Small particles in a viscous fluid

Course in three parts

1. A quick course in micro-hydrodynamics
2. Sedimentation of particles
3. Rheology of suspensions

Good textbook for parts 1 & 2:
A Physical Introduction to Suspension Dynamics
by Elisabeth Guazzelli, Jeffrey F. Morris and Sylvie Pic
(Cambridge Texts in Applied Mathematics 2012).

Part 2. Sedimentation of particles

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An isolated particle
- A sphere
- Stokes settling velocity
- A spherical viscous drop
- Flow past an ellipsoid

Two particles

Finite clouds

Suspensions

Fluctuations in velocity of particles

Sedimentation of an isolated sphere

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$$\underset{\text{no inertia}}{0} = \underset{\text{weight}}{\rho_s \frac{4\pi}{3} a^3 g} - \underset{\text{buoyancy}}{\rho_f \frac{4\pi}{3} a^3 g} - \underset{\text{Stokes drag}}{6\pi\mu a\mathbf{U}}$$

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Drag on a spherical drop $2\pi\mu a\frac{2\mu+3\mu_i}{\mu+\mu_i}$.

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Important conclusion Drag $\approx 6\pi\mu \times$ (largest diameter).

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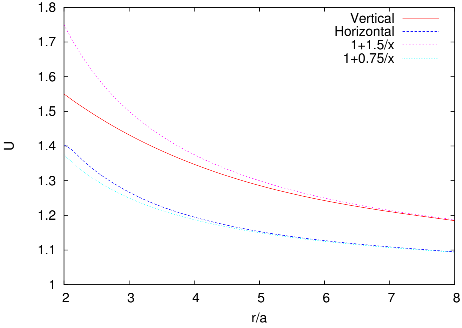

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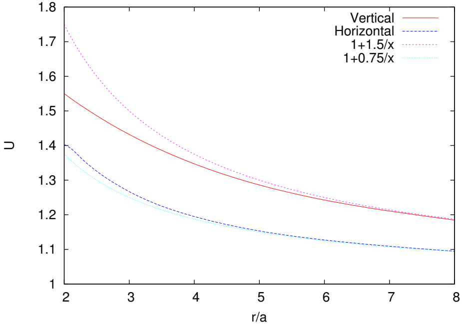

Approximations for large separations

$$U = U_S\left(1+\frac{3a}{2r}+\frac{a^3}{r^3}\right), \quad U_S\left(1+\frac{3a}{4r}+\frac{a^3}{r^3}\right)$$

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Method of Reflections

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Can use computer algebra to go 100 reflections.

Method of Refections: zeroth approximation

Zeroth approximation: each sphere falls as effectively isolated

$$\mathbf{u}^0 = \mathbf{u}^{01} + \mathbf{u}^{02}$$

$$\mathbf{u}^{01}(\mathbf{x}) = \mathbf{U}_1 \left(\frac{3a_1}{4r_1} + \frac{a_1^3}{4r_1^3} \right) + \frac{(\mathbf{U}_1 \cdot \mathbf{r}_1)\mathbf{r}_1}{r_1^2} \left(\frac{3a_1}{4r_1} - \frac{3a_1^3}{4r_1^3} \right)$$

with $\mathbf{U}_1$ isolated fall speed of 1, centred at $\mathbf{x}_1$, $\mathbf{r}_1 = \mathbf{x} - \mathbf{x}_1$.

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And $\mathbf{u}^{02}(\mathbf{x})$ similar.

Method of Refections: first reflection

First refection: Sphere 2 sees $\mathbf{u}^{01}(\mathbf{x})$ in neighbourhood of $\mathbf{x} = \mathbf{x}_2$

$$\mathbf{u}^{01}(\mathbf{x}) = \underset{(1)}{\mathbf{u}^{01}(\mathbf{x}_2)} + \underset{(2)}{\mathbf{r}_2 \cdot \nabla \mathbf{u}^{01}|_{\mathbf{x}_2}} + \underset{(3)}{\mathbf{r}_2\mathbf{r}_2 : \nabla\nabla \mathbf{u}^{01}|_{\mathbf{x}_2}} + \ldots$$

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(3) even smaller, a quadratic flow. Reacts with force quadrupole $O(a_2^2\, \mu a_2\, a_2^2 U_1 a_1/r_{21}^3)$.
Addition to fall speed by Faxen $\frac{1}{6} a_2^2 \nabla^2 u^{01} = O(a_2^2 U_1 a_1/r_{21}^3)$.

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Collecting contributions to fall speed

$$U_1 + U_2 \frac{a_2}{r_{12}} + U_2 \frac{a_2^3}{r_{12}^3} + U_2 \frac{a_1^2 a_2}{r_{12}^3} + U_1 \frac{a_1 a_2^2}{r_{21}^2 r_{12}^2} + U_1 \frac{a_1 a_2^5}{r_{21}^3 r_{12}^3}$$

$+1/r^8 + \ldots$ from 2nd reflection $+1/r^7 + \ldots$ from 3rd + more.

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This loss produces a toroid. In a very tall tank, breaks up into two drops, process repeating (Metzner, Nicolas & Guazzelli 2007).

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http://youtu.be/6I5PNdAHlLI

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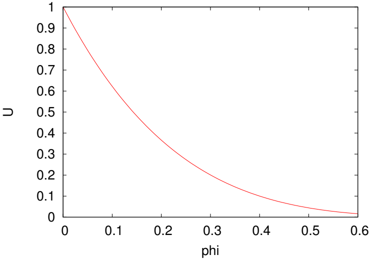

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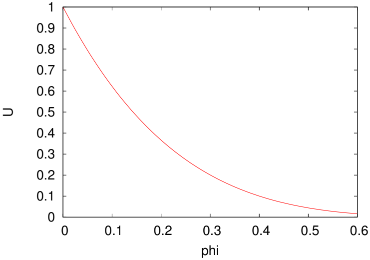

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Note near maximum packing reduced to $\frac{1}{20}$ – like porous media.

Summing interactions with other particles in a dilute suspension

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with Stokes velocity for isolated sphere $U_0 = 2\Delta\rho g a^2/9\mu$

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Far-field form from reflections

$$\begin{aligned}
\Delta U(r)/U_0 \quad = \quad & \tfrac{a}{r} + \tfrac{a^3}{r^3} && \text{1st reflection} \\
& + \tfrac{a^4}{r^4} + \tfrac{a^6}{r^6} + \ldots && \text{2nd reflection} \\
& + \tfrac{a^7}{r^7} + \tfrac{a^9}{r^9} + \ldots && \text{3rd reflection} \\
& + \ldots
\end{aligned}$$

Divergence in naive sum

Naive pairwise addition of disturbances within large domain $r \leq R$, with n spheres per unit volume

$$\langle \Delta U \rangle = \int_{r=2a}^{R} U_0 \left(\frac{a}{r} + \dots \right) n \, dV$$

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$$\langle \Delta U \rangle = \int_{r=2a}^{R} U_0 \left(\frac{a}{r} + \dots \right) n \, dV = O\left(U_0 \phi \frac{R^2}{a^2} \right)$$

$\phi = \frac{4\pi}{3} n a^3$ the volume fraction.

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$$\langle \Delta U \rangle = \int_{r=2a}^{R} U_0 \left(\frac{a}{r} + \dots \right) n \, dV = O\left(U_0 \phi \frac{R^2}{a^2} \right)$$

$\phi = \frac{4\pi}{3} n a^3$ the volume fraction.

The divergence problem:

- Does mean settling velocity depend on size of domain?

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- Does mean settling velocity depend on size of domain?
- Or is it an intrinsic property independent of domain?
  i.e. is pairwise addition naive?

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Hence

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Hence practical result (Salin 1986)

$$\langle U \rangle = U_0(1 - 5.6\phi)$$

Part 2. Sedimentation of particles