Part III
Geological Fluid Mechanics
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SHEET III

1. Pure liquid gold at temperature T_∞, which is greater than the solidification temperature T_S, is poured into a large pot with insulated sides. After the liquid has settled, the base of the pot is maintained at temperature T_B ($< T_S < T_\infty$).

   Calculate an expression for the rate of solidification, on the assumption that the values of all physical properties in the solid and liquid alloy are identical.

   Evaluate your result in the limit of large and small values of the two parameters $r = (T_S - T_B) / (T_\infty - T_S)$ and $S = L/c_p (T_\infty - T_S)$. Explain physically what your results represent.

2. An infinite medium of fluid is initially at uniform temperature T_o. The solidification temperature of the fluid is T_S, which is greater than T_o. At time $t = 0$ a speck of dust is introduced into the medium, which causes the fluid to grow a crystal on the speck.

   Assuming the crystal to be spherically symmetrical and neglecting convection effects, calculate the rate of growth of the crystal.

   Comment on the validity of the assumptions of spherical symmetry and zero convective motion.

3. Sketch the phase diagram of a typical two-component system. Briefly explain the various regions that appear in the phase diagram.

   Consider a binary alloy which has vertical solidii and a liquidus given by $T = -mC$, where T is temperature, C is composition and $m > 0$. An infinitely deep strip of solid alloy of zero composition at zero temperature initially occupies the region $z > 0$. At time $t = 0$ the solid is overlain ($z < 0$) by an infinitely deep layer of molten alloy at uniform temperature T_∞ and uniform composition C_∞. The values of all physical properties of solid and melt are identical.

   On the assumption that the interface between solid and liquid is given by $a = 2\lambda(Dt)^{1/2}$, where D is the compositional diffusivity, and neglecting any fluid motions in the molten alloy, show that the temperature and composition of the interface, T_i and C_i, are given by

$$T_i \left[\frac{1}{f(-\epsilon\lambda)} - \frac{1}{f(\epsilon\lambda)} \right] = \frac{T_\infty}{f(-\epsilon\lambda)} + \frac{L}{c} \qquad (1)$$

and

$$C_i = C_\infty \left[1 + f(-\lambda) \right]^{-1}, \qquad (2)$$

where L and c are the latent heat and specific heat respectively, $\epsilon = (D/\kappa)^{1/2}$ and

$$f(x) = \pi^{1/2} x \epsilon^{x^2} erfc(x). \qquad (3)$$

Write down the eigenvalue relationship for λ.

Sketch the solution for temperature and concentration on a phase diagram using z as an arc-length parameter along the curve.

Show that, in the limit $\epsilon \to 0$, if $T_\infty > 0$ then $\lambda = 0(\epsilon^{-1})$. Show that this indicates that the depth of melt produced is independent of the diffusion of composition and that the melt region has almost uniform composition of approximately

$$\frac{C_\infty}{2\pi^{1/2}} \ \frac{\epsilon^{-\lambda^2}}{\lambda}. \tag{4}$$

What determines λ?

Alternatively, still in the limit $\epsilon \to 0$, deduce that if $T_\infty < 0$ then $\lambda = 0(1)$ and $T_i \approx 1/2 \ \ T_\infty$. What causes the solid to be transformed to melt in this case?

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