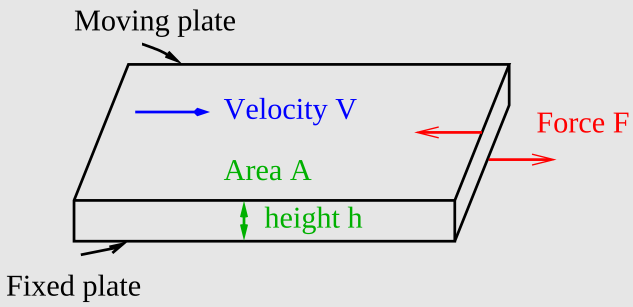
# Lecture 2

Rheometry

- Simple shear devices
- Steady shear viscosity
- Normal stresses
- Oscillating shear
- Extensional viscosity
- Scalings
- Nondimensional parameter

# Simple shear devices

Conceptual device for simple shear




Shear rate $\dot{\gamma} = \frac{V}{h}$

Tangential shear stress $\sigma_{xy} = \frac{F}{A}$

Shear viscosity $\mu = \frac{\sigma_{xy}}{\dot{\gamma}} = \frac{Fh}{AV}$

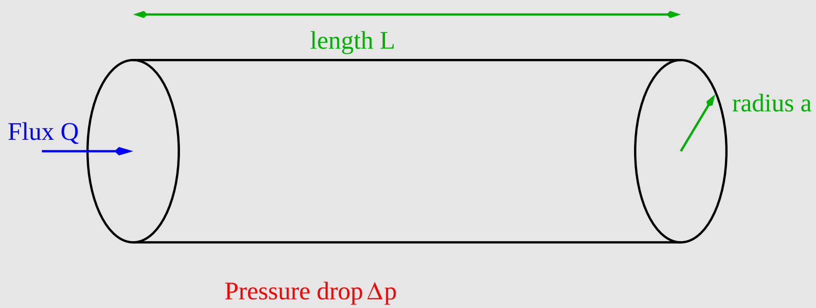
– works for heavy tars

# Simple shear devices 2

Viscometric flows: one layer of fluid slides over another

Need $\nabla(u^2)$ orthogonal to $\mathbf{u}$, i.e. $\mathbf{u} \cdot \nabla\mathbf{u} \cdot \mathbf{u} = 0$

Capillary tube – use for low $\mu$ and for high $\dot{\gamma}$

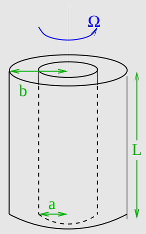



$$\mu = \frac{\pi a^4 \Delta p}{8QL}$$

Student Exercise

# Simple shear devices 3

Couette experiments in Paris for viscosity of gases, device found in Loire garage.
Unstable if rotate inner too fast.



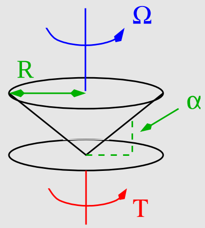

Inner rotating at angular velocity $\Omega$. Torque $T$.

$$\mu = \frac{T(b^2 - a^2)}{4\pi\Omega a^2 b^2 L}$$

Student Exercise

## Simple shear devices 4

Cone-and-plate. Typically angle $\alpha = 2°$.




Has shear rate independent of position – useful if $\mu(\dot{\gamma})$.

Angular velocity of top cone $\Omega$. Torque (on lower plate) $T$.

$$\mu = \frac{3T\alpha}{2\pi\Omega R^3}$$

Student Exercise

## Steady shear viscosity

- $\mu$ in Pa s
  - air $10^{-5}$
  - water $10^{-3}$
  - golden syrup $10^2$
  - molten polymer $10^{3\to5}$
  - molten glass $10^{12\to15}$
- $\dot{\gamma}$ in $s^{-1}$
  - sedimenting fines $10^{-5}$,
  - chewing food 10,
  - mixing $10^2$,
  - painting $10^3$,
  - lubrication $10^{3\to7}$.

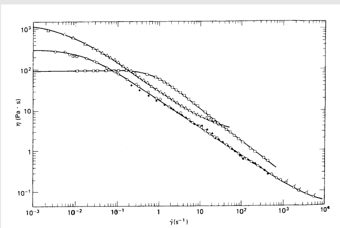
Typically has range of power-law shear-thinning

$$\mu(\dot{\gamma}) = k\dot{\gamma}^{n-1}$$

n: 0.6 molten polymer, 0.3 toothpaste, 0.1 grease.

## Steady shear viscosity 2

Two polymer solutions and an aluminium soap solution



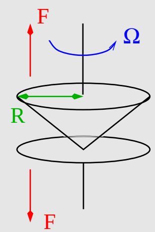
Decades of power-law shear-thinning

## Normal stresses

$$\mathbf{u} = (\dot{\gamma}y, 0, 0) \qquad \begin{cases} N_1 = \sigma_{xx} - \sigma_{yy} \\ N_2 = \sigma_{zz} - \sigma_{yy} \end{cases}$$

Stress differences to eliminate incompressibility's isotropic pressure

First normal stress difference from axial thrust on plate $F$.




Axial thrust on plate $F$.

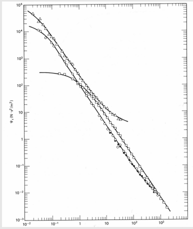
$$N_1 = \frac{2F}{\pi R^2}$$

Student Exercise

## Normal stresses 2

Plot $\Psi_1 = N_1/\dot{\gamma}^2$, as $\propto \dot{\gamma}^2$ at low $\dot{\gamma}$ (indpt sign/direction).

Two polymer solutions and an aluminium soap solution



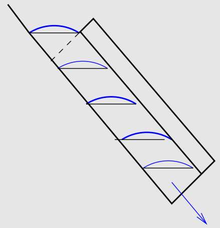
Decades of power-law behaviour.

At low $\dot{\gamma}$, $N \ll \sigma_{xy}$, but at high can be $100\times$.

## Normal stresses 3

$N_2$ normally small and negative.

- Pressure variation over disk
- Axial thrusts from plate-plate if know $N_1$
- Rod climbing if know $N_1$.
- Bowing of free surface in Tanner's tilted trough



## Oscillating shear

Shear:

$$\gamma = \gamma_0 e^{i\omega t} \qquad \text{(real part understood)}$$

Small amplitude: $\gamma_0 < 0.1$.

Stress $\sigma$ in terms of a
complex elastic modulus $G^*$ or complex (dynamic) viscosity $\mu^*$:

$$\sigma = G^* \gamma = \mu^* \dot{\gamma} = \mu^* i\omega\gamma$$

Storage modulus $G'$ and loss modulus $G''$.

$$G^* = G' + iG''$$

## Oscillating shear 2

Oscillating because wider range of frequencies, $10^{-3}$ to $10^5\,\mathrm{s}^{-1}$, than steady shear rates.

Low $\omega$: viscous response

$$\mu' = G''/\omega \to \text{const} , \qquad G' \quad \text{smaller.}$$

High $\omega$: elastic response

$$G' = \mu''\omega \to \text{const} , \qquad G'' \quad \text{smaller.}$$

Power law behaviour at intermediate $\omega$ – probes small scale structure.

## Oscillating shear 3

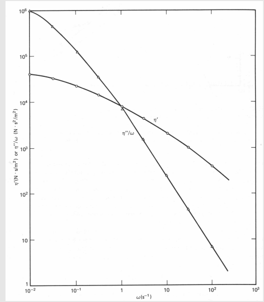
Other unsteady shear flows in modern computer controlled rheometers.

- Switch on stress, measure transient creep
- Switch off stress, measure transient recoil
- Switch on flow, measure build up of stress
- Switch off flow, measure relaxation of stress

Student Exercise: Connection between these and $G^*(\omega)$?

## Oscillating shear 4

Dynamic viscosity $\mu^* = \mu' - i\mu''$



Polyethylene melt (IUPAC Sample A)

At low $\omega$, $\mu'$ tends to a constant, and $\mu''$ is smaller by a factor of $\omega$

## Extensional viscosity

Uni-axial (axisymmetric) pure straining motion

$$\mathbf{u} = \dot{\epsilon}(x, -\tfrac{1}{2}y, -\tfrac{1}{2}z)$$

Calculate an extensional viscosity

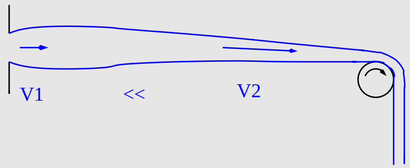
$$\mu_{\text{ext}} = \frac{\sigma_{xx} - \frac{1}{2}\sigma_{yy} - \frac{1}{2}\sigma_{zz}}{3\dot{\epsilon}}$$

Without 3 have confusing Trouton Viscosity.

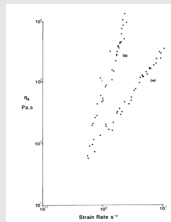
Cannot be steady in time and constant in space, so devices are not perfect.

## Extensional viscosity 2

Spinline

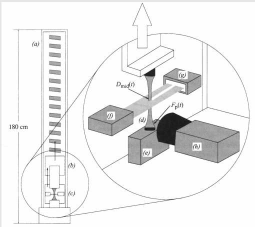


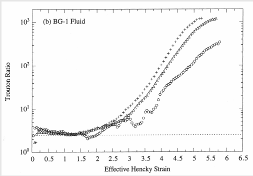

Measure tension $T$ & area $A(x)$ gives stress $\sigma_{xx}(x) = T/A$.
Velocity change & length gives strain-rate $\dot{\epsilon} = (v_2 - v_1)/L$.



## Extensional viscosity 3

Filament stretching – Cogswell, Meissner, Sridhar

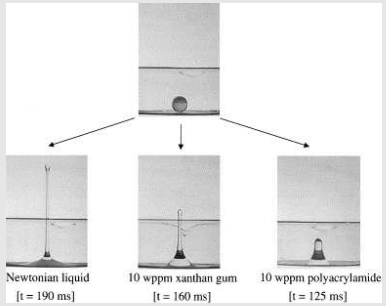





BG-1 Boger fluid: $\dot{\epsilon} = 1.0$, 3.0 and 5.0.

## Extensional viscosity 4

Solid sphere hits free surface producing a Worthington jet




Needs theory to interpret splash height.

## Extensional viscosity 5

More devices – uniaxial

- Moscow capillary squeezing – cheap, uncontrolled strain rate
- Four-roll mill: good $\mathbf{u}$, how to measure $\sigma$?
- Opposed jets: less good $\mathbf{u}$, can measure $\Delta p$.

Biaxial extensions

- Film blowing: have $\Delta p$, measure $r(t)$.
- Meissner film stretch with 8 tractors

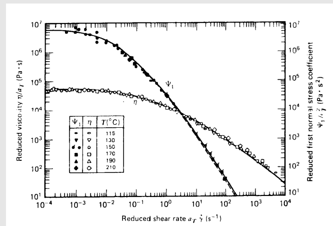
Other

- Sphere in a tube - common for Newtonian
- Squeeze film
- Sag of heap of cement

## Temperature scaling

Plot reduced viscosity $\mu_r$ as function of reduced shear-rate $\dot{\gamma}_r$

$$\mu_r = \mu(\dot{\gamma}, T)\frac{\mu(0, T_*)}{\mu(0, T)}, \qquad \dot{\gamma}_r = \dot{\gamma}\frac{\mu(0, T)}{\mu(0, T_*)}\frac{T_*\rho_*}{T\rho}$$

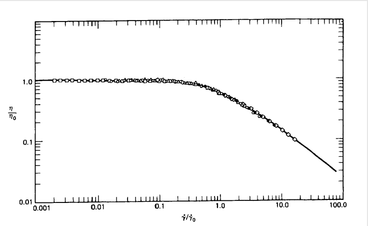


Low density polyethylene melt, reference temp 423K

$\mu(0, T)$ has activation energy around 4000°K

## Concentration scaling

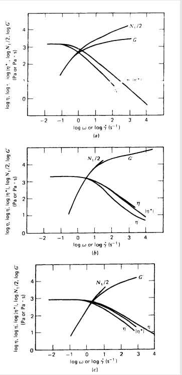
Plot *intrinsic viscosity* $= \mu(c, \gamma/\gamma_0)/\mu(0,0)$




## Cox-Merz 'rule'

'Ad hoc' approximation linking steady and oscillating response, based on oscillation seen if rotate with vorticity in a steady shear.

$$\mu_{\text{steady}}(\dot\gamma) \approx |\mu_{\text{osc}}(\omega = \dot\gamma)|, \qquad N_1(\dot\gamma) \approx 2G'(\omega = \dot\gamma)$$

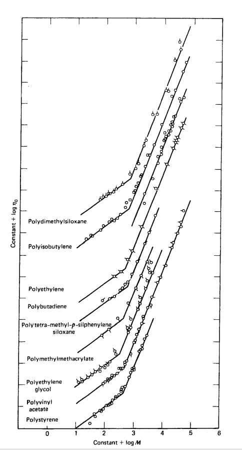


Solutions of polystyrene in 1-chloronaphalene

## Molecular weight scaling

At low molecular weight $M$,
$\mu \propto M^1$

At high molecular weight $M$,
$\mu \propto M^{3.4}$




## Nondimensional parameter

Materials have a time constant $\tau$

- $\mu_{\text{steady}}(\dot\gamma)$ plateau ends at $\dot\gamma = 1/\tau$,
- $\mu_{\text{osc}}(\omega)$ plateau ends at $\omega = 1/\tau$

Strength of shear rate

$$\text{Weissenberg} \quad Wi = \dot\gamma\tau$$

Speed of change

$$\text{Deborah} \quad De = \frac{U\tau}{L}$$

$De \ll 1$ – fully relaxed, liquid-like behaviour, viscosity
$De \gg 1$ – little relaxed, solid-like behaviour, elasticity