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Example Sheet 3

1. *Constraint.* A particle starts is released from rest very near to the top of a smooth sphere. At what angle to the vertical does the particle leave the surface of the sphere?

2. *Impulse.* A kangaroo of mass $m_1$ stands on a platform of mass $m_2$ which is supported by a rope passing over a pulley and attached to a counterweight $m_1 + m_2$. If the kangaroo jumps off the platform applying an impulse $I$, show that the greatest height of the kangaroo above the platform is

$$\frac{2(m_1 + m_2)}{m_1 + 2m_2} h \; ,$$

where $h$ is the height the kangaroo rises over a fixed platform with the same impulse.

3. *Collision.* Two solid steel balls of radii 1 and 2 cm are released simultaneously some distance from a horizontal plane, with the smaller vertically above the larger at a small separation. (Separation constant before impact ignoring air drag?) The larger bounces off the plane, followed shortly by the smaller bouncing off the upward moving larger. Assume perfectly elastic collisions. Show that the smaller rises to 6.5 times the original height while the larger rises to 0.31 times the original height.

4. *Collision.* In 1932 Chadwick obtained an estimate for the mass of the neutron by studying elastic collisions of fast neutrons with nuclei of hydrogen and nitrogen. He observed a maximum recoil velocity of hydrogen (initially stationary) of $3.3 \times 10^7\,\mathrm{ms}^{-1}$, and of nitrogen $4.7 \times 10^6\,\mathrm{ms}^{-1}$ with an uncertainty of 10%. Take the mass of an $H$ nucleus to be 1 amu and $N$ 14 amu. Estimate the mass of a neutron and their initial velocity, taking into account the uncertainty in the $N$ measurement.

5. *Collision.* A line of $n$ railway wagons are stationary and slightly separated. A new wagon impacts the line at velocity $V$. If the coefficient of restitution is $e$, show that if the wagons first hit one another in strict sequence (optional computer project to investigate when true) then the end one pops off at velocity $V((1+e)/2)^n$.

6. *Conservation of?* An ice skater (i.e. ignore friction) is attached to a thick circular pole of radius $a$ by an inextensible rope which is kept taut. Why does the $v$ speed of the skater remain constant? Show that as the rope becomes wrapped around the pole the free length to the skater varies as $l = \sqrt{l_0^2 - 2avt}$. Find the total change in angular momentum of the skater about the centre of the pole by integrating in time the couple due to the tension $T = mv^2/l$ with moment arm $a$, starting from when the rope begins to wrap around the pole.

A second skater is attached by a taut rope to a small fixed hole in a device which *slowly* reduces the free length $l(t)$ of the rope. What does conservation of angular momentum of the skater about the hole say about the variation of the speed of the skater $v(t)$? Now integrate the work done against the tension $T$ above.

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7. *Coriolis.* Write down Newton's equations of motion in a rotating frame. Consider a projectile launched at velocity $V$ towards the East at an inclination of $\alpha$ to the horizontal from a position at 45° latitude on the Earth. Ignore air drag. Find to a first approximation to the velocity by ignoring rotation. Substitute this approximation into the Coriolis term in order to find the deflection to the South in a second approximation. If $V = 400\,\mathrm{ms}^{-1}$ and $\alpha = 45°$, what is the range in the first approximation, and the deflection South in the second approximation?

8. *Gravity* Consider a point particle of mass $M$ at a distance $r$ from the centre of a large sphere of radius $R$ ($< r$) of mass $m$ in which density is constant. By integrating the gravitational potential energy $-GM\delta m/s$ over the different rings within the large sphere of mass $\delta m$ at distance $s$ from the point particle, show that the force on the point particle is as if the mass of the sphere were concentrated at its centre. Now consider the case $R > r$ and find the corresponding formula for the gravitational potential energy and thence the force.

How far must one go down a mine shaft for a pendulum to lose 1 second in a day (treat the Earth as uniform density)? How many seconds would a pendulum clock lose during a 6 hour flight at 10 km above the surface of the Earth (ignore the motion of the aircraft)?

9. *Orbits, energy* For a planet in a circular orbit about the Sun, show that the gravitational potential energy (relative to zero energy at great separation) is twice the total (negative) energy and that the kinetic energy is minus the total energy.

Now consider a comet in a parabolic orbit. Show that when the comet crosses the circular orbit of the planet it is moving $\sqrt{2}$ faster.

10. *Orbits, energy, transfer* Consider a planet on an elliptical orbit about the Sun. Show that the semi-major diameter (sum of the minimum and maximum radii) depends on the energy and is independent of the angular momentum.

A spacecraft has escaped the Earth's gravitation field and is orbiting the Sun on a circular orbit with the same radius as the Earth's. How much must the velocity of the spacecraft be increased for its orbit just to reach out to Mars? Take the radii of the orbits of the Earth and Mars to be $1.495 \times 10^{11}$ m and $2.279 \times 10^{11}$ m and an Earth year to be 365.25 days.

11. *Orbit, force* A spacecraft finds itself being drawn towards a strange star by a new force. It is observed that the craft is on a circular path which *passes through* the star, and that the vector from the star to the spacecraft sweeps out area at a constant rate. Show that the force is directed towards the star with an $1/r^5$ law. [Use the $u_{\theta\theta}$ equation.]

12. *Scattering* An $\alpha$-particle of mass $m$ and electric charge $2e$ moving at speed $v$ along the line $y = b$ and $z = 0$ is scattered through an angle $\Phi$ by a heavy (immobile) nucleus of electric charge $Ze$. What is the change in the momentum the $x$-direction? Treating the nucleus as a point, integrate the $x$-component of the repulsive force over the duration of the collision, using $dt = r^2 d\theta/vb$. Hence show that $\tan(\Phi/2) = 2Ze^2/4\pi\epsilon_0 mv^2 b$.

For Geiger & Marsden's 1910 experiment, take $m = 6.44 \times 10^{-27}$ kg, $e = 1.60 \times 10^{-19}$ C, $Z = 79$ (Gold foil), $1/4\pi\epsilon_0 = 8.99 \times 10^9\,\mathrm{NC^{-2}m^2}$ and $v = 2 \times 10^7\,\mathrm{ms}^{-1}$. Find $b$ for $\Phi = 90°$. Compare your answer with the size of an atom $3 \times 10^{-10}$ m and the size of the nucleus $5 \times 10^{-15}$ m.