Part III: GALAXIES

Examples Sheet 4: Galaxy Evolution and Essay Discussion

Lent Term 2011

1. LIMITS ON STAR FORMATION AND GALAXY EVOLUTION:

(a) The maximum rate that a galaxy can convert gas into stars is set by the dynamical time of the system, i.e., the maximum rate would invoke conversion of the entire intial gas mass of the galaxy into stars in a free-fall time. Derive an expression for this maximal "Monolithic Collapse" star formation rate in terms of the mass, radius, and/or density of the system.

(b) Show that this maximal SFR can be expressed solely as a function of the circular velocity of the protogalaxy. How does the slope of this dependence compare to that of the Faber-Jackson or Tully-Fisher relations? (To simplify the calculation you may assume that the gas dominates the total mass of the system within the radii of interest.)

(c) Calculate the maximal SFR for a protogalaxy with mass 10^{11} M$_\odot$ and radius 50 kpc.

(d) Estimate the bolometric luminosity of the protogalaxy, averaged over the free-fall time. For this calculation you may assume that the stars of relevance convert 10% of their mass into energy with an efficiency 0.01 mc^2 (this includes a contribution from supernovae). Assume a Salpeter IMF and the mass-luminosity relation from the last examples sheet to determine which stars are relevant. How does this luminosity compare to the values observed today for star-forming galaxies?

(e) Compare the total energy released in the primordial starburst to the gravitational binding energy of the original gas cloud. How does the ratio of these energies scale with galaxy mass or circular velocity?

2. CHEMICAL EVOLUTION MODELLING:

(a) Show that if the star formation rate is proportional to the mass of gas, then in the limit of no gas inflow or outflow, the star formation rate evolves as $SFR(t) \propto e^{-t}$. What is the total stellar mass formed as a function of time? Repeat for a star formation rate proportional to $M_{gas}^{3/2}$

(b) For a closed-box model the metal abundance of a stellar population that evolves from zero metallicity is determined solely by the remaining fractions of mass in gas and stars:

$$Z = y \ln \frac{M}{M_{gas}} = y \ln \frac{1}{\mu} \tag{1}$$

where y is the metal yield, and μ is the gas mass fraction of the system (or alternately the ratio of the current to original gas mass).

In the case of an exponentially declining SFR, how does the metallicity evolve with time? What about the case of a constant SFR?

(d) Show that at any time (and for any star-formation history) the distribution of stellar metallicity in the system (i.e. the number of stars with metallicity between Z and dZ) can be written

$$\frac{dN(Z)}{dZ} = e^{-Z/y} \tag{2}$$

(e) Show that in the limit that all of the gas in converted into stars, the mean stellar metal abundance is equal to the yield y.

3. FORMATION OF A DISC GALAXY:

For details, see

www.ast.cam.ac.uk/~schapman/Part3Galaxies/examples4.pdf

This problem will be fully discussed in the examples class in Easter term.

4. ESSAY EXAMPLE: THE GROWTH OF GALAXIES

An underlying theme throughout the course was the comparison of observed properties of galaxies with expectations from the current ΛCDM hierarchical model, which integrates a picture for the growth of galaxies with the buildup of the large scale structure of the Universe itself.

(a) Describe, in roughly a paragraph each, 5 general observations of galaxies (that can include the Milky Way), which provide empirical support for the hierarchical assembly of galaxies. Exclude the obvious one, i.e. the observation of occasional instances of galaxies merging at the present time. In each case explain why the same observations are not as readily understood in the traditional formation/evolution model, which proposed that the formation of galaxies was largely completed in single rapid collapse events more than 10 Gyr ago.

(b) Describe in the same way 3 general observations of galaxies (that can include the Milky Way) which cannot be reproduced by current models or simulations of galaxies within the ΛCDM framework. For each case explain whether these observations can be understood more readily in the old instantaneous collapse picture, and if so how. You should include at least one observation that is more consistent with the rapid collapse theory.