Introduction

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Mobile particulate systems take many forms. As a first approximation we choose to divide them into three broad classes :– (i) dry granular materials with no fluid between the grains, (ii) fluidised beds with a low viscosity fluid between the grains, and (iii) suspensions with a viscous fluid between the grains. Many phenomena however transcend this crude division. For example the sedimentation of particles in a concentrated suspension is very little different to the relative motion of a liquid through a liquid fluidised bed of fine particles. On the other hand in a gas fluidised bed with coarse particles, the momentum transfer between the particles as they bounce off one another – producing a 'particle pressure' – is much the same as in a fast flowing granular material.

Mobile particulate systems occur in many forms under a variety of names. Colloidal dispersions, emulsions and bubbly liquids are examples of suspensions. While suspended mud in an estuary is a suspension, the mathematical description of its turbulent transport has much in common with that of fluidised beds. Dusty gases, used for example in sand blasting, are similar to fluidised beds. Pneumatic transport for long distance conveyance of particles can be thought of as a horizontal fluidised bed or as a granular material with a liquid. Similarly pastes, slurries and the bed load transport of sand up and down a beach are intermediate between granular materials with a little liquid and dense suspensions. Finally two closely related immobile systems are soils, which share many of the features of static granular materials (although the cohesive nature of the grains introduces important differences), and flow through a porous rock, which is essentially the same as flow through a dense suspension.

1. Suspensions

Suspensions are found in a wide variety of situations in nature and in engineering. Examples include aerosols (suspensions of particles in air) such as sprays, mists, coal dust and particulate air pollution; biological fluids such as blood and milk; household fluids such as paints and emulsions in food; and in industry the processing of fibre composites and paper pulp. A general introduction to suspensions is given by Davis in chapter 2.

The distinguishing feature of a suspension is that the particles in the fluid are small, typically 1 μm in diameter. At this size the particles sediment slowly so that they remain in suspension over the time scale of interest.

The small size of the particles means that the fluid flow around the particles is viscous, i.e. that the ‘particle Reynolds number’ (based on the particle size and the difference in velocities in the immediate neighbourhood of one particle) is small. The linearity of the Stokes viscous flow problem allows much analytic progress (see chapter 3 by Davis) and much computational progress. The linearity also means that the bulk motion of the suspension may rigorously be described by an effective continuum, although the detailed calculation of the effective properties such as the value of viscosity may require approximations.

The small size of the particles also means the colloidal forces such as Brownian motion, van der Waals’, electrical double layer and capillary (possibly variable with surfactants) are often important. For example a 1 μm sphere in water will diffuse in Brownian motion through its diameter in about 1 second.

Compared with granular materials and fluidised beds, the understanding of the behaviour and the theoretical description of suspensions are more advanced. This is partly a consequence of the good comprehension of the colloidal forces and also the linearity of the viscous flow problem. The following review of the behaviour of suspensions is grouped under the different possible motion of the particles :– translation, rotation, deformation and interactions.

1.1. TRANSLATION

Sedimentation. Particles in a suspension sediment at a velocity $mg/6\pi\mu a$ (balancing the weight against the Stokes drag), where a is the size of the particle, equal to the radius of spheres (see chapter 3 by Davis). Thus a 1 μm sphere in water will fall at 1 μms^{-1}. Sedimentation is used as an inexpensive way to clarify liquids or alternatively to recover particles. It can also be used to separate (fractionate) particles of different sizes. Experiments reported by Hoyos* suggest, however, that in a dense suspension (volume concentration in excess of 42%) the particles become interlocked and so all sediment at the same velocity.

An important geophysical application of sedimentation is turbulent transport of a sediment in an estuary, in which the turbulence velocities must exceed the sedimentation velocity in order to keep the particles in suspension. This application was being studied with a variety of augmented turbulence models (models quite similar to those used to describe fluidised beds) by Davies* and by Villaret*, who noted differences with observations when an eddy is shed from the bottom as the flow reverses. Such ejections or bursts from the viscous boundary layer were being modelled by Hogg*.

Migration. In addition to sedimentation, particles may move relative to the flow, or migrate, through a number of different effects. In chapter 7, Acrivos describes a general theory and some particular calculations for migration across laminar stream-

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* Denotes a short contribution, see list on page 257.

lines caused by gradients in concentration (shear-induced diffusion with diffusivity $O(a^2\gamma)$) and caused by gradients in shear-rate (with velocity $O(a^3\nabla\gamma)$). Very crudely, the larger number of 'impacts' from the side with the higher concentration and higher shear pushes the particles to regions of lower concentration and lower shear rates. Stokesian dynamics computer simulations of this phenomenon were described by Morris*.

Other effects which can make the particles migrate in a pipe or channel are :– (i) small but non-zero inertia in the fluid, which drives the particles to an equilibrium distance roughly halfway between the centreline and the wall; (ii) deformation of particles if they are droplets of a second immiscible liquid, which causes the particles to migrate towards the centreline; and (iii) non-Newtonian effects in a slightly elastic liquid, which are little understood, and which may send the particle to the centreline. As an application of the latter, Unwin* described the transport of small particles along a fracture in an oil well, where the migration might lead to a faster settling and so blocking of the fracture, but might also lead to transporting a greater distance in the faster flow of the centreline.

Resuspension. An extreme form of migration is the resuspension of particles which have sedimented to the bottom of a horizontal pipe, as occurs in pneumatic transport when the flow is temporarily shut down. Shaflinger* described experimental observations of turbulent resuspension, and also a laminar wave instability which is not yet understood. Eames* described how a large particle bouncing off the ground could entrain fine dust into the air.

1.2. ROTATION

Many studies of suspensions have consider only spherical particles, perhaps too many studies, theoretical, computational and experimental. The rotation of a spherical particle is not very interesting compared with that of rods and disks.

In sedimentation, non-spherical particles will rotate so that their centre of weight (corrected for buoyancy) is below the centre of drag. If the particle is symmetric so that these two centres coincide, the particles do not rotate in isolation in Stokes flow (see chapter 3 by Davis). The accumulation of small effects can however lead to a slow reorientation, e.g. small inertia of the fluid makes rods turn to the horizontal, small non-Newtonian effects make a rod turn to the vertical, and it was suggested by Shaqfeh in chapter 26 that interactions would make rods turn to the vertical.

In shearing flows disks and rods rotate in a nonuniform way (see chapter 23 by Blanc), spending much of their time nearly aligned with the flow (rods pointing in the direction of the flow and disks in the plane of the flow) and just infrequently they flip over quickly. The preferred alignment leads to an anisotropy of the suspension in its mechanical and other properties, something often sought in manufacturing composite materials.

Fibres (and platelets, although they are less well studied) are used in composite materials because a small volume fraction can have a large effect on the properties (see chapter 24 by Shaqfeh). The fibres have a small effect only in very very dilute suspensions, when $nl^3 \ll 1$ where n is the number of fibres per unit volume and l is their length. In this dilute regime, spheres containing each fibre do not overlap. In the so called 'semi-dilute' regime, when $nla^2 \ll 1 \ll nl^3$ where a is the thickness of the fibres, the volume concentration is small but the presence of the fibres can produce a large effect. The suspension becomes concentrated only when $nla^2 = O(1)$. However in the middle of the semi-dilute regime, when $1 < nl^2a$, the fibres cannot remain randomly orientated and in a nematic phase transition they adopt some considerable degree of alignment in order to pack into the suspension at this concentration (see chapter 23 by Blanc).

1.3. DEFORMATION

In an emulsion, the suspended droplets will deform in a shear flow. If the viscous stress $\mu\gamma$ exceeds the capillary stress Γ/a with surface tension Γ, the drop will be stretched and will break (except for very slippery low viscosity drops). Thus one can calculate how fast one needs to stir in order to ensure that the drops are smaller than a required size, or equally that the surface area for reactions is sufficiently large. Manga* reported some experiments and computer simulations of two drops coalescing in sedimentation as a result of the deformation.

1.4. INTERACTIONS

Hydrodynamic. The linearity of the viscous Stokes flow problem makes analytic progress in studying the hydrodynamic interaction between two particles possible, if algebraically difficult. Davis describes in chapter 3 how the interaction between two spheres can be calculated using bispherical polar coordinates, a method of reflections for far field interactions and a lubrication theory for close separations. There is also the Stokesian dynamics approximation by Brady, multipole-collocation methods reported by Bławzdziewicz* and boundary integral methods. Feuillebois* also showed how the reciprocal theorem can be used to obtain results for slightly deformed spheres from the solution for undeformed spheres.

When trying to sum the effects of many particles in a suspension, the long range nature of hydrodynamic interactions (like r^{-1} in sedimentation and r^{-2} in a shear flow) can lead to divergent integrals which must be renormalised, i.e. one must recognise the existence of strictly multiparticle effects such as back flow in sedimentation which hinders the settling.

Aggregation. Colloidal forces, in particular van der Waals', can make the particles in a suspension stick together if they come sufficiently close. They can come into one anothers' neighbourhood either through Brownian motion (giving Diffusion Limited Aggregation) for submicron particles, and for larger particles through differential sedimentation (see chapter 18 by Davis) or a shearing flow. After forming pairs, larger aggregates, often fractal, grow rapidly in time. Large aggregates sediment faster. Thus 'flocculants' are often added to a cloudy suspension in order to clarify the liquid quickly, although there is a compromise in that a sediment of large fractal aggregates is mostly 'holes' between the particles, i.e. there is liquid still to be recovered. To stop colloidal particles sticking together, i.e. to make what is known as a 'stable colloid', the particles can be given a repulsive electric charge or a hairy coating of polymer which gives a stearic repulsion. Gondret* reported some experiments in oscillating shear where small inertial forces produced long chains of particles parallel to the vorticity axis.

Filtration. Filtration can be thought of as an interaction between the particles and a large object, the fibres of a filter. The filtering of very fine particulate matter is increasingly important for ‘clean rooms’ in certain industries. The problem with filtering is that the carrying fluid goes around the obstacles and often takes the particles with it. Thus as a result of hydrodynamic interactions the collision efficiency (the cross-section of the incoming flow with particles which will be captured divided by the cross-section of the obstacle) can easily be less than 10% (see chapter 18 by Davis). Small submicron particles deviate from the streamlines through Brownian motion and so are captured more efficiently, while particles of larger than $10\,\mu$m in air deviate through inertia. Cigarette smoke lies in the gap which is not filtered efficiently.

Filters operate in a number of modes. Davis describes in chapter 18 cross-flow filtration. Ghadaglia* and de Arcangelis* described some curious observations in deep-bed filtration in which a packet of particles penetrates further than the same number released one at a time, perhaps due to relaunchable ‘hydrodynamic captures’ in addition to ‘geometric capture’ when the particles are too big to pass through the pores. Pereira Gomes* described a filtering device to measure air pollution. Related to filtration, Hoyos* reported some concentration effects in Field Flow Fractionation.

1.5. RHEOLOGY

It can be shown that a suspension can be described by an effective continuum with properties such as a viscosity. Acrivos describes in chapter 6 that the viscosity cannot be a universal function of just the volume fraction, even for large non-colloidal spherical particles. Shaqfeh describes in chapter 24 how fibres can have a very large effect on the viscosity (increasing like nl^3) with an example of a 0.1% volume concentration of fibres 600 times longer than their thickness producing an 18 fold increase in the viscosity. This can produce large vortices upstream of an orifice, as reported by Mongruet*. Blanc in chapter 25 describes how pastes can have a yield stress, below which they do not flow, and above which they can show both a shear thinning and thickening. The thinning is due to the flow destroying aggregates and also to aligning the microstructure with the flow. The thickening can be due either to an order-disorder transition or to solid contacts between particles in clusters which become jammed across the apparatus, as reported by Boersma*. Finally Shaqfeh describes in chapter 26 some non-local, i.e. non-continuum, effects when the length of the fibres is comparable to the height of the layer in which they are flowing. Some dynamic light scattering measurements in a thin film of size comparable to the particles was reported by Lobry*.

1.6. HYDRODYNAMIC DISPERSION

In non-Brownian systems there can still be fluctuations in the velocity of the particles due to the constantly changing configuration of the neighbouring particles. In chapter 22 Davis describes how this leads to a spreading of the front between a sedimenting suspension and the clear fluid above it, with a gradient diffusivity $O(7U_0a)$, where U_0 is the sedimentation velocity of an isolated particle. Salin* and Martin* suggested that an alternative analysis of the same data could give $O(20U_0a)$. In the interior of the sedimenting suspension, Guazzelli* reported observations of large fluctuations $O(2U_0)$ which produced a tracer diffusivity $O(U_0a)$.

Cunha* showed some computer simulations of this, which reproduced the experimental correlation time and anisotropy, but which had the size of the fluctuations increasing proportional to the size of the box. Nicolai* reported that experimentally the fluctuations were independent of the size of the container. This leaves unsolved the difference between placing the particles at random in the computer simulations and stirring the suspension in the laboratory. Finally Rutgers* presented experimental observations of very large spatial structures of fluctuations in sedimentation even at the very low concentration of 0.05%.

2. Fluidised beds

Fluidised beds are used extensively in catalytic chemical reactors for gases because of their large surface area for reactions and ease for removing and rejuvenating the catalyst. They are also used in combustion of coal and turned horizontally in pneumatic transport of particles. A general introduction to fluidised beds is given by Davidson in chapter 4.

The essential feature of a fluidised bed is that a fluid, liquid or gas, moves upward through the particles so that the bed can remain stationary even though the particles are falling under gravity relative to the flow. Usually the Reynolds number of the flow is high, being small only in liquid fluidised beds of fine particles.

Although the bed remains stationary, the particles within the bed are in active motion, frequently bouncing off one another. This random motion is sometimes described by a 'particle temperature'. As the particles collide, they exchange momentum, and this has been described by a 'particle pressure'.

2.1. GOVERNING EQUATIONS

Our understanding of fluidised beds comes mainly from experiments. A complete theory for fluidised beds is not yet fully established, although there are some successful fragments. Thus it is clear that one needs conservation equations for the mass and momentum for the two separate intermingled phases, the particles and the fluid, see for example chapter 11 by Sergeev. There is a good empirical relation, the Richardson-Zaki 'law', for the drag forces between the phases in terms of their relative motion (although Cody* reported some curious discrepancies when changing from small catalytic particles to glass spheres of the same size). It is not clear, however, how to describe the particle pressure, whether it is just a local function of the relative flow and the volume concentration of particles, or whether it should be found from an additional traditional temperature equation. Equally the need for viscosities and diffusivities is not completely certain, although Davidson reported that moving paddles through a fluidised bed met with a resistance corresponding to a viscosity of 1 Pa s. It is to be hoped that experiments can be designed to test in isolation components of candidate governing equations. Poletto* reported some inconclusive attempts to fit an effective density and effective viscosity. Perhaps one could use forced small amplitude waves in narrow (and therefore one dimensional) beds. Some preliminary results along these lines were presented by Nicolas* for unforced instabilities. It is worth noting that, for the different system of a bubbly liquid, Lammers* demonstrated that the KdV equation could predict quantitatively details of shock waves. Experimental techniques in fluidised beds are not easy, although Demon* reported an electrical capacitance method in a tomographic mode to measure spatial distributions of particle concentrations, and Poletto* reported the use of hot-wire methods. At this time, it seems to me that the extensive and untestable analyses of weakly nonlinear one-dimensional waves (see chapter 14 by Sergeev) cannot be justified.

2.2. FLUIDISATION

To fluidise a bed of particles, a fluid is pumped in at the bottom of the bed through a porous plate carefully designed to produce a uniform flow, see chapter 4 by Davidson. At low flow rates nothing happens. As the flow U is increased, the bed expands a little, with part of the weight of a particle being supported by the fluid drag and part through particle contacts. At a critical flow U_m, the minimum fluidisation velocity, all the weight of the particles can be supported by the fluid drag, and at this point the bed starts to behave like a fluid in that it can be stirred and that a heavy body will sink through it. Now because the drag is higher for a dense packing of the particles, as the flow increases further the fluidised bed expands (up to the point where the fluid drag on an isolated particle exceeds its weight when the bed is blown out of the container). This behaviour is seen in liquid fluidised beds and in gas fluidised beds of fine particles (less that $100\,\mu$m, called group A). Note very fine particles can be cohesive and need to be vibrated to assist fluidisation, as reported by Russo*. Uniformly fluidised beds are however not common, because they suffer an instability which forms bubbles. The onset of bubbling occurs at the minimum fluidisation flow with group B particles (larger than $100\,\mu$m) in air.

2.3. INSTABILITY

Uniformly fluidised beds are unstable. Consider a small disturbance to the velocity of the particles with variations only in the vertical. Now at those locations just above upward velocity disturbances and just below downward disturbances, the concentration of the particles must be increasing by mass conservation. If one first ignores inertia in the momentum equation for the particles, then the increase in concentration would cause an increase in the drag and hence an increase in the velocity of the particles. Such an increase in time at locations where the velocity disturbance vanishes at that instant has precisely the correct phase to produce an upward propagating kinematic wave which does not change amplitude. Growth in amplitude comes from the neglected inertia. Without inertia, the drag law makes the velocity disturbances of the particles proportional to the instantaneous concentration disturbance. With inertia, the velocity disturbances lag a little. Thus the maximum rate of increase in the velocity occurs not when the velocity disturbance vanishes but a little later, i.e. when it is already positive. Hence the amplitude grows.

The above argument has neglected the effects of particle pressure in the momentum equation. One would expect the particle pressure to increase with concentration, through more impacts and stronger fluctuations of the fluid drag, at least at low and moderate concentrations. A region of enhanced concentration would then be at a higher pressure, and this would drive a flow to even out the disturbance, i.e. particle pressure has a stabilising effect. It has been suggested that the particle pressure might decrease at high concnetrations, which would be destabilising.

A linear stability analysis finds that short waves grow faster. A shortwave cut off can be obtained by introducing viscous terms in the momentum equation for the particles.

If the instability remains one-dimensional, then it grows until the regions of high concentration become 'open random' packed with the particles moving upwards relatively slowly. In between these regions, particle rain down without much interaction. The instability remains one-dimensional, however, only in narrow pipes. In wider containers, there is a secondary instability of a gravitational overturning caused by a dense heavy region of the bed finding itself above a light region.

Jackson (private communication) has recently suggested that liquid fluidised beds differ from gas ones only through the later development of this secondary instability, i.e. both types of bed suffer the same primary one-dimensional instability of a uniformly fluidised bed, and both go on to suffer the same secondary overturning. The difference occurs only in that density disturbances continue to increase in the gas fluidised bed until a full bubble develops, whereas there is no intensification of the density disturbances in a liquid fluidised bed and so no bubbles. This difference arises through the slower growth rate of the primary one-dimensional instability in liquid beds, which is due to the lower fluidisation velocity required, which is due to the higher fluid density giving greater drag. Note that the primary instability has the mechanism to intensify the density disturbances whereas the secondary gravitational overturning only rearranges these density disturbances.

2.4. BUBBLES

Instabilities in fluidised beds rapidly develop into bubbles, which adopt a spherical cap form in wide beds and become slugs which fill narrow pipes, see chapter 13 by Davidson. They can be observed in the interior of a bed using X-rays. Some time-dependent two-dimensional computations of certain two-phase equations reported by Balzer* also developed bubbles. Bubbles in fluidised beds are very similar to bubbles in a normal liquid with a low surface tension. It is found experimentally that spherical cap bubbles of radius R rise at $0.71\sqrt{gR}$ and that slugs in a pipe of diameter D rise at $0.35\sqrt{gD}$. While the reacting gases are passing preferentially through the bubbles, they are not near the catalytic particles, and this makes the bed inefficient. The bubbles do however stir the particles efficiently. The slugs also enhance pneumatic transport. In an industrial plant, the bubbles coalesce and can grow to 1 m thus producing vibration forces of 1 tonne weight as they break the surface. Bubbles breaking the top surface can also concentrate their momentum into a relatively small volume, which leads to a few particles being flung very high into the 'freeboard' above.

2.5. OTHER

A novel two-phase model of fluidised beds was proposed by Davidson (chapter 13), consisting of one phase of the bubbles and a second phase of a bed at minimum fluidisation. This model suggests that all the extra fluid flow above the minimum for fluidisation goes directly into bubbles.

Sergeev (chapter 16) described how an axial magnetic field applied to a bed of magnetisable particles can increase the stability of the bed, permitting much higher flow rates through a uniform bed. The effect of the magnetic field is to create an effective pressure which increases with concentration. Then a region of higher concentration is at a higher pressure, which drives a flow away from this region, which reduces the concentration.

In fast flowing turbulent beds, particles can migrate so as to be more concentrated near to the wall, as described by Isle*. This can be thought of as a result of more violent turbulence in the centre throwing more particles to the boundary than the quieter region near the walls return.

In an investigation of pressure disturbances in the air, Davidson (chapter 4) reported low velocities of propagation $10\,\mathrm{ms}^{-1}$, because the compressibility of the air has to move the inertia of the particles, i.e. $c = \sqrt{\kappa_{air}/\phi\rho_{particles}}$ with ϕ the volume fraction of particles.

3. Granular materials

Examples of mobile granular materials can be found in the pharmaceutical industry where 80% of the products are processed as dry powders and pills; in materials processing with the preparation of ceramics; in the food industry with powders and grains; in civil engineering with sand piles, dredging, landslides and seismic liquification of soils; in geology in sorting layers of boulders and stones; and in log-jams of logs on rivers and in ice-floes on the sea. A general introduction to granular material is given by Savage in chapter 5.

3.1. CONTACT FORCES

Granular materials have been studied with computer simulations and experiments. Some experiments have been performed in model two-dimensional systems which permit an examination of the interior. For computer simulations it is often necessary to have a good model of the contact forces between the particles. In chapter 10, Walton describes how real contacts start with a (nonlinear) Hertzian elastic response and change over to a plastic deformation (with hysteresis) which leads to an energy loss and thus a coefficient of restitution e less than unity. As the particles will be rotating, there is a complicated tangential force with some regions of the contact sliding and others rolling. For collisions between pairs of spheres, the overall change between the initial and final rotational and translational velocities is shown to be described well by a simple three parameter semi-empirical relation. An accurate description of the simultaneous collision of more than two spheres, as well as the interaction between two non-spheres, is not available.

3.2. RAPID GRANULAR FLOWS

Traditionally the motion of granular materials is divided into rapid granular flow and quasi-static motions. It may however be more convenient to consider fast, slow and static materials. For rapid granular flows in which particles are bouncing off one another in an excited random motion, a theory has been developed by adapting the classical kinetic theory of a gas with particles replacing molecules, see chapter 9 by Savage. The validity of this approach can be questioned because the particles have fluctuating velocities comparable with the variations of the mean flow over the particle size (and not much larger, as for molecules in a gas), and because the mean free path is comparable to the depth of the flowing layer (and not much smaller, as for a molecular gas). The theory is however successful at predicting some experiments, and in chapter 15 Walton shows that it agrees with numerical simulations if the particles have a high coefficient of restitution, $e > 0.8$, and are at a moderate concentration $0.1 < \phi < 0.5$. If the particles have a high loss, $e < 0.8$, or if the system is too dense, $\phi > 0.5$, then the fluctuations are suppressed and the basis of the theory collapses. Because the fluctuations are due to the shearing of the material, they have velocities $O(a\dot{\gamma})$, which yields a particle pressure and other stresses $O(\rho a^2 \dot{\gamma}^2)$, as found by Bagnold.

3.3. DURATION OF THE COLLISION

An essential feature of the kinetic theory for rapid granular flow is that two particles collide in isolation and exchange momentum with a small loss of energy. A loss is necessary to achieve an equilibrium with a constant supply of new energy from gravity or mean shear. A large energy loss however would eliminate the excited fluctuations. The size of the energy loss is controlled clearly by the coefficient of restitution, and more subtly by the ratio of a duration of the collision compared with the interval between collisions. If the collisions last a long time, then more than two particles will be involved in each collision. Momentum then tends to be distributed equally between the interacting group, and this means that there is a considerable degeneration of reusable impact energy. The fluctuations are thus suppressed, and large regions of the grains move together roughly as a rigid block. We thus move into the quasi-static regime. This change in behaviour was demonstrated by Luding* in a one-dimensional computer simulation. Note in higher dimensions one would expect it to be even more difficult to reconcentrate momentum into a single particle after it had been distributed amongst an interacting group. A graphic illustration was provided on the beach near to the Summer School: if one dropped a small round pebble onto a large massive one, it would bounce off to something like the original height, showing that the coefficient of restitution was near to unity; whereas if one dropped the same pebble onto a pile of similar pebbles, it would stop dead and not bounce at all.

3.4. QUASI-STATIC

In quasi-static granular flow, most of the grains are in regions which are moving in solid body motion, with shearing motion restricted to thin gaps only a few particles wide between these regions. Herrmann describes in chapter 21 a classical plasticity approach for dividing the full domain into different regions separated by slip lines or failures. A non-classical aspect, discussed by Herrmann in chapter 21 and by Fauvre in chapter 17, is the size of these slips in time, e.g. the spectrum of the sizes of avalanches, of landslides, of surges as a bulldozer pushes a pile of soil, of pulses in the flow of grains down a pipe and of fluctuations in forces on hoppers due to internal avalanches. Another non-classical behaviour is the separation and segregation of particles of different sizes, in both rapid and quasi-static granular flows.

3.5. STATICS

Stress-localisation. In order to understand the quasi-static motion, it is helpful to look at the static behaviour of granular materials. If pressure is applied to the face of a granular material, the stress is transmitted through the interior on a sparse network of interconnected particles. This phenomenon of stress-localisation can be demonstrated using photoelastic plastic rods in a two-dimensional packing. As the total load increases, more routes for the force-lines become active, and this contributes to a very nonlinear response, more nonlinear than that of the contact between two particles. This has a large effect on the electrical conductivity of the pack of grains, see chapter 12 by Goddard. It also effects the propagation of sound, see chapter 19 by Herrmann. Herrmann further suggested that the stress-localisation leads to fractures in rocks with the greatest stress in the middle of the fracture and not at the tips as in cracks.

Spreading stress. In addition to being highly localised, stress spreads out rapidly in three-dimensions on account of the random geometry of the particle contacts and the tangential friction force at the contacts. This feature is the basis of the construction of roads in which hard core helps to distribute the load of a vehicle over a wide area. It also means that the weight of the contents of a silo is supported mainly by the side walls (see chapter 5 by Savage), so that the flow out of the bottom is independent of the height of the contents, quite unlike a liquid where there would be a hydrostatic pressure $\rho g h$ driving a flow with a velocity $\sqrt{2gh}$. This constancy of the flow through an hourglass helps graduate time intervals. The velocity of grains exiting a hopper (or silo or hourglass) is approximately $\sqrt{gd}$, where d is the diameter of the neck (assumed to be several times the size of the grains), i.e. the grains are in free fall from about one diameter above the neck.

Angle of repose. When a pile of grains is tilted, the first motion is described by a critical angle of repose. Similarly when sand is sprinkled onto a pile, small avalanches maintain the angle near to the critical value. As discussed by Fauvre in chapter 17, there is a small hysteresis between the critical angle for motion to start and for it to stop. The angle of repose is partly set by the Coulomb friction angle for the constituent material of the grains, but is more influenced by the shape of the grains which controls the geometry of the surface and the possibilities of fitting a further particle snugly or precariously onto the surface, see chapter 15 by Walton.

Dilatancy. A confined (compacted) static granular material will not in general flow unless it is given some extra volume to enable some grains to pass over others (or unless some grains are destroyed by excessive force). This phenomenon is called 'dilatancy' and is described, along with Bagnold's simple demonstration, by Goddard in chapters 8 and 12. Goddard also suggests that there might be some connection between the non-monotonicity of the dilatancy and the stress-localisation described above, although I personally would be happier to associate the dilatancy with the localised slip lines between blocks of grains moving rigidly together in quasi-static flows. The thickness and friction laws of these slip lines are not yet understood well, although Tillemans* reported some numerical simulations of polygons showing shear bands and Wang* reported a stability analysis of a kinetic theory of rapid granular flow in Couette flow which showed the development of shear bands. In some geometries, such as the convergence into the exit of a hopper, a little dilation cannot be arranged to stop the particles from locking together, forming an arch which totally stops the flow when the exit is too narrow.

3.6. ROTATING DRUM

A drum half full of grains rotating about its axis in the horizontal can be found in a variety of industries, e.g. for drying grain, for crushing ores with added steel balls, and for polishing crystals used in accurate electronic oscillators. As described by Fauvre in chapter 17, the charge of grains can move in three regimes. At low speeds the grains fall intermittently in avalanches down the inclined surface, with the majority of the grains in solid body rotation with the rough drum. At intermediate speeds, there is a continuous motion of grains falling down the inclined surface with some small quasi-static motion in the interior. At very high speeds the grains are thrown by centrifugal forces onto the outside, and there is little relative motion, which is not very helpful in most applications. If the grains are a mixture of sizes, the smaller ones become concentrated in the central core after one complete revolution of the drum, as reported by Lebec*, by Clement* and by Oger*. The segregation, which may or may not be useful in different applications, arises from the surface appearing rougher to small grains so that they become lodged before reaching the bottom, whereas the large grains roll completely to the bottom on what appears to them to be a smooth inclined surface. In long drums (some of the model experiments have been with very short drums which permit just a monolayer of particles), Fauvre describes in chapter 20 an axial instability which leads to a standing wave pattern along the drum. There can also be an axial segregation of particles of different sizes along a long drum, reported by Nakagawa* using a NMR technique.

3.7. CHUTE FLOW

The flow of a granular material down an inclined chute has been studied extensively numerically, mostly in two-dimensions. Walton in chapter 10 describes how the flow at high angles of inclination can be rapid and described well by the kinetic theory. At lower angles of inclination the flow collapses into a quasi-static motion with a solid block moving over a thin slip layer only a few grains wide on the boundary. One vivid example of the latter was provided by some simulations by Renault* of small icebergs in an ice floe being held against a shore by Coriolis forces and moving up the coast as a solid mass with a thin slip layer next to the coast. Williams* described how different sizes of particles segregate in a chute flow that leads to grading (sorting) of boulders in some geological layers. Pouliquen* showed an experiment of a very slow chute where avalanches arrange themselves into a curious fingering instability across the inclined plane.

3.8. TUBE FLOWS

As reported for simulations by Herrmann in chapter 21 and Schäfer* and for two-dimensional experiments by Bideau* and for three-dimensional experiments by

Veje*, the flow of dry grains down a rough and narrow pipe (less than a dozen grains wide) suffers a kinematic wave instability. The amplitude grows until most of the grains are virtually stationary in a dense packing, and just a few are raining sparsely from one packed region down to the next packed region. This is very reminiscent of bubbles in fluidised beds with a densely packed bed surrounding a nearly empty bubble. The mechanism which drives the amplitude to this extreme state needs to be understood, both for fluidised beds and granular materials. The disturbances in the the flow of the granular materials propagate at the standard kinematic wave velocity. The disturbances do however merge and split producing a wide spectrum of sizes.

3.9. VIBRATING TABLE

Heaping. When a container of powder is placed on a table which is vibrating vertically, the surface of the powder does not remain horizontal but forms a heap. It is necessary for the maximum downward acceleration of the table to exceed gravity. There has been considerable controversy over the cause of this instability, although it now seems clear that at least four proposed mechanisms are appropriate to different circumstances. As pointed out by Fauvre in chapter 17, the table must remain accurately horizontal as the load of powder shifts to one side, for else the lightly loaded side would vibrate more strongly which would further deplete that side. This effect was also demonstrated by Herrmann in chapter 19. For a deep layer of powder, friction of the side walls can drive a circulation down at the walls, returning up at the centre, as reported by Rosato*, Knight* and Herrmann. In two-dimensional systems of monosized disks, crystals can form. During the vibrations, Clement* showed that micro-cracks can appear in this crystal when a horizontal line of contacts becomes jammed between the side walls, and this drives a circulation in the corner at the top of the bed near to the wall. Note the need for a horizontal line of contacts, which Clement* demonstrated was essential by suppressing the heaping when the crystal structure was rotated by 90°, the crystal then having a vertical line of contacts and only a zig-zag line of contacts horizontally. For thin layers of grains, Fauvre shows in chapter 20 that air in the gap under the lifted heap is important, because the effect disappeared in a vacuum. The layer must be sufficiently thin so that, when all the grains are used in a triangular heap at the angle of repose, the heap does not touch the sides, and thus the side wall friction cannot play any role. The air in the gap can escape more easily at the edges of the heap where the bed is thin. This permits the freely falling bed to land on the table first at its edges. The next part of the heap to land finds a compact region at the edges and so is nudged towards the centre. This produces a circulation inwards to the centre along the bottom, and hence upwards under the centre. Finally at extremely high accelerations (maximum downwards gravity exceeding $7g$), Melo* reported that a thin layer of grains would turn into a rapid granular flow, and being liquid-like could suffer the same instabilities as a liquid on a vibrating table, including elaborate patterns of standing waves.

A variant of the heaping instability was demonstrated by Herrmann where sand in a vibrating U-tube would rise to a higher level in one arm.

Segregation (Brazil nut). If one large particle is introduced into a vibrating bed of grains, it will normally rise to the surface. Similarly a mixture of small and large particles will segregate with the larger grains going to the top, even if the larger grains are more dense. It should be noted that any increase in potential energy is negligible compared with the large energy introduced by vibrating the table, energy dissipated by particle collisions. The separation process has applications in industry, e.g. processing ores, and can be demonstrated in the home by shaking a glass jar of muesli. As in heaping, there are a number of mechanisms which cause the segregation. Friction of the side walls can drive a circulation (reported by Knight*), which tends to be restricted to the level of the large particle and above. In a two-dimensional system of monosized disks, micro-cracking of the crystal structure is concentrated around the faults radiating out horizontally from the particle (reported by Clement*). With smooth walls, computer simulations with monosized disks (Dippel*) recover Jullien's mechanism of a single grain jumping into a lower lattice site through the gap between the crystal and the temporarily elevated large disk. This mechanism does not operate unless the diameter of the large disk is about three times that of the small disks, and then gives a discontinuous motion (no progress on some shakes) until the diameter ratio exceeds about 13. These delicate mechanisms may be dominated by something much simpler in three dimensions with real non-monosized non-spherical grains. It seems to me that starting from one configuration, there are many local arrangements involving only a small number of grains which require the large particle to rise, whereas there are comparatively few global rearrangements involving the coordinated motion of a large number of grains which lower the large particle.

As well as vertical vibration, Herrmann shows in chapter 19 that pharmaceutical pills can be transported horizontally by including a horizontal component to the oscillatory vibration.

3.10. OTHER

Drake*, Davies* and Conley* describe the transport up and down a beach of sand and small pebbles on the surface induced by the oscillating fluid drag from waves. Ristow* and Schwarzer* reported attempts to include some fluid interactions between grains in a granular material. Troadec* showed how small disks form linear chains in between large disks in a dense slightly agitated packing on an air table. Jacques* reported some experimental measurements of the velocity distribution of particles above a vibrating table, finding approximately the Maxwell-Boltzmann distribution but with anisotropy. Kertesz*, Oger* and Herrmann in chapter 21 described the possibility of cellular automata for studying granular materials. Particularly intriguing was the possibility of including dissipation through lattice collisions with a loss of energy and dilatancy through freezing particles which do not have a vacant site to move on to. Bideau* described the different regimes of a sphere rolling down a very rough inclined plane.

4. Common themes

4.1. MIXING AND SEPARATION

Mixing and de-mixing occur in each type of mobile particulate systems. Almost any motion of a dry granular material causes some separation of the grains of different sizes. In suspensions differential sedimentation or migration is used to separate particles of different types, not necessarily just different sizes. The efficiency of these separation processes is reduced by diffusion, e.g. the hydrodynamically induced diffusion during sedimentation, while in fluidised beds the bubbles provide an important mixing.

4.2. WAVES AND INSTABILITIES

The classical kinematic wave is found in tube flow of granular materials, in one-dimensional motion of a fluidised bed, and in a suspension of dense bubbles (as in creamy beers – observe that the waves propagate downwards while the bubbles rise). The waves are unstable in fluidised beds and in granular materials, where in both the amplitude grows until the grains are mostly in closely packed regions separated by relatively empty regions. Perhaps it is the possible reduction of the particle pressure/temperature above a certain concentration (which might occur when the duraction of the collisions exceeds the time between collisions) which leads to the formation of these dense phases.

4.3. SOUND

In gas fluidised beds and in liquids filled with bubbles, the speed of sound is drastically reduced as the compressibility of the air must move the inertia of the dense phase, the solid particles in the fluidised beds and the liquid in the bubble mixture. The propagation of sound through a granular material is complicated by the nonlinear contact forces between the particles, and is not yet fully understood.

5. Open problems

5.1. SHAPE OF PARTICLES

Far too many studies have been misled by the preference of theoreticians to consider only spherical particles. Some mechanisms for heaping and segregation of granular materials on a vibrating table may crucially depend on the crystal structure which occurs with monosized spheres and disks. The difficult question of the tangential friction force exerted between touching spheres in a granular material may become much simpler for angular particles. In suspensions, there has been some progress away from spheres, in particular to rod-like particles, with disk-like particles still in need of further study. In a discussion at the end of the Summer School, Etienne Guyon raised the problem of finding some non-spherical particles which could be adopted as a standard to be used by all laboratories.

5.2. GRANULAR MATERIALS

There are many features of granular materials which I do not understand. What is the connection if any between stress–localisation, dilatancy and shear bands in quasi-static flows? What is the thickness and the friction law for these shear bands? What determines the sizes of avalanches, their volume, depth and velocity? What is the instability mechanism for the axial waves in a rotating drum, for the fingering of avalanches in slow chute flows, and for the unbalanced arms of a vibrating U-tube? (From the state of understanding of the vibrating table, rotating drum and rough inclined plane, it seems to me that one should now be able to explain these instabilities.) How does one introduce the effect of fluid between the grains? Can the contact forces be represented simply when many particles collide simultaneously.

5.3. FLUIDISED BEDS

The most pressing problem in this area seem to be uncertainties in the governing equations. To what extent is the particle pressure determined locally by continual fluctuations in the drag force and how much is it determined by the bulk shearing motion (as in the kinetic theory of granular materials)? If an effective viscosity is required, what does its value depend on? Does one additionally require a particle diffusivity? Experiments are required which isolate and measure the individual terms. Does particle shape have any effect on the operation of a fluidised bed?

5.4. SUSPENSIONS

The unsolved problems in suspensions are now rather technical. One however is simple to pose and reveals a serious theoretical difficiency and this concerns fluctuations in sedimentation. The experiments show that the fluctuations are independent of the size of the container, while the computer simulations predict fluctuations growing with the size of the container. The question is what is the difference between randomly positioning the non-overlapping particles on the computer and the experimental procedure of stirring thoroughly.

5.5. CONTINUUM THEORIES

For suspensions, it is known that their behaviour can be described by a continuum theory so long as the scale of the flow exceeds that of the particles. Calculating the non-Newtonian rheology should be viewed as a challenging technical problem, often more easily solved by experimental measurement. There seems to follow an expectation that fluidised beds and granular materials can also be described by continuum theories. For fluidised beds, the flow events of interest (e.g. bubbles) do seem to be larger that the particle size, and so perhaps a continuum theory will be found, although there remain uncertainties as to the correct form. Perhaps the only difficult question is whether for some systems the homogeneous fluidised bed ever exists for which one writes down the governing equation. For rapid granular flows, the kinetic theory does seem to work, despite many of the events being of a similar size to the particles. For quasi-static flows, the controlling events are the shear bands which are only a few particles thick, for which one cannot hope to have a continuum theory. However the global structure of rigid blocks separated by such slip bands may be described by a plasticity theory, augmented by a friction law characterising the large scale effect of the slip bands.

5.6. COMPUTER SIMULATIONS

There is at present a healthy diversity of approaches to computer simulations. For example in suspensions, there is Stokesian dynamics and also multipole-collocation, which are fine for rigid spheres, but deformable particles require something like the boundary integral method. For example in granular materials, the programs can be event driven or have fixed time-steps, an assortment of contact forces have been used, the lattice gas (cellular automata) would be fast and sufficient if there existed any 'universal' behaviour which did not depend on particle scale details, and the lattice Boltzmann gas (again very fast on parallel machines) has been used to include fluid between the grains (although I myself cannot see its advantage over the Navier-Stokes equations with less than half the number of unknowns). It would be useful to have a 'best-buy' guide. It would also be useful if calculations could be made in non-dimensional units; this remark applies particularly to the current practice in fluidised beds and granular materials.

5.7. EXPERIMENTAL TECHNIQUES

At the end of the Summer School, Etienne Guyon lead a discussion of future directions. In particular, he suggested the need for more experimental techniques to measure the velocity of particles in the middle of a fluidised bed or granular material, the mean velocity and the fluctuations, correlations in time, correlations in space with nearby particles, and similarly details of the velocity of any fluid. There were photo-bleaching and photochromism techniques which had not been employed. He also suggested that future studies might consider reacting particles and geophysical applications such as turbidity currents and sedimentology.

5.8. INDUSTRY

Finally I should observe that many studies seem to have drifted away from the industrial applications, so that important features and important quantities to be predicted have been overlooked. Of course the primary aim must be to generate some scientific insight into the phenomena, and this insight must be able to be cast in the form of a simple mathematic model which then permits quantitative predictions. But it would be good to able to predict something of interest in the real engineering world.