd-B results for low inertia

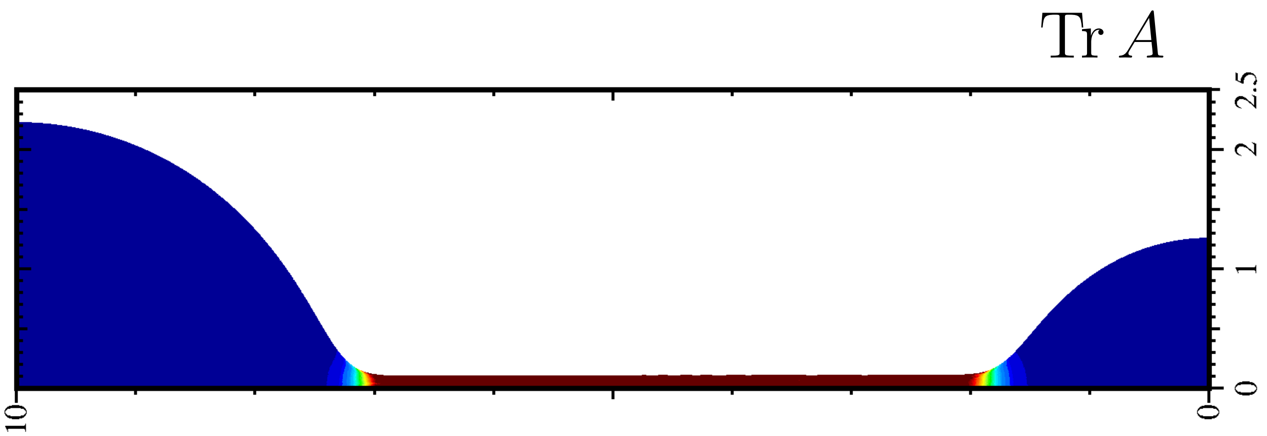


Slow relaxation: $De = \tau \sqrt{\dfrac{\gamma}{\rho R^3}} = 95$

Concentration: $c = \dfrac{G\tau}{\mu} = 3$

Ohnesorge (capillary Reynolds): $Oh = \dfrac{\sqrt{\rho \gamma R}}{\mu} = 3.2$

Oldroyd-B results for high inertia



Slower relaxation: $De = \tau\sqrt{\frac{\gamma}{\rho R^3}} = 300$

Concentration: $c = \frac{G\tau}{\mu} = 3$

Ohnesorge (capillary Reynolds): $Oh = \frac{\sqrt{\rho\gamma R}}{\mu} = 10$

Oldroyd-B thinning

Mass $$\dot{a} = -\frac{1}{2}Ea$$

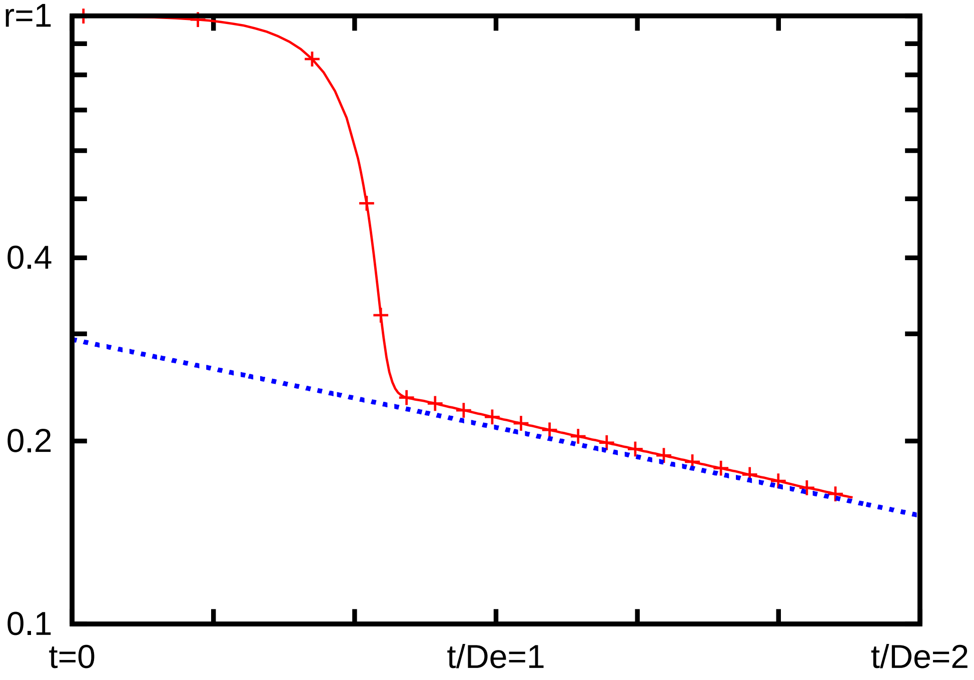
Momentum $$\frac{\chi}{a} = 3\mu_0 E + G(A_{zz} - A_{rr})$$

Microstructure $$\dot{A}_{zz} = 2EA_{zz} - \frac{1}{\tau}(A_{zz} - 1)$$

Solution

$$a(t) = a(0)e^{-t/3\tau}$$

Never breaks!



FENE modification

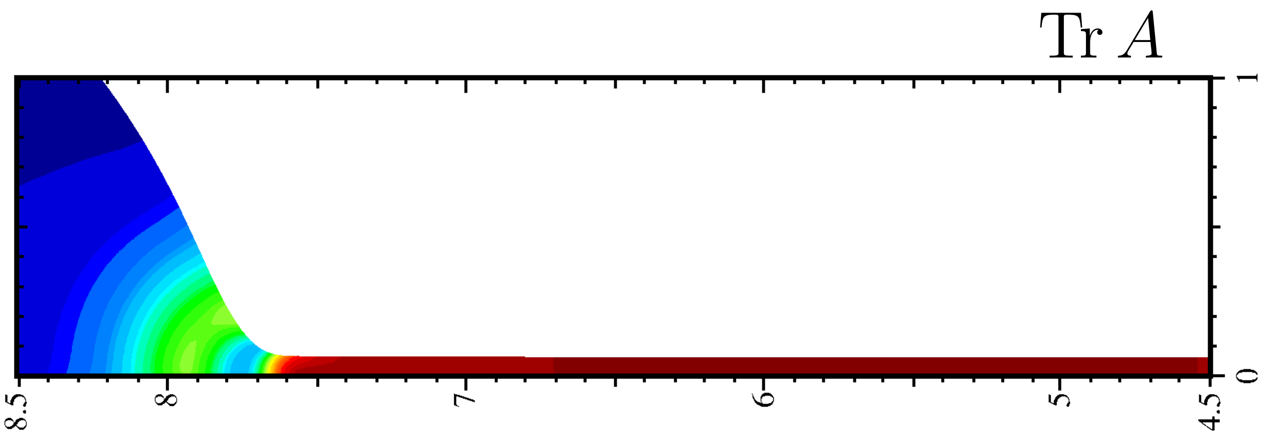
Finite Eextension Nonlinear Elasticity

$$\frac{DA}{Dt} = A \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot A - \frac{f}{\tau}(A - \mathbf{I})$$

$$\sigma = -p\mathbf{I} + 2\mu_0 E + G f A$$

$$f = \frac{L^2}{L^2 - \mathrm{trace}\, A}$$ keeps $A < L^2$

FENE results



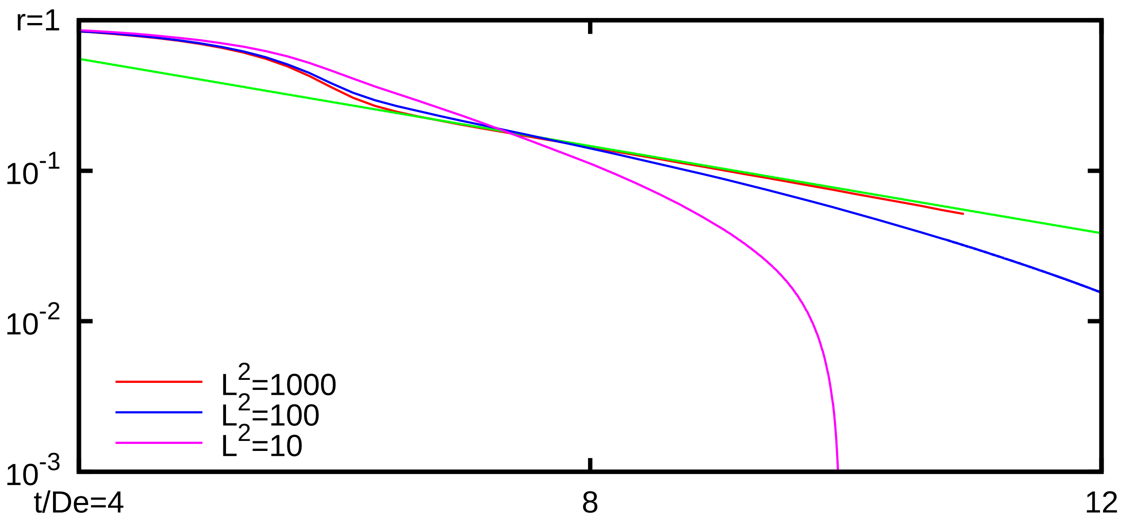
Slower relaxation: $De = \tau\sqrt{\frac{\gamma}{\rho R^3}} = 30$

Concentration: $c = \frac{G\tau}{\mu} = 1.5$

Ohnesorge (capillary Reynolds): $Oh = \frac{\sqrt{\rho\gamma R}}{\mu} = 2.5$

FENE thinning

Now breaks:



Hence design of inks:

Need $\text{concentration} \times \text{molecularweight}^2$ less than critical.