Example Sheet 1

1. *Overtaking.* Two cars are travelling at 50 mph (how fast in ms^{-1}?) at a separation of 30 m. The rear car pulls out, accelerates to 70 mph which it holds until it is 30 m ahead, when it pulls in. Assume that the acceleration is constant, at such a value to change from 0 to 60 mph in 10 s (what fraction of g is this?). How far does the rear car travel during the overtaking? If a third car comes from the other direction at 60 mph, how far away should it be initially for safety?

2. *Time of travel.* Two cyclists, each travelling at speed u, cycle towards one each other. Initially they are d apart. A bee flies at a constant speed $4u$ from one cyclist to the other, instantly reversing direction on colliding with the nose of a cyclist. Write down (too easy) how far does the bee travel before the cyclists collide.

3. *Range up an incline.* Show that the trajectory of a cricket ball launched with speed V at an angle α to the horizontal may be expressed in polar coordinates as

$$r = \frac{2V^2 \cos\alpha \sin(\alpha - \theta)}{g \cos^2\theta} \ .$$

If the ground slopes up with an angle β to the horizontal, what angle α maximizes the range?

4. *Centripetal accelerations.* Calculate the following accelerations as fractions or multiples of $g = 9.81\,\mathrm{ms}^{-2}$.

(a) The acceleration towards the Earth's axis of Cambridge (at 52.2° N). The Earth's radius is 6378 km.

(b) The acceleration of the Moon towards the Earth. The radius of the Moon's orbit is 384399 km and its period is 27.3 days.

(c) The acceleration of an electron moving around a proton at a speed $0.007c$ (where c is the speed of light) in an orbit of radius 0.05 nm (the first Bohr orbit of the H atom).

(d) The acceleration of the outside of a car tyre of radius 30 cm when the car is travelling at 70 mph.

5. *Newton II.* To bump-start an old car it is necessary to attain a speed of 10 mph. Assume that the car weighs 1 tonne (10^3 kg) and that the road surface allows a person (mass 80 kg) to exert a force equal to half their weight. Starting from rest, how far does the car have to be pushed?

6. *Forces.* A person of mass 80 kg jumps from a height of 0.5 m. The knees are not bent and the motion is arrested in about 2 cm. What is the average force on the bone structure?

Now the person jumps from a height of 1.5 m but does bend the knees. How much must the knees be bent (*i.e.* how far does the centre of gravity descend) in order that the average force is kept less than five times the normal weight?

Please notify all errors to `E.J.Hinch@damtp.cam.ac.uk`.

7. *Gravity.* A malevolent deity suddenly stops the Moon. Calculate how long it takes to crash into the Earth. Treat the Earth as a fixed point mass M and the Moon as initially at a great distance R from the surface of the Earth. Use dimensional analysis to show that the time is given by

$$T = CG^{-1/2}M^{-1/2}R^{3/2}$$

where G is the gravitational constant and C is a dimensionless constant. Now integrate the equation of motion for the Moon to deduce $C = \pi/2\sqrt{2}$. Finally substitute the numerical values of the parameters and find the number of days to disaster.

8. *Gravity.* Find that radius of the geostationary orbit, i.e. the orbit with period 24 hours. Use only the data that $g = 9.81\,\mathrm{ms}^{-2}$ and the radius of the Earth is 6378 km.

9. *Electrostatics.* Return to question 4(c) and show that the acceleration is provided by the Coulombic electrostatic attraction.

10. *Magnetic field.* An electron would move in the magnetic field of a (hypothetical) magnet monopole according to

$$\ddot{\mathbf{x}} = \frac{k}{r^3}\dot{\mathbf{x}}\wedge\mathbf{x}$$

where $r^2 = \mathbf{x}\cdot\mathbf{x}$ and k is a constant. Show that the electron would move at a constant speed, v say, and that

$$r^2 = r_0^2 + v^2(t - t_0)^2$$

where r_0 and t_0 are constants. [Hint: consider $d^2(\mathbf{x}\cdot\mathbf{x})/dt^2$.]

Difficult optional part. Show that $\mathbf{x}\wedge\dot{\mathbf{x}} - k\mathbf{x}/r$ is constant vector, $\mathbf{L}$ say. By considering $\mathbf{L}\cdot\mathbf{x}$, show that the motion is on a cone with axis $\mathbf{L}$.

11. *Small air drag.* At Pisa, Galileo simultaneously dropped from a height of 15 m two iron spheres of radii 2 and 10 cm. Did they land simultaneously? First ignore air resistance and write down expressions for the velocity and height as functions of time. Then substitute this approximation for the velocity into the air drag $0.84a^2v^2$ Newtons, where a is the radius and v the speed. Now calculate a second approximation for the position, and hence the time of flight. Take the density of iron to be $7500\,\mathrm{kg\,m}^{-3}$.

12. *Air drag.* In the Millikan experiment to measure the charge of an electron, a small oil drop (density $900\,\mathrm{kg\,m}^{-3}$) is observed to fall under gravity at a terminal velocity of 1 cm in 1 minute. Find the radius a of the drop, using the air drag law for small slow spheres $3.5 \times 10^{-4}av$ Newtons. A vertical electric field of 1.46 kV/cm is able to hold the drop fixed. When the drop picks up another charge from the ionised air, a field of 1.1 kV/cm is required. Estimate the size of the quantum of charge.

13. *Strength of propeller.* Consider a cylinder of length $2L$, of small cross-sectional area A and density ρ, rotating at angular velocity ω about an axis perpendicular to its own axis through its centre. Find the tension in the cylinder by integrating the force per unit length ($\rho A\omega^2 z$ with z distance from rotation axis) required to maintain the circular motion (tension vanishes at end). Find the angular velocity for this tension to exceed AS where S is the stress to break the material. What is the maximum number of revolutions per minute for a wooden propeller with $L = 1\,\mathrm{m}$, $\rho = 10^3\,\mathrm{kg\,m}^{-3}$ and $S = 10^8\,\mathrm{Nm}^{-2}$?