

String Theory: Example Sheet 2

Malcolm Perry, February 2017

1. Starting from

$$L_0 = \frac{1}{2} \sum \alpha_n \cdot \alpha_{-n}$$

construct the normal ordered version of this expression. Use the zeta function technique to compute the shift in the vacuum energy for the case of NN and ND strings.

Now compute the commutator $[L_m, L_n]$.

Take the anticommutation rules for the ghost fields and the expression for $L_n^{(ghost)}$ and find the commutator $[L_m^{(ghost)}, L_n^{(ghost)}]$. Convince yourself that this means the critical dimension of the bosonic string is 26.

2. Assuming the commutation relations for x^a , p^a , α_n^a and $\tilde{\alpha}_n^a$ in the mode expansion, compute the commutation relations for X^a and Π^a for the case of the closed string. Repeat the calculation for the ND open string.

3. Construct the oriented open string NN states at level 2 in the lightcone formalism and determine their representations under the little group $SO(25)$. Construct the states at level 3. Show that they fit into a traceless symmetric-3-tensor and an anti-symmetric-2-tensor representation of $SO(25)$.

Now repeat this calculation for the ND states and DD states.

What happens if you chose the unoriented open string instead?

4. Suppose that one takes the commutator of two consecutive variations of the fermions in the superstring in the gauge where the worldsheet metric is fixed to $\eta_{\mu\nu}$ and the supersymmetric parameters ϵ_1 and ϵ_2 are taken to be constants. Show that this is an infinitesimal translation by an amount that is the same as that for bosons, namely a translation through $2\bar{\epsilon}_2 \gamma^\nu \epsilon_1$.

What happens if you repeat this calculation for local superconformal invariance?

Hint: It is possible to do this by using a Fierz transformation. However, it is easier, since we are in dimension two, to write each of the spinors explicitly in terms of their components and reassemble the answer into the correct form.

5. Consider the action for the superstring, including the gravitinos, as given in lectures. Show that the action is invariant under global supersymmetry, local Weyl transformations and local superconformal transformations. Verify that the transformation rules

for rigid supersymmetry

If you are brave or foolhardy, try this for local supersymmetry using the expressions given in the lectures.

6. Find the Hagedorn temperature for the closed bosonic string.