Part IIb. A seminar on fluctuations in sedimentation

May 13, 2014

Fluctuations in the velocities of sedimenting particles

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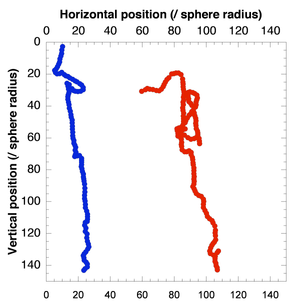
and their students

Guazzelli & Hinch (2011) Ann. Rev. Fluid Mech. **43**, 97–116

Fluctuating velocities

Particles do no fall at a constant speed in a suspension

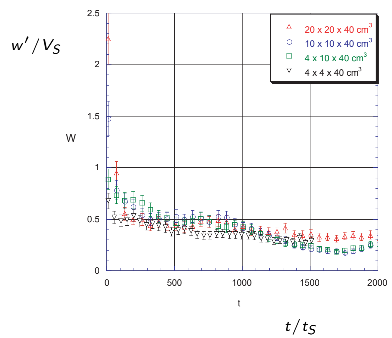
Trajectories of two spheres at $\phi = 0.3$



Nicolai, Herzhaft, Hinch, Oger & Guazzelli. (1995) Phys. Fluids **7**, 12–23.

The divergence paradox

- Theory: depend on size L of box $\qquad w' = V_S\sqrt{\phi\frac{L}{a}}$
- Experiments: no such dependence



Nicolai & Guazzelli. (1995) Phys. Fluids **7**, 3–5.

Theory of scaling

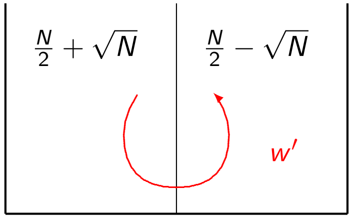
- Dilute: pair separated by r have $w' \sim V_S \frac{a}{r}$,
  so averaging with $p \sim n$ (const)

$$\int w'^2 p\, dV \quad \text{diverges like} \quad V_S^2 \phi \frac{L}{a}$$

Caflisch & Luke (1985) Phys. Fluids **28**, 759-60.

- Explanation

Hinch (1988) *Disorder and Mixing* 153–60



$$w' = \frac{\sqrt{N} mg}{6\pi\mu L} = V_S \sqrt{\phi \frac{L}{a}}$$

'Poisson' value

Big effect of a little stratification

Bławdziewicz c1995, private communication - ignored.

Luke (2000) Phys. Fluids **12**, 1619–21.

- If vertical change in density exceeds statistical fluctuation, then heavy side sinks only to level of neutral buoyancy.
- Blobs smaller than ℓ unaffected, with ℓ given by

$$\ell \frac{\partial n \ell^3}{\partial z} = \sqrt{n\ell^3} \quad \text{so} \quad \ell = n^{1/5} \left(-\frac{\partial n}{\partial z} \right)^{-2/5}$$

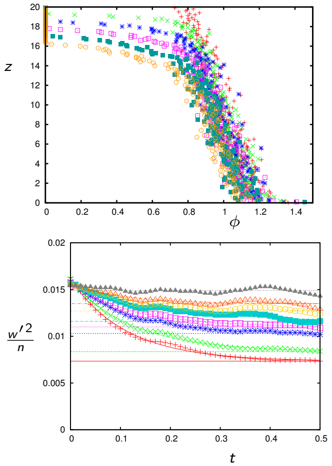
Hence

$$w' = V_s \sqrt{\phi \frac{\ell}{a}} = V_S \phi^{3/5} \left(-a \frac{\partial \phi}{\partial z} \right)^{-1/5}$$

Tee, Mucha, Cipelletti, Manley & Brenner (2002) PRL **89**:054501

Computer simulations to test effect of stratification

Initially stratified



Concentration profile at different times

$\Delta\phi/\phi = 0.4$, 2500 particles, average over 40 realisations

Velocity fluctuations for $\Delta\phi/\phi = 0, \ldots, 0.4$

10^4 particles, $h = 10$

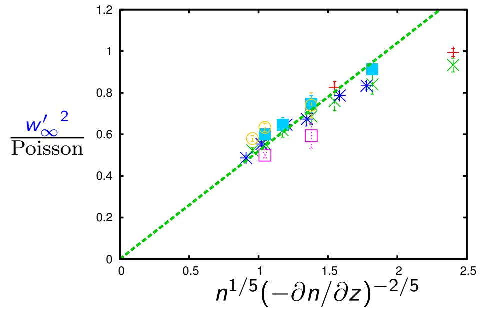
Decay to a plateau value w'_∞

Chehata Gómez, Bergougnoux, Guazzelli & Hinch (2009) Phys. Fluids **21**: 093304

Computer simulations to test effect of stratification

Initially stratified

Plateau value ${w'_\infty}^2$ plotted against stratification



Different n & δx

- Hence

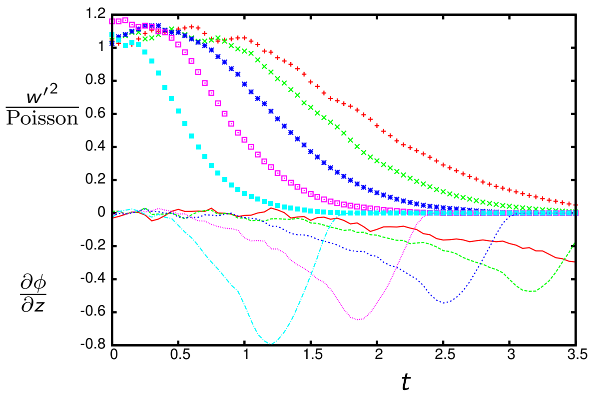
$$w'_\infty = 0.94 V_S \phi^{3/5} \left(-a \frac{\partial \phi}{\partial z} \right)^{-1/5}$$

Chehata Gómez, Bergougnoux, Guazzelli & Hinch (2009) Phys. Fluids **21**: 093304

Computer simulations to test effect of stratification

Initially uniform - stratified in descending front

Viewed in windows at different heights: top, bottom

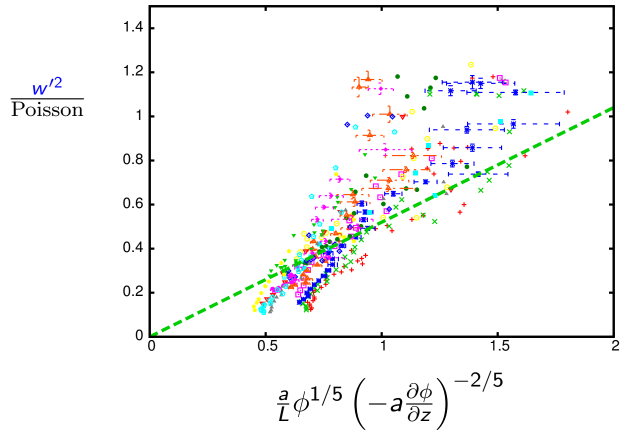


Velocity fluctuations reduced when front arrives in window

Computer simulations to test effect of stratification

Initially uniform - stratified in descending front

w'^2 in front plotted against stratification

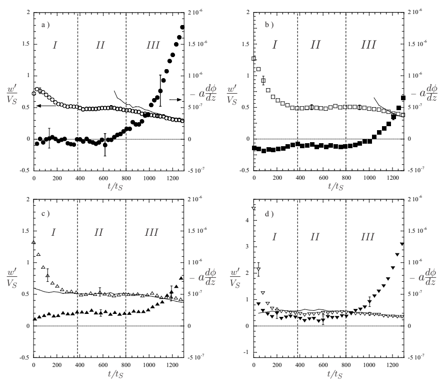


Fair agreement only, but recall time delay for initial value to decay

Experiments

Initially uniform - stratified in descending front

- Four experiments at $\phi = 0.3\%$, with different box size and different particle sizes and densities. View in fixed window.
- Open symbols w'/V_S. Filled symbols $-a\partial\phi/\partial z$ (difficult).

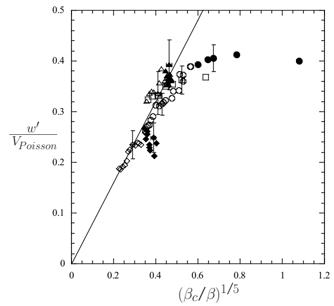


I – Decay of initial state, II – plateau, III – in front

Experiments

Initially uniform - stratified in descending front

- Velocity fluctuations inhibited by stratification



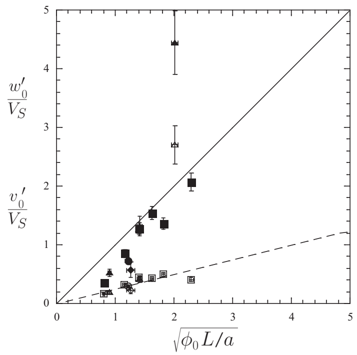
Filled symbols on plateau (II), open in front (I).

Chehata Gómez, Bergougnoux, Guazzelli & Hinch (2009) Phys. Fluids **21**: 093304

Experiments

And initial values are the old divergent scaling

$$w_0' = V_S \sqrt{\phi \frac{L}{a}}$$



Chehata Gómez, Bergougnoux, Guazzelli & Hinch (2009) Phys. Fluids **21**: 093304

Diffusing front

- Does front between top of suspension and clear fluid diffuse?
- Self-diffusivity $D = w'\ell = 2.75 V_s a \phi^{4/5} \left(-a\partial\phi/\partial z\right)^{-3/5}$
- Nonlinear diffusion equation

$$\frac{\partial \phi}{\partial t} - \frac{\partial (V_s \phi)}{\partial z} = \frac{\partial}{\partial z}\left(2.75 V_s a^{2/5} \phi^{4/5} \left(-\frac{\partial \phi}{\partial z}\right)^{2/5}\right)$$

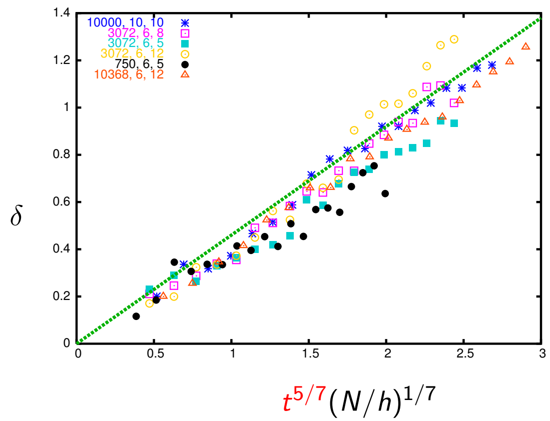
Mucha & Brenner (2003) Phys. Fluids **15**: 1305-13

- Numerical value 2.75 of diffusivity from similarity solution . . .

Diffusing front

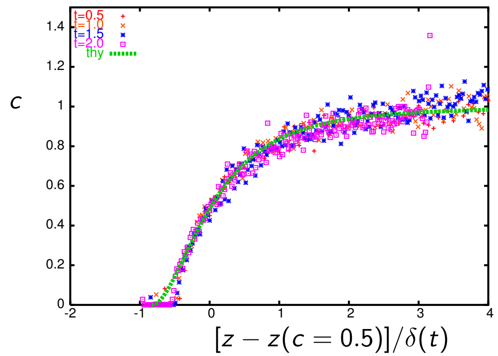
- Similarity thickness of front

$$\delta = 3.07 a \phi^{1/7} (V_s t/a)^{5/7}$$



Diffusing front

Similarity plot of concentration profile



- Nonlinear diffusion equation predicts concentration profile in diffusing front at top of suspension