3P7b Supersymmetry: Example Sheet 2 Lent 2016

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1. Provide a concise answer to each of the following questions:

   (a) Write up each of the components of a massive $\mathcal{N}=1$ supersymmetric multiplet for superspin $y=1/2$ clearly specifying each of the labels differentiating the members of the multiplet. Compare these components with those of massless multiplets with maximal helicities $|\lambda|=1/2$ (chiral multiplet) and $|\lambda|=1$ (vector multiplet). Given this information explain how the Higgs mechanism can be realised in $\mathcal{N}=1$ supersymmetry.

   (b) Write up the massless states of the $\mathcal{N}=4$ vector multiplet. Decompose this multiplet in terms of $\mathcal{N}=2$ vector multiplets and hypermultiplets. Decompose it in terms of $\mathcal{N}=1$ chiral and vector multiplets.

   (c) Write up explicitly the components of the $|\lambda|\leq 2$ multiplets of $\mathcal{N}=7$ supersymmetry. Compare with the $\mathcal{N}=8$ case. Compare also the vector multiplets of $\mathcal{N}=3$ and $\mathcal{N}=4$ supersymmetries.

   (d) Write up all the components of the multiplets with smallest possible helicities for $\mathcal{N}=9$ and $\mathcal{N}=10$ supersymmetries. Provide three undesirable physical properties of these multiplets.

   (e) Describe the most general structure of massive states for $\mathcal{N}=4$ supersymmetry. Consider separately the cases with central charges $Z^{AB}=0$ and $Z^{AB}\neq 0$. In this case derive the BPS condition and describe all the options for the existence and the sizes of the massive multiplets.

2. Show that

$$\begin{aligned} P_\mu &= -i\partial_\mu \\ \mathcal{Q}_\alpha &= -i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu)_{\alpha\dot\beta}\,\bar\theta^{\dot\beta}\,\partial_\mu \\ \bar{\mathcal{Q}}_{\dot\alpha} &= i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \theta^\gamma\,(\sigma^\mu)_{\gamma\dot\alpha}\,\partial_\mu \end{aligned}$$

   satisfy the $N=1$ supersymmetry algebra.

3. Given the general scalar superfield:

$$S(x,\theta,\bar\theta)=\varphi(x)+\theta\psi(x)+\bar\theta\bar\chi(x)+\theta\theta M(x)+\bar\theta\bar\theta N(x)$$

$$+(\theta\sigma^\mu\bar\theta)V_\mu(x)+(\theta\theta)\bar\theta\bar\lambda(x)+(\bar\theta\bar\theta)\theta\rho(x)+\theta\theta\bar\theta\bar\theta D(x)$$

   Show that under a supersymmetry transformation

$$\delta S = i(\epsilon\mathcal{Q}+\bar\epsilon\bar{\mathcal{Q}})S$$

   the components of the superfield transform as:

$$\delta\varphi=\epsilon\psi+\bar\epsilon\bar\chi, \qquad \delta\psi=2\epsilon M+(\sigma^\mu\bar\epsilon)(i\partial_\mu\varphi+V_\mu)$$

$$\delta\bar\chi=2\bar\epsilon N-(\epsilon\sigma^\mu)(i\partial_\mu\varphi-V_\mu) \qquad \delta M=\bar\epsilon\bar\lambda-\frac{i}{2}\partial_\mu\psi\sigma^\mu\bar\epsilon$$

$$\delta V_\mu = \epsilon\sigma_\mu\bar{\lambda} + \rho\sigma_\mu\bar{\epsilon} + \frac{i}{2}\left(\partial^\nu\psi\sigma_\mu\bar{\sigma}_\nu\epsilon - \bar{\epsilon}\bar{\sigma}_\nu\sigma_\mu\partial^\nu\bar{\chi}\right) \qquad \delta N = \epsilon\rho + \frac{i}{2}\epsilon\sigma^\mu\partial_\mu\bar{\chi}$$

$$\delta\bar{\lambda} = 2\bar{\epsilon}D + \frac{i}{2}\left(\bar{\sigma}^\nu\sigma^\mu\bar{\epsilon}\right)\partial_\mu V_\nu + i(\bar{\sigma}^\mu\epsilon)\partial_\mu M \qquad \delta D = \frac{i}{2}\partial_\mu(\epsilon\sigma^\mu\bar{\lambda} - \rho\sigma^\mu\bar{\epsilon})$$

$$\delta\rho = 2\epsilon D - \frac{i}{2}\left(\sigma^\nu\bar{\sigma}^\mu\epsilon\right)\partial_\mu V_\nu + i(\sigma^\mu\bar{\epsilon})\partial_\mu N$$

Notice in particular that the transformation of the field D is a total derivative.

4. Prove that the Lagrangian:

$$\mathcal{L} \;=\; (\partial_\mu\varphi\;\partial^\mu\varphi^*) - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + FF^* + m[\varphi F - \frac{1}{2}\psi\psi + h.c.]$$

is invariant, up to a total derivative, under the supersymmetry transformations:

$$\begin{aligned}
\delta\varphi &= \sqrt{2}\epsilon\;\psi \\
\delta\psi &= i\sqrt{2}\;\sigma^\mu\bar{\epsilon}\;\partial_\mu\varphi \;+\; \sqrt{2}\;\epsilon F \\
\delta F &= \sqrt{2}\;i\;\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi
\end{aligned} \qquad (1)$$

where ϵ is a Grassmann-valued parameter and m a constant (mass). Here $h.c.$ stands for complex or hermitian conjugate.