Part IIb. A seminar on fluctuations in sedimentation

May 13, 2014

Fluctuations in the velocities of sedimenting particles

John Hinch

DAMTP, Cambridge

May 13, 2014

In collaboration with Élisabeth Guazzelli & Laurence Bergougnoux

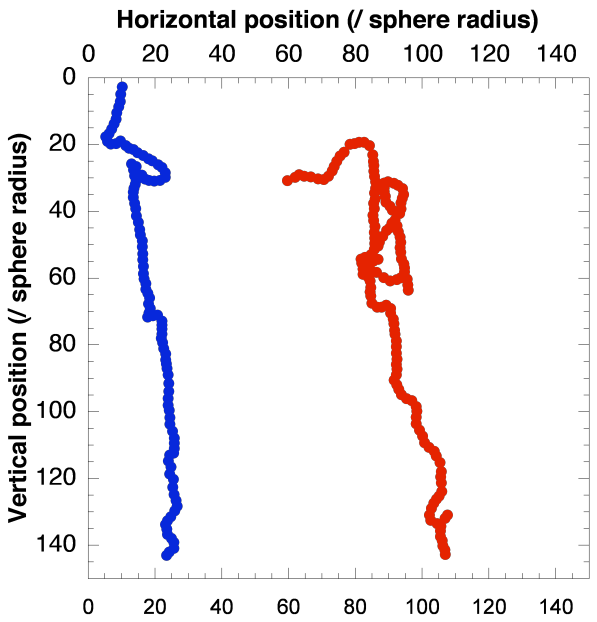
and their students

Guazzelli & Hinch (2011) Ann. Rev. Fluid Mech. **43**, 97–116

Fluctuating velocities

Particles do no fall at a constant speed in a suspension

Trajectories of two spheres at $\phi = 0.3$



Nicolai, Herzhaft, Hinch, Oger & Guazzelli. (1995) Phys. Fluids **7**, 12–23.

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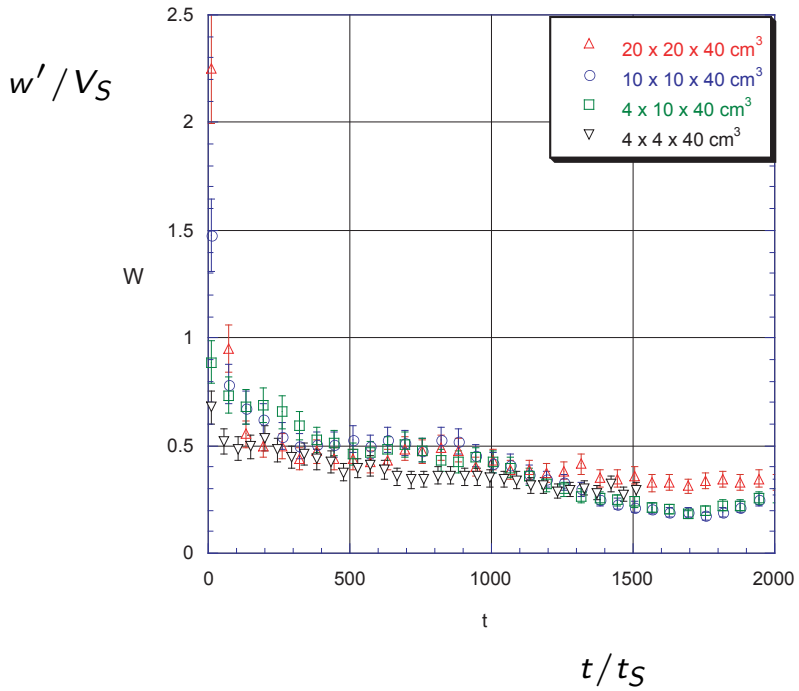
- Experiments: no such dependence



Nicolai & Guazzelli. (1995) Phys. Fluids **7**, 3–5.

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Caflisch & Luke (1985) Phys. Fluids **28**, 759-60.

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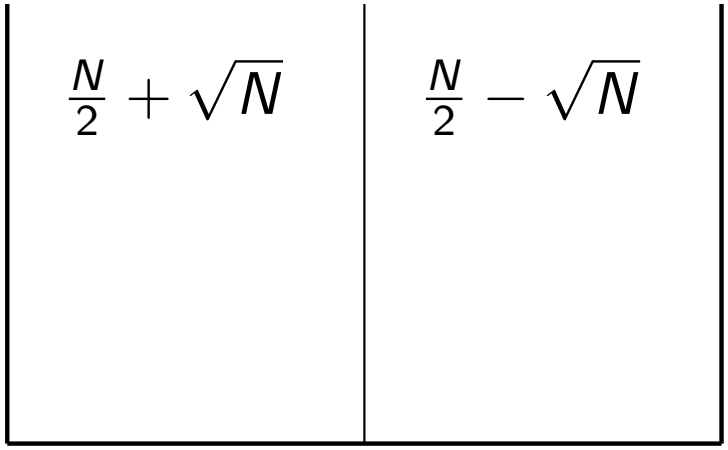

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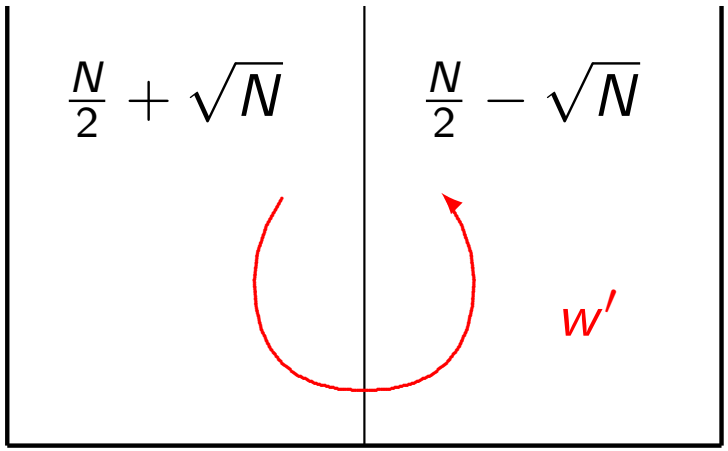

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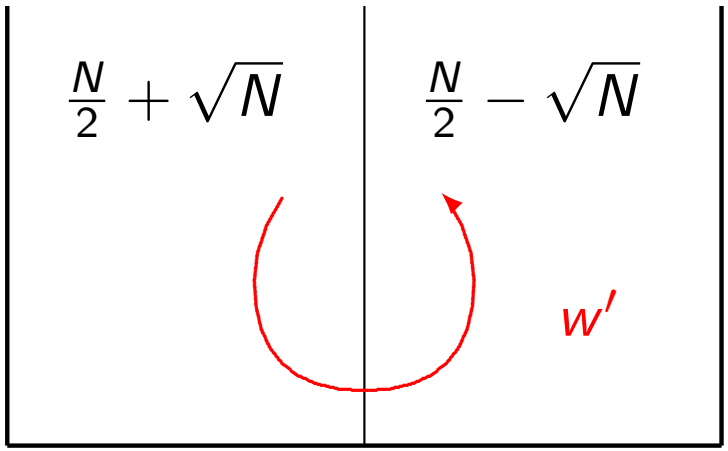

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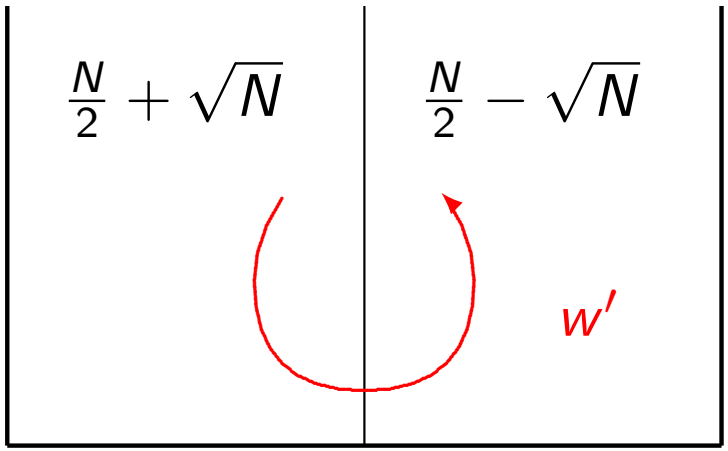

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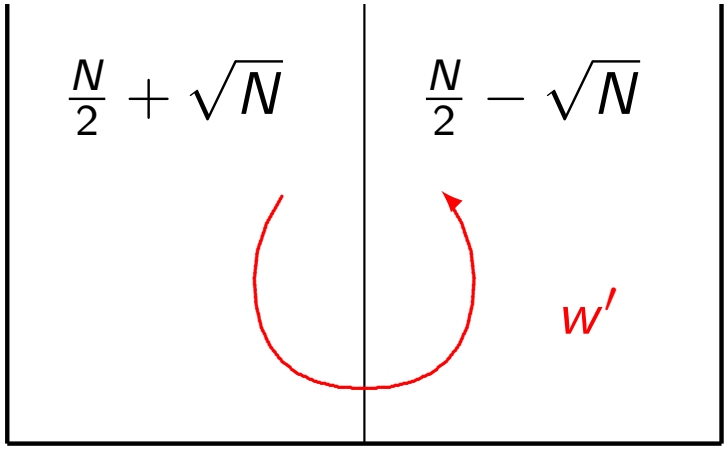

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'Poisson' value

Big effect of a little stratification

Bławdziewicz c1995, private communication - ignored.

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Luke (2000) Phys. Fluids **12**, 1619–21.

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Tee, Mucha, Cipelletti, Manley & Brenner (2002) PRL **89**:054501

Computer simulations to test effect of stratification

Initially stratified

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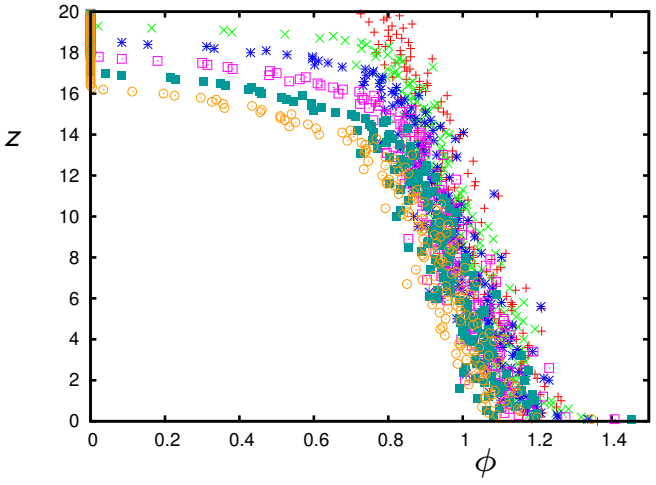

Concentration profile
at different times

$\Delta\phi/\phi = 0.4$, 2500 particles,
average over 40 realisations

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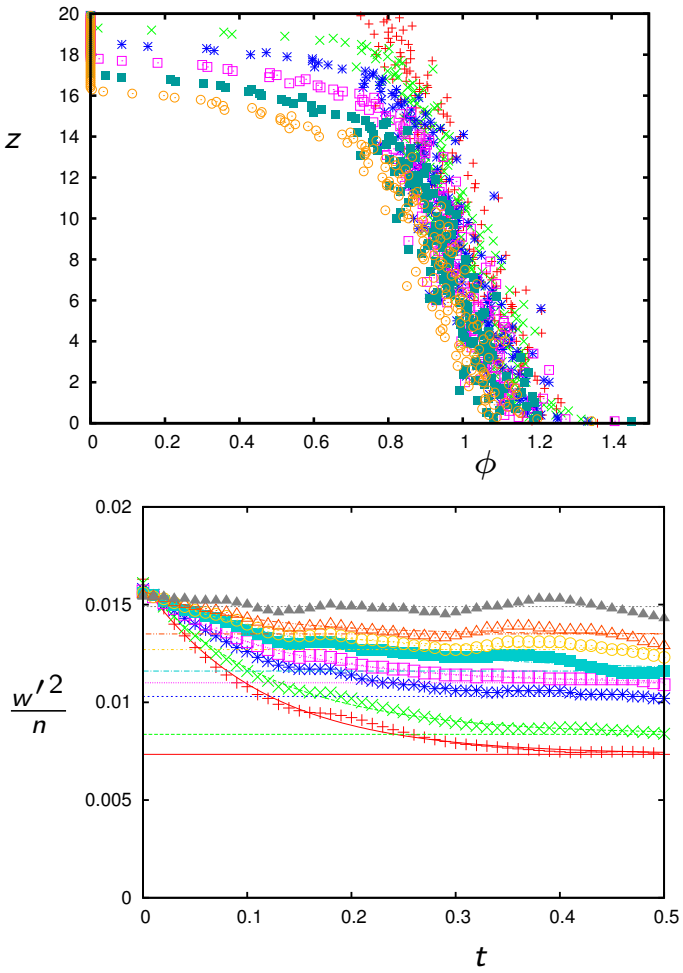

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Velocity fluctuations for $\Delta\phi/\phi = 0,\ldots,0.4$

10^4 particles, $h = 10$

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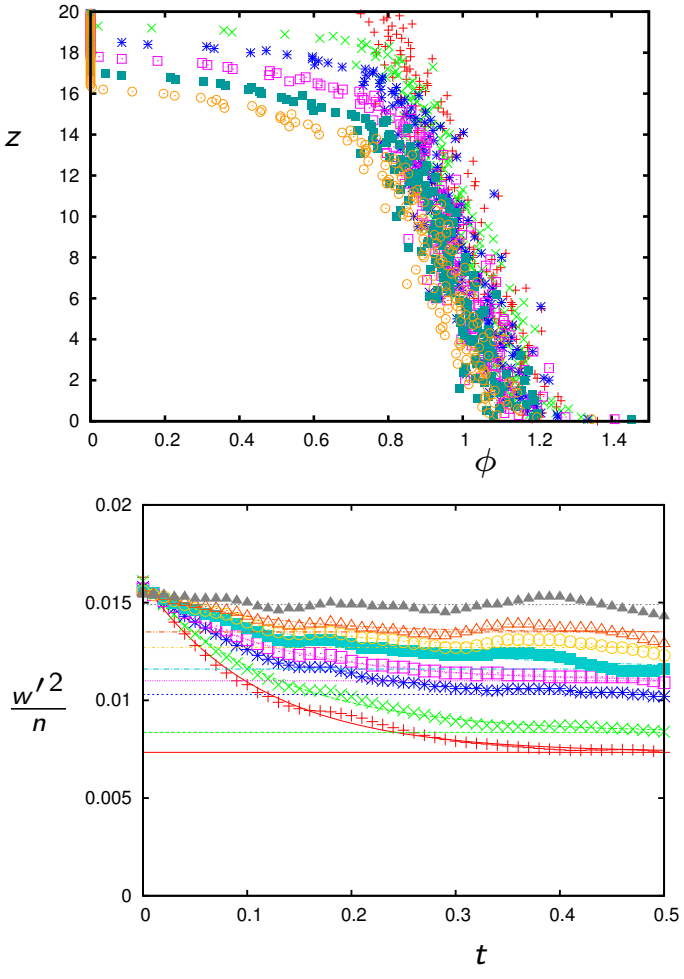

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Decay to a plateau value w'_∞

Chehata Gómez, Bergougnoux, Guazzelli & Hinch (2009) Phys. Fluids **21**: 093304

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Plateau value ${w'_\infty}^2$ plotted against stratification

Computer simulations to test effect of stratification

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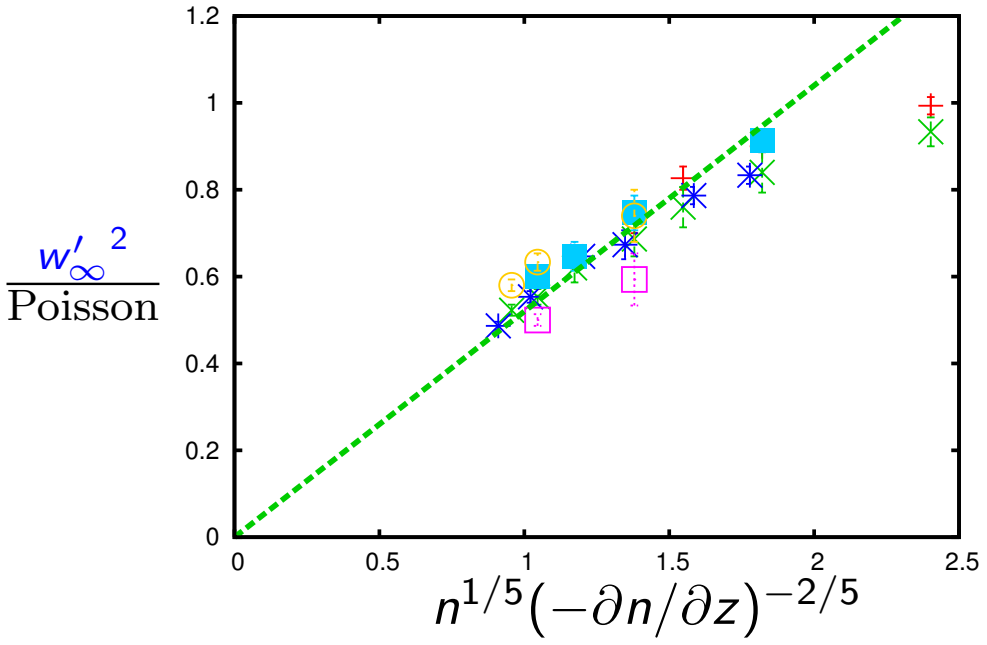
Plateau value w'^2_∞ plotted against stratification



Different n & δx

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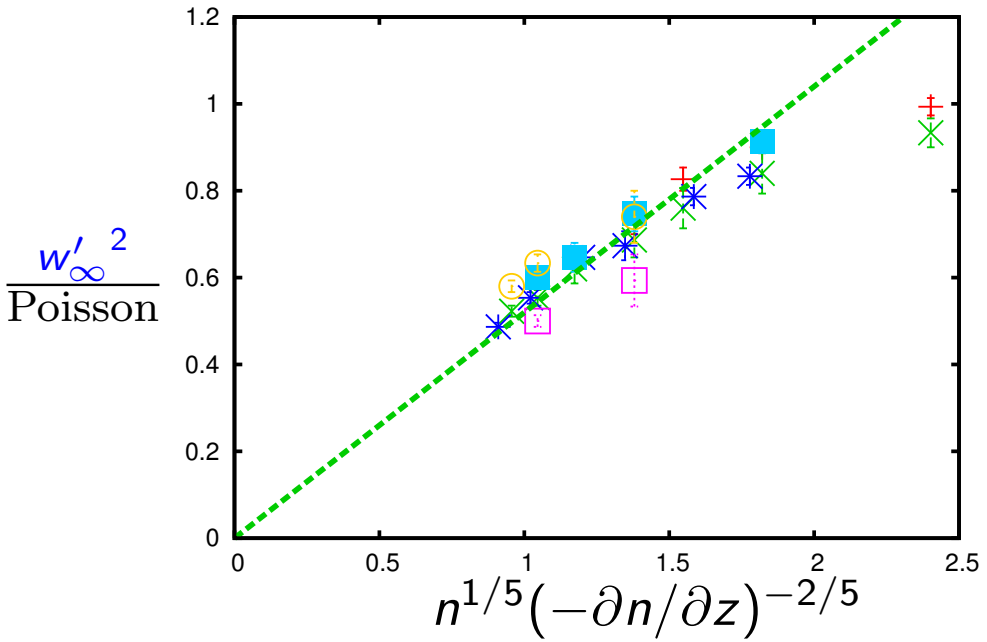

Different n & δx

- Hence

$$w'_\infty = 0.94 V_S \phi^{3/5} \left(-a \frac{\partial \phi}{\partial z} \right)^{-1/5}$$

Chehata Gómez, Bergougnoux, Guazzelli & Hinch (2009) Phys. Fluids **21**: 093304

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Initially uniform - stratified in descending front

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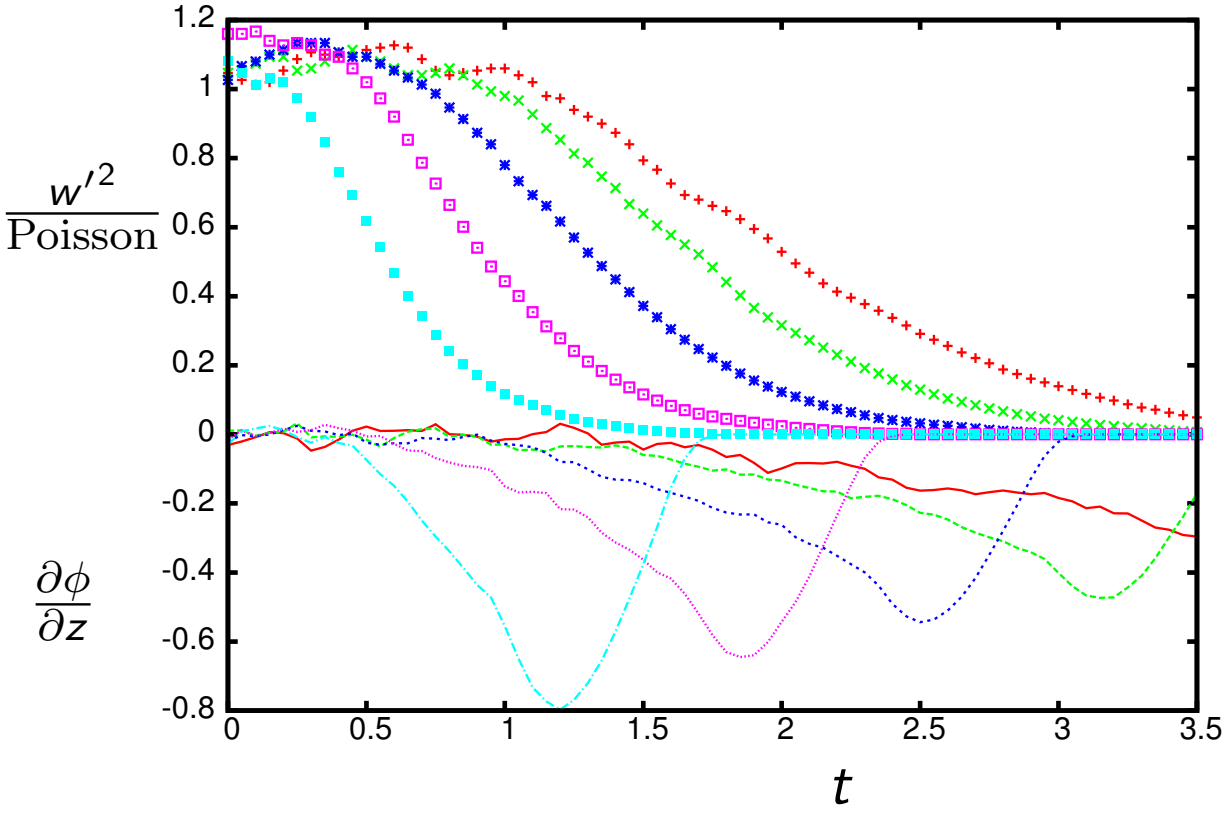
Viewed in windows at different heights: top, bottom



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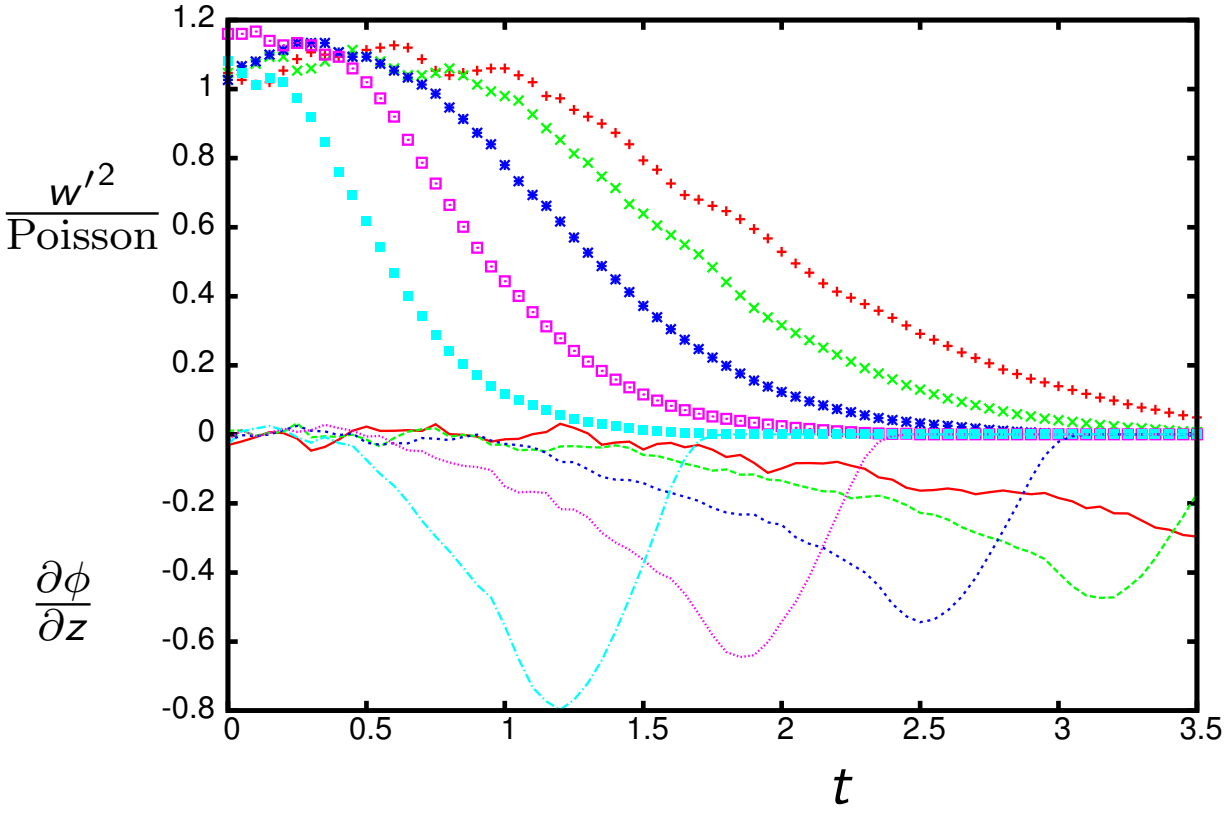

Velocity fluctuations reduced when front arrives in window

Computer simulations to test effect of stratification

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w'^2 in front plotted against stratification

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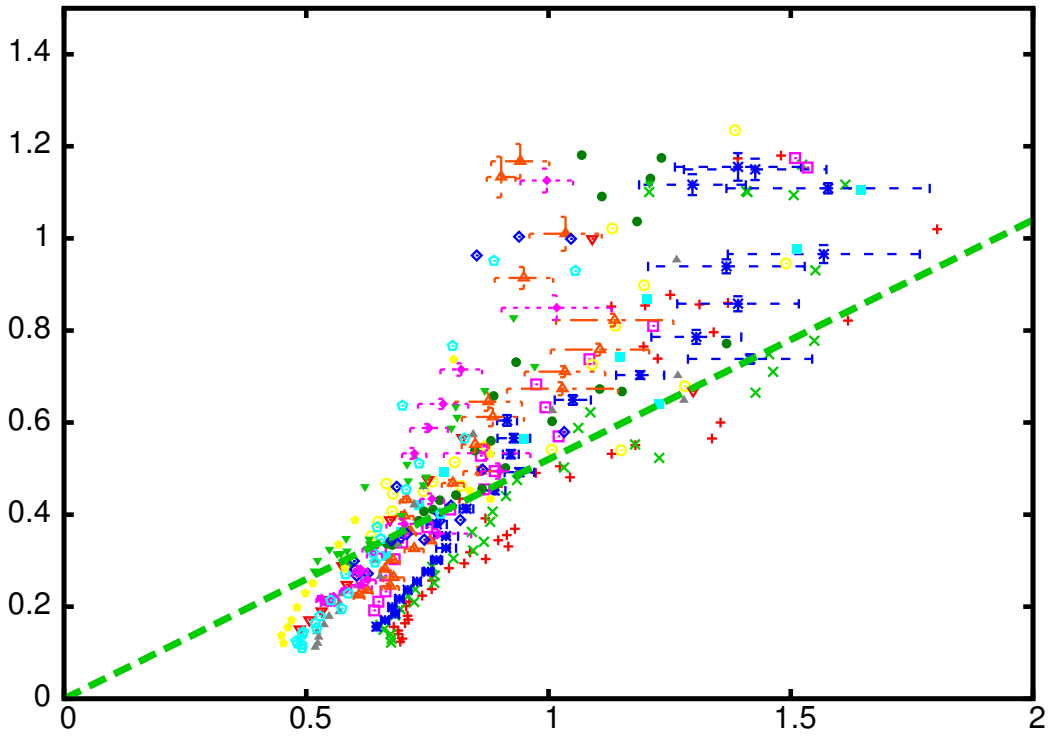
$$\frac{w'^2}{\text{Poisson}}$$



$$\frac{a}{L}\phi^{1/5}\left(-a\frac{\partial\phi}{\partial z}\right)^{-2/5}$$

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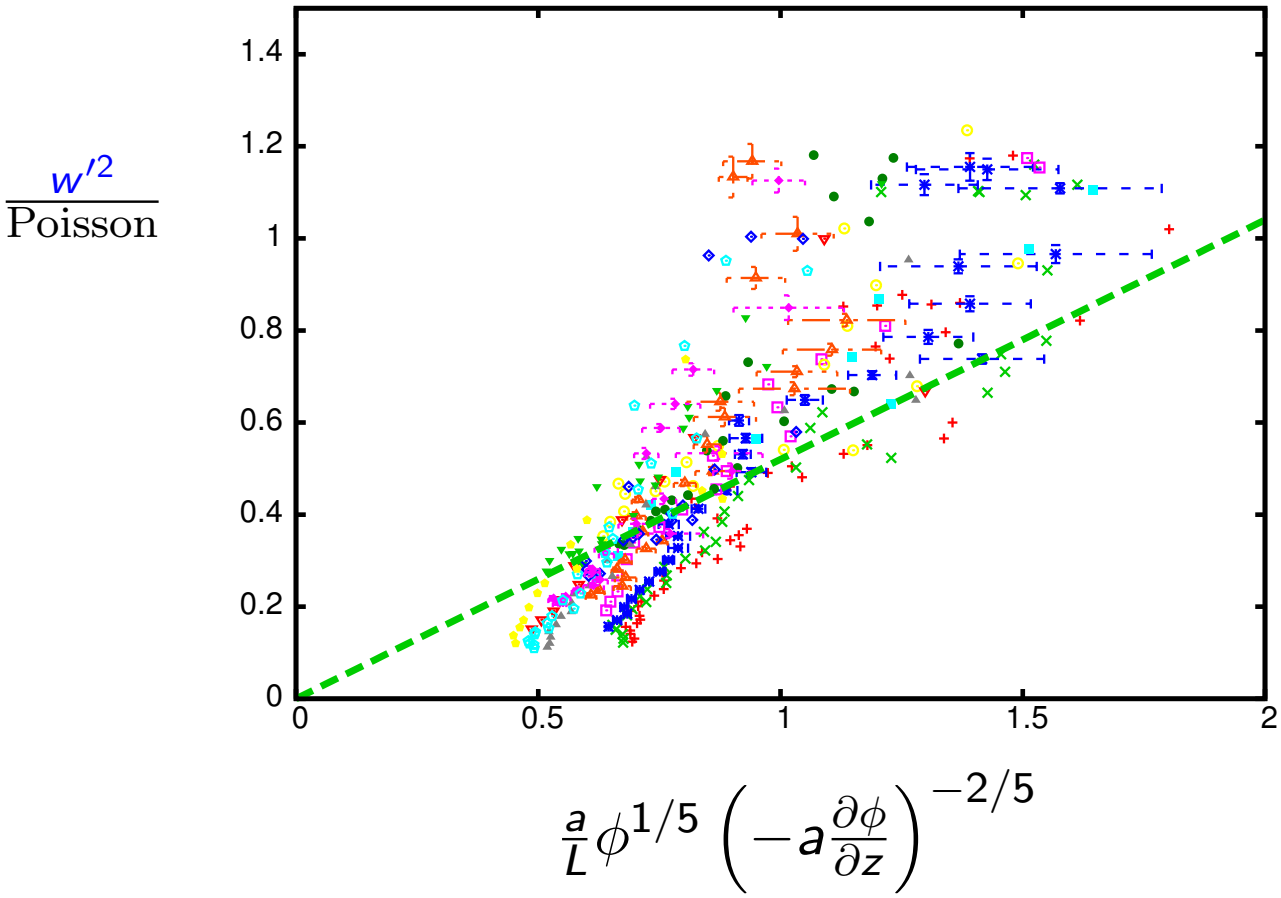
w'^2 in front plotted against stratification



Fair agreement only, but recall time delay for initial value to decay

Experiments

Initially uniform - stratified in descending front

- Four experiments at $\phi = 0.3\%$, with different box size and different particle sizes and densities. View in fixed window.

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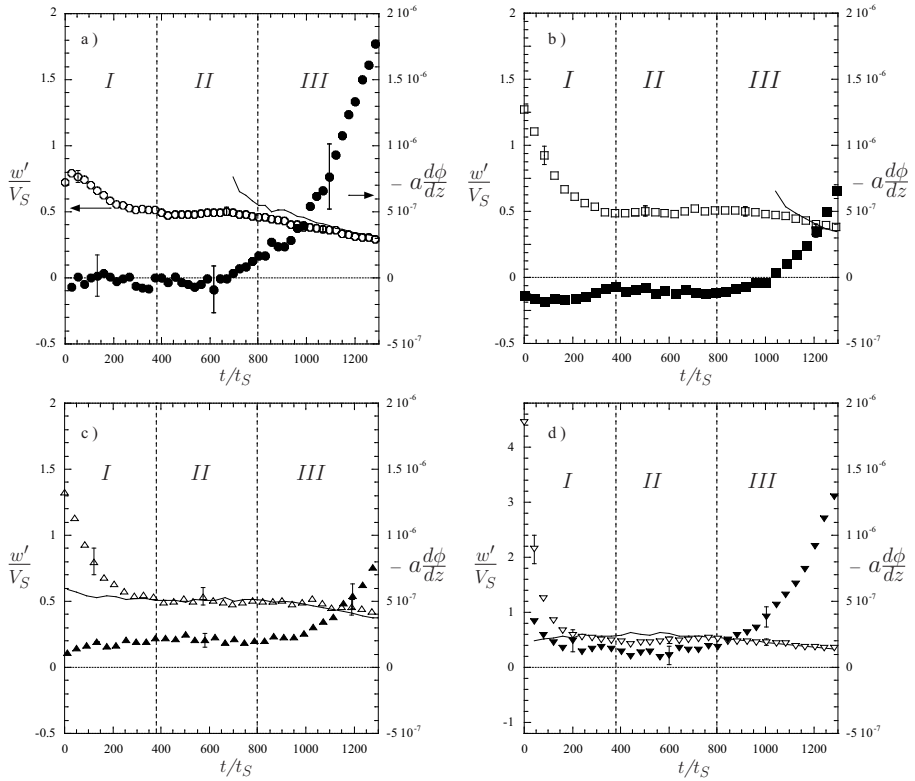
- Open symbols w'/V_S. Filled symbols $-a\partial\phi/\partial z$ (difficult).



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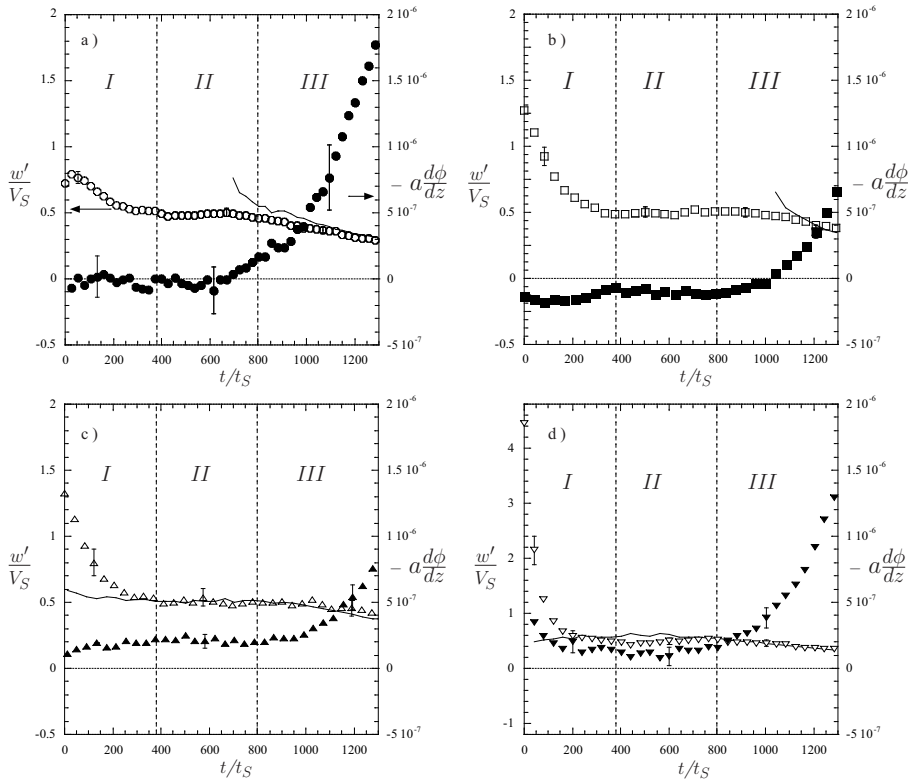

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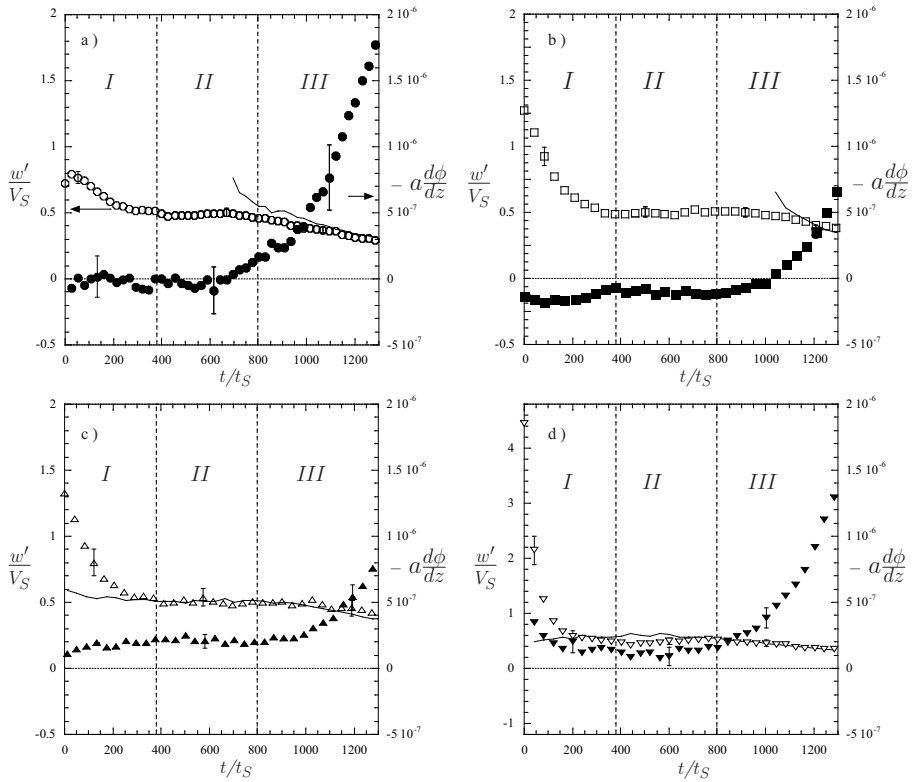

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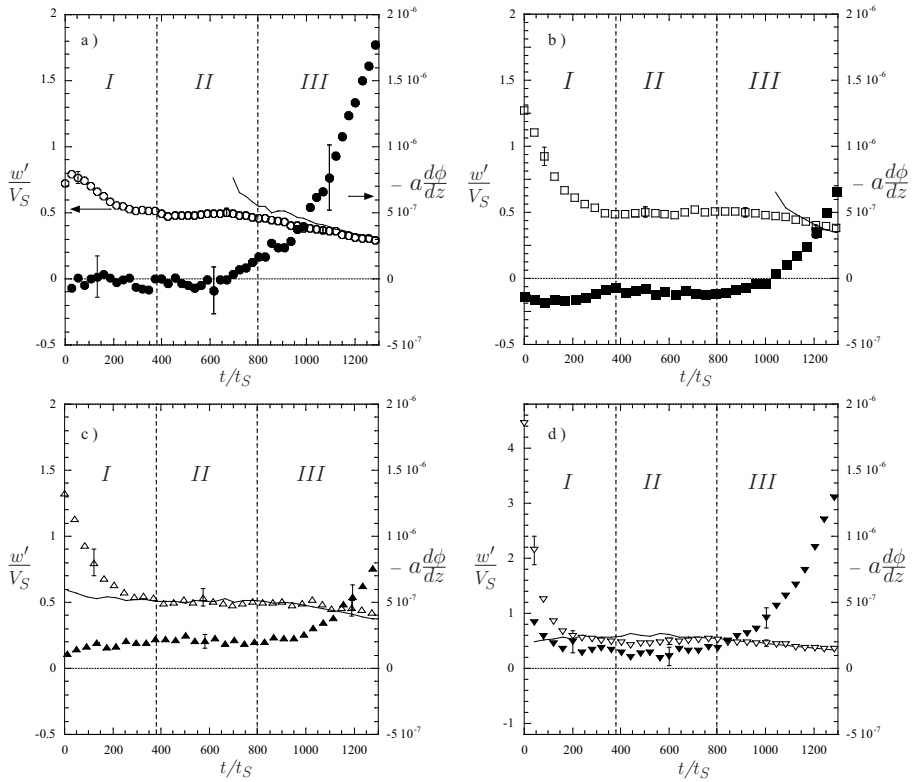
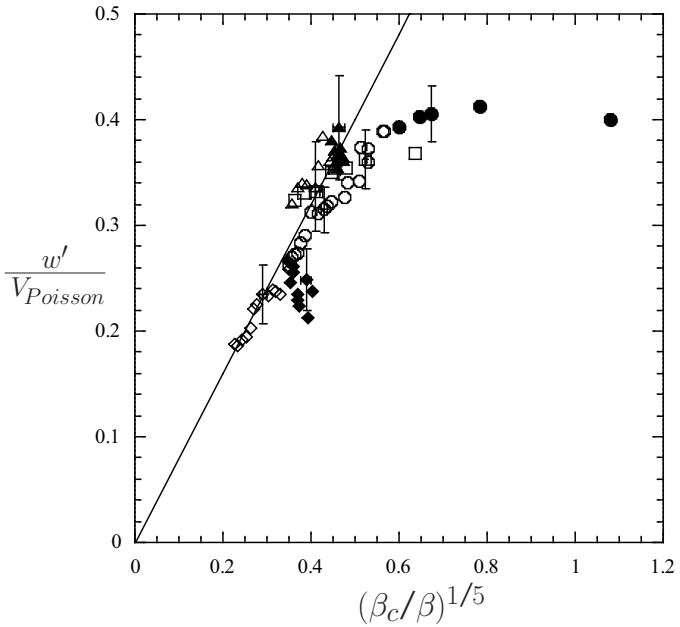


I – Decay of initial state, II – plateau, III – in front

Experiments

Initially uniform - stratified in descending front

- Velocity fluctuations inhibited by stratification



Filled symbols on plateau (II), open in front (I).

Chehata Gómez, Bergougnoux, Guazzelli & Hinch (2009) Phys. Fluids **21**: 093304

Experiments

And initial values are the old divergent scaling

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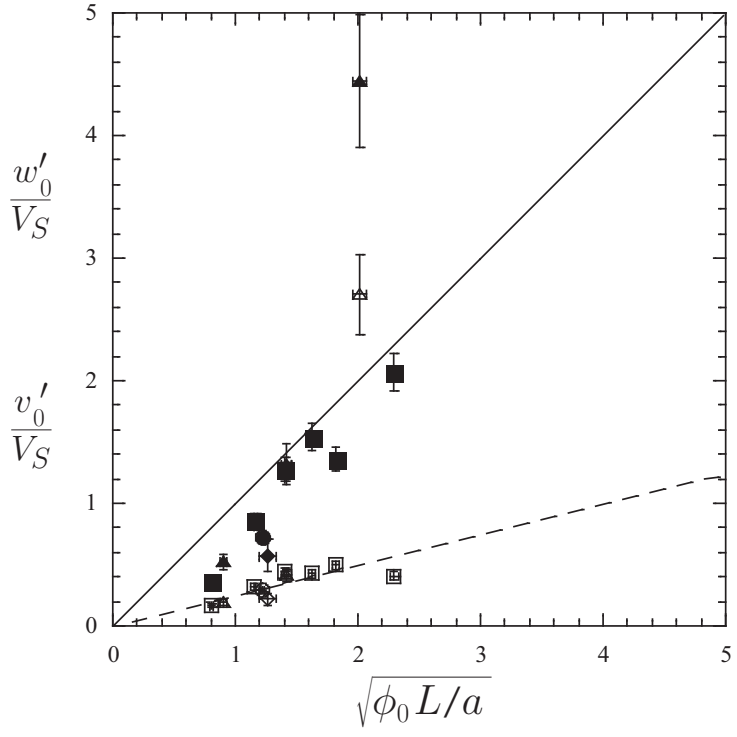

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Mucha & Brenner (2003) Phys. Fluids **15**: 1305-13

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Mucha & Brenner (2003) Phys. Fluids **15**: 1305-13

- Numerical value 2.75 of diffusivity from similarity solution . . .

Diffusing front

- Similarity thickness of front

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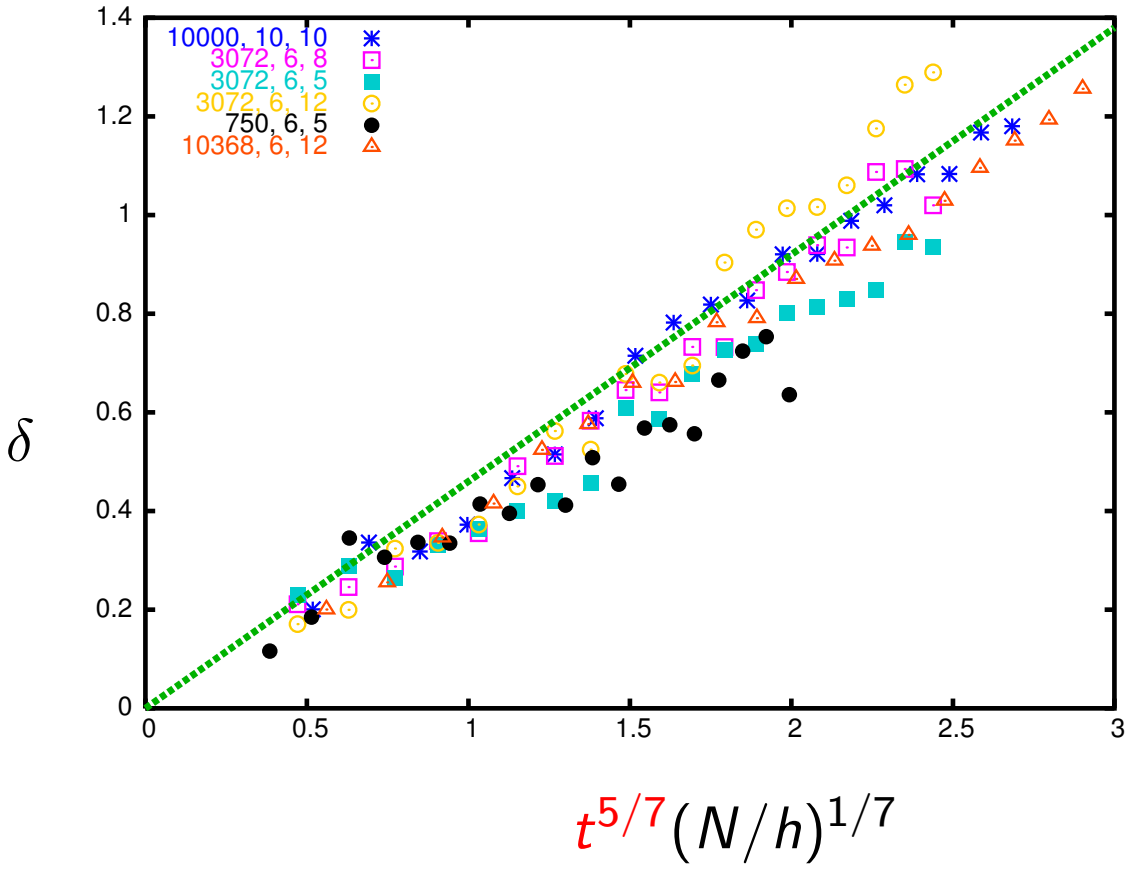
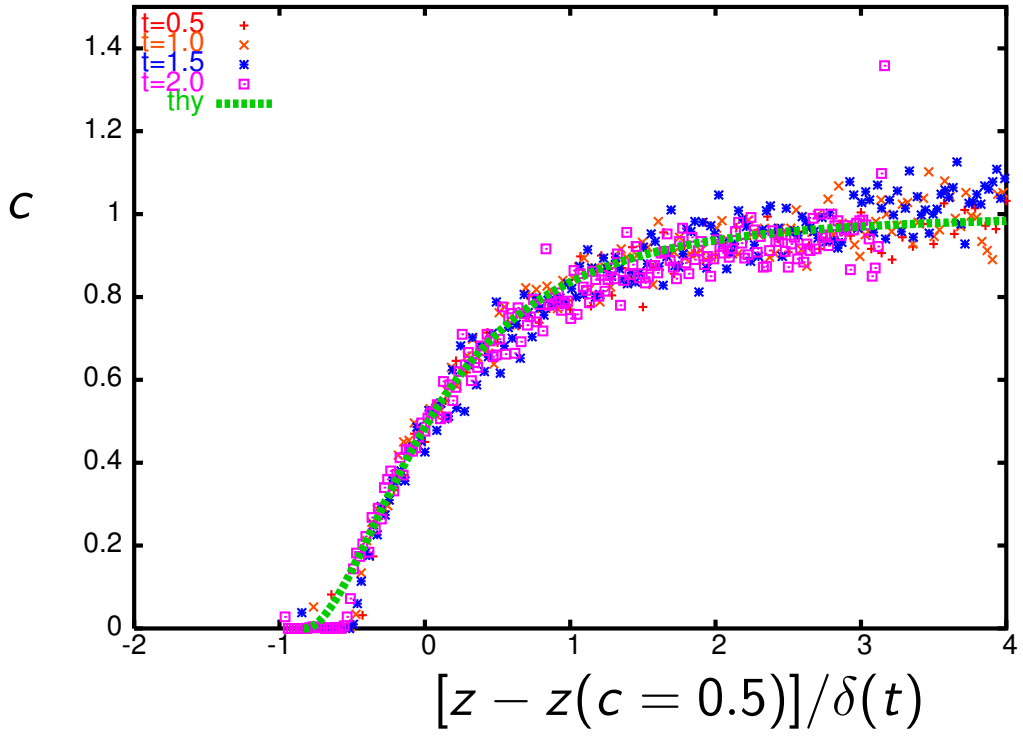


Diffusing front

Similarity plot of concentration profile



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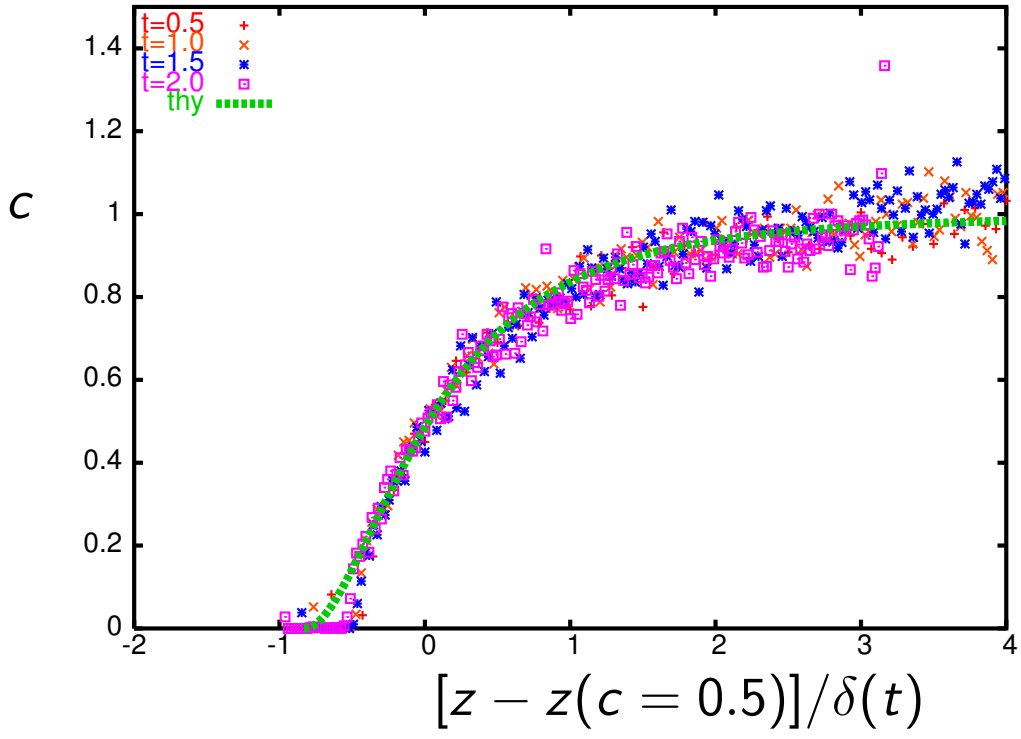

- Nonlinear diffusion equation predicts concentration profile in diffusing front at top of suspension

Open question: effect of small inertia

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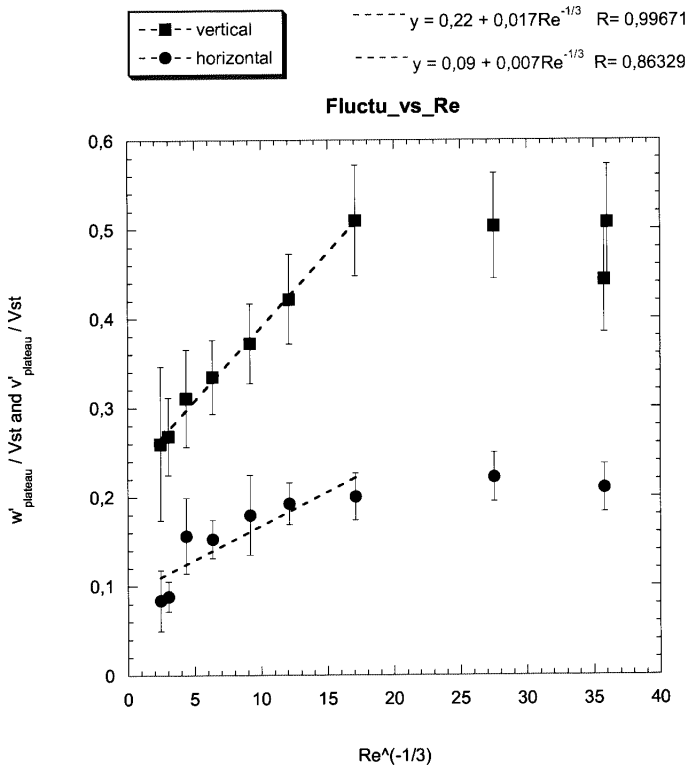

Recent preliminary experiments (Bergougnoux 2011)