Extrasolar Planets - Atmospheres and Interiors

Cambridge Part-III Mathematics/Astrophysics, Michaelmas 2015

Example Sheet 3

Problem 1: Clouds and Hazes

(a) What is the difference between the geometric albedo (A_g) and the bond albedo (A_B) of a planet? The planet-star flux ratio in the visible at 0.5 μm for a Jupiter-like planet at full phase is 5×10^{-9}. What is its geometric albedo? For the same geometric albedo, wavelength, and phase, what would be the planet-star flux ratio for a typical hot Jupiter?

(b) Derive the expression for the equilibrium temperature of a planet. *Hint:* The function should be of the form: $T_{\rm eq} = T_s[(1 - A_B)(1 - f_r)R_s^2/(2a^2)]^{1/4}$. Calculate the equilibrium temperatures for Jupiter and a typical hot Jupiter orbiting a sun-like star, assuming efficient day-night redistribution (i.e. $f_r \sim 0.5$) and a Bond albedo of 0.3 in both cases. Repeat the calculation for inefficient day-night redistribution and compare with the efficient case.

(c) What are the observable signatures of clouds/hazes in exoplanetary atmospheres in transmission and emission spectra in visible and infrared wavelengths? Note: List signatures in each of the four combinations (i.e. transmission/emission and visible/infrared).

(d) The transmission spectrum of a transiting hot exo-Neptune observed recently revealed no spectral features, i.e. a flat spectrum was observed. Assuming the planetary atmosphere has a solar elemental composition one would have expected to see strong water absorption features in the spectrum of a clear atmosphere. The observed flat spectrum therefore suggested the presence of a thick cloud or haze layer in the atmosphere. Estimate the atmospheric pressure at the top of the cloud deck, assuming the base of the observable atmosphere is at $\sim$1 bar. More generally, derive the expression for the change in amplitude of an absorption feature in an infrared transmission spectrum due to the presence of a cloud deck of a given thickness in the atmosphere. Make any needed assumptions and state them.

(e) What are the prominent types of scattering phenomena possible in exoplanetary atmospheres? What are their spectral signatures?

Problem 2: Atmospheric Modeling Methods and Atmospheric Dynamics

(a) What are the two one-dimensional modeling paradigms and how are they different? Which particular equations are solved differently between them, and why?

(b) What are the key observable signatures of atmospheric dynamics in exoplanets? Which of the signatures have already been observed using current telescopes, and for which kinds of planets?

(c) Why are extremely irradiated hot Jupiters expected to have larger day-night temperature contrasts compared to modestly irradiated hot Jupiters? How do the day-night temperature contrasts on hot Jupiters compare against those on solar system planets?

(d) Images of Jupiter reveal distinct bands of clouds that indicate jet structures caused by atmospheric circulation in the planet. How are the bands different in number from those of other planets in the solar system and those expected for hot Jupiters?

Problem 3: Planetary Interiors and Mass-Radius Relations

(a) Write down the equations of planetary structure for a spherically symmetric object. How are they different from stellar structure equations.

(b) Assuming a polytropic pressure-density relation and spherical symmetry, derive the Lane-Emden equation for a polytropic index n. Derive the mass-radius (M-R) relation from the same assumptions. What physical situations in planetary interiors can be described by polytropic solutions with indices n = 0 and n=3/2?

(c) Draw a plot with qualitative sketches of M-R relations expected for zero-temperature spheres of planetary masses, for four different elemental compositions: H, He, solar composition, and Fe. Show Jupiter and Saturn on the plot. Identify three key trends in the M-R curves, and explain them. You might find it useful to consider polytropic solutions to explain some of the trends.

(d) Draw a rough sketch of the M-R curve predicted by detailed theoretical models for solar composition bodies with masses ranging from gas giants to low-mass stars. Identify and explain the main trends in the curve. Identify and explain the transitions between different object types (e.g. planets, brown dwarfs, and stars).

Problem 4: Interiors of Giant Planets

(a) Starting with the appropriate structure equation(s) show that a newly formed planet starts hot and cools during its evolutionary lifetime, assuming no significant external source of energy. Given that the emergent flux from Jupiter's interior is $\sim 5 \times 10^3$ ergs/cm^2/s, estimate the approximate age of Jupiter by which it will exhaust its interior energy supply if it radiates at the same rate until then.

(b) The luminosity (L) of a giant planet evolves approximately as:

$$L \propto t^{-\alpha} M^{\beta} \kappa^{\gamma}, \text{with } \alpha \sim 5/4, \beta \sim 5/2, \text{and } \gamma \sim 2/5, \quad (1)$$

where, M is the mass of the planet, t is its age, and κ is the atmospheric opacity at its photosphere. The current luminosity of Jupiter is about a billion times fainter than that of the Sun, which means the planet-star flux contrast for a Jupiter-like planet around a sun-like star is too small to be detected in thermal emission with existing telescopes. Yet, in recent years several giant exoplanets have been detected with orbital separations longer than Jupiter's 5.2 AU orbit. How is this possible? What kind of detection method is used to find such planets? What are the characteristic masses, radii, and temperatures of such planets?

(c) What is the “Inflated hot Jupiters” problem? What are the two broad classes of solutions considered to solve the problem? List 2-3 specific solutions in each class.

(d) A planet orbiting very close to its host star on an eccentric orbit is subjected to strong tidal forces from the star due to which its orbit will be circularized on a secular time scale. The dissipation rate of the planet’s eccentric orbital energy is given by

$$\dot{E}_{tidal} \sim 2 \times 10^{28} e^2 \left(\frac{M_\star}{M_\odot}\right)^{5/2} \left(\frac{a}{10R_\odot}\right)^{-15/2} \left(\frac{R_p}{R_J}\right)^{5} \left(\frac{Q_p}{10^6}\right)^{-1} \text{ergs/s}, \tag{2}$$

where the symbols represent quantities you should be able to guess easily. Consider an inflated hot Jupiter HD 209458b with a mass of 0.71 M_J, radius of 1.35 R_J, and orbital separation of 0.045 AU, orbiting a sun-like star. Theoretical models suggest that extra energy from an external source is required to be deposited in the interior of the planet at the rate of 8×10^{28} ergs/s to be able to explain the observed radius of the planet. The orbit of the planet is very slightly eccentric ($e \sim 0.015$). Can tidal dissipation explain the observed radius of the planet? If so, for what values of Q_p? Are those Q_p values reasonable for this planet? Could tidal dissipation be a generic mechanism to explain all inflated hot Jupiters? Why? You might find it useful to know that the timescale for tidally damping the eccentricity of the orbit is given by

$$\tau_{e,tidal} = 0.63 \left(\frac{M_\star}{M_\odot}\right)^{-3/2} \left(\frac{M_p}{M_J}\right) \left(\frac{a}{10R_\odot}\right)^{13/2} \left(\frac{R_p}{R_J}\right)^{-5} \left(\frac{Q_p}{10^6}\right) \text{Gyr}. \tag{3}$$