Part III: Galaxies
Lent Term 2011
Examples Sheet # 2: Kinematics/Dynamics

1. DISTRIBUTION FUNCTIONS: One of the most useful distribution functions for astrophysical applications is the Maxwellian velocity distribution:

$$F(v) = Ne^{-v^2/2\sigma^2},$$

where σ is the one dimensional velocity dispersion.

For such a distribution function show that:

(a) the mean particle speed is $\langle V \rangle = \sqrt{8}/\pi\sigma$;

(b) the mean squared speed is $\langle V \rangle^2 = 3\sigma^2$;

(c) the mean squared speed of any single cartesian component of velocity is $\langle V \rangle_x^2 = \sigma^2$;

(d) the fraction of stars with $V^2 \geq 4\langle V \rangle^2$ is 1/136 (0.00738).

(e) What is the physical significance of the last result?

2. SCALING LAWS: Spheroids and discs share a dynamical scaling law by which the total luminosity of the galaxy scales as the fourth power of velocity dispersion or rotation speed, respectively. Show that this can be derived from the virial theorum, if M/L is constant (mass traces light), and the luminosity scales as a power of the characteristic radius of the system (derive the scaling). What is the observational significance of the latter result?

3. LUMINOSITY FUNCTIONS: The distribution of galaxy luminosities is well described by the Schechter function:

$$n(L)dL = (\phi^*/L^*)(L/L^*)^\alpha e^{-L/L^*} dL,$$

(a) What is the luminosity density of the universe in terms of ϕ^*, L^*, and α? What is the luminosity of galaxies that contribute the most to the luminosity density of the universe?

Observations suggest that the optical luminosty function has the values $\phi^* = 10^{-2}h^3$ Mpc^{-3}, $L^* = 10^{10.8}h^2 L_\odot$, and $\alpha \sim -1.2$.

(b) What is the number density of galaxies in the local universe assuming $h = 0.7$? What is the optical luminosity density of the universe? The faint end slope is often very poorly constrained, repeat for $\alpha = 1, 0, -2$.

(c) For $\alpha = -0.5$ give the mean luminosity of a galaxy and the fraction of the luminosity density contributed by galaxies with $L > L^*$.

(d) The number density of galaxies with luminosties between L' and $L' + dL'$ is measured at two epochs t_1 and t_2. For $L' = L^*(t_1)/100$ and $L' = 10L^*(t_1)$, what is the fractional change in the measured number density assuming
(i) the luminosity of all galaxies increases from t_1 to t_2 by a factor of 3, and
(ii) ϕ^* increases by a factor of 3.

4. FLAT ROTATION CURVES AND DARK MATTER: To get a feel for the role of dark matter in the dynamics of typical galaxies refer to the rotation curve of the spiral galaxy NGC 3198, published in Table 2 of van Albada et al. (1985), ApJ, 295, 305.

(a) Derive the predicted rotation curve for the galaxy, assuming that all of the gravitation arises from an exponential disc with luminosity, scale length and distance measured (see Table 1 of the paper), and compare it to the observed rotation curve. You can assume that M/L is constant.

(b) What is the maximum value of M/L that is consistent with the data? What is the corresponding maximum baryonic mass of the disc, and how does it compare to the total dynamical mass within the last measured velocity?

(c) Assuming that the remaining mass is in the form of a spherical dark matter halo, plot the enclosed halo mass as a function of radius for your chosen disc M/L ratio.

5. THE NFW PROFILE: The NFW Profile (Navarro, Frenk and White 1996, ApJ 462, 563) is the most commonly used modern density profile for galaxies and is based on the results of CDM simulations:

$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2},$$

where $r_s = r_{200}/c$ is a characteristic radius and $p_{crit} = 3H^2/\pi G$ is the cosmological critical density. The mass of the so described halo is

$$M_{200} = 200\rho_{crit}(4\pi/3)r_{200}^3,$$

and the definition of r_{200} is that the mean density within r_{200} is $200\rho_{crit}$, with

$$\delta_c = \frac{200}{3}\frac{c^3}{[ln(1 + c) - c/(1 + c)]},$$

δ_c can be considered the characteristic overdensity of the halo, r_s is its scale radius, and c is its concentration.

(a) Consider an ensemble of test particles in circular orbit within this potential. Derive the the enclosed halo mass and rotation curve (circular velocity vs. radius) for this halo.

(b) Compare the enclosed halo mass as a function of radius to the results from 3(c). What parameters for the NFW halo give a good match?