Collapse of a granular column

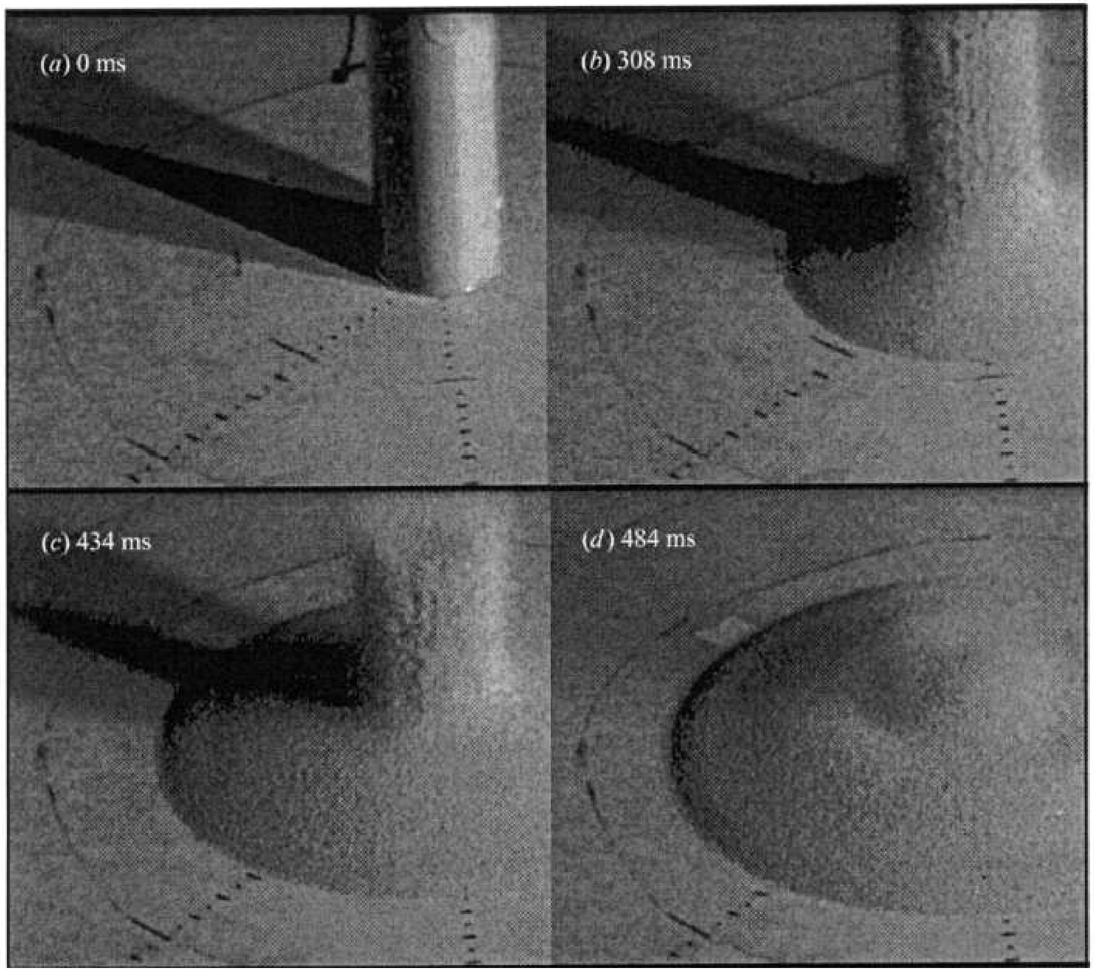
John Hinch

in collaboration with Lydie Staron & Emeline Larrieu

and Chris Cawthorne

inspired by Herbert Huppert

Collapse of a granular column

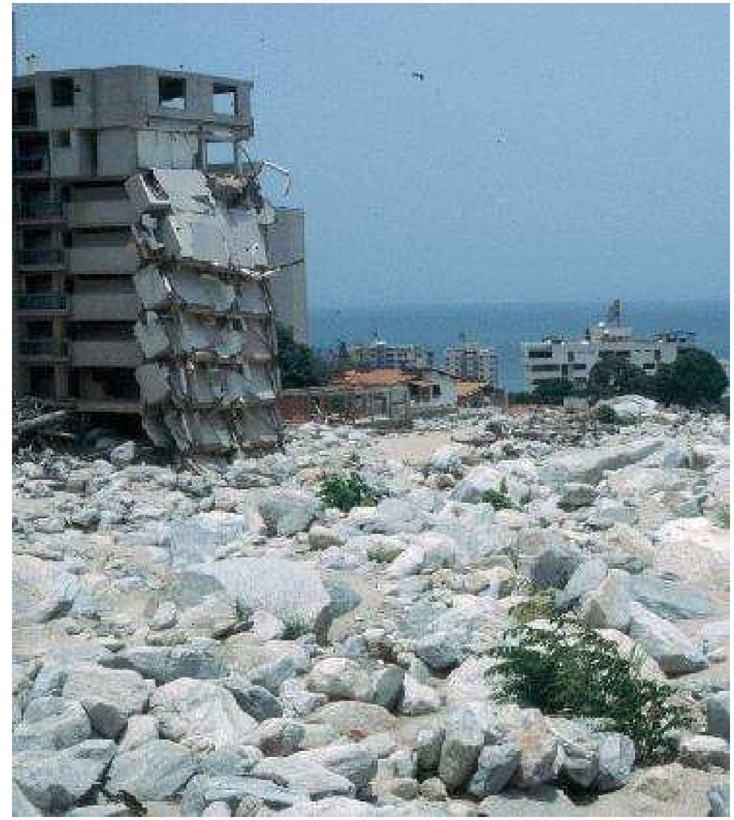


Lube, Huppert, Sparks & Hallworth 2004 JFM

Idealisation of geophysical events

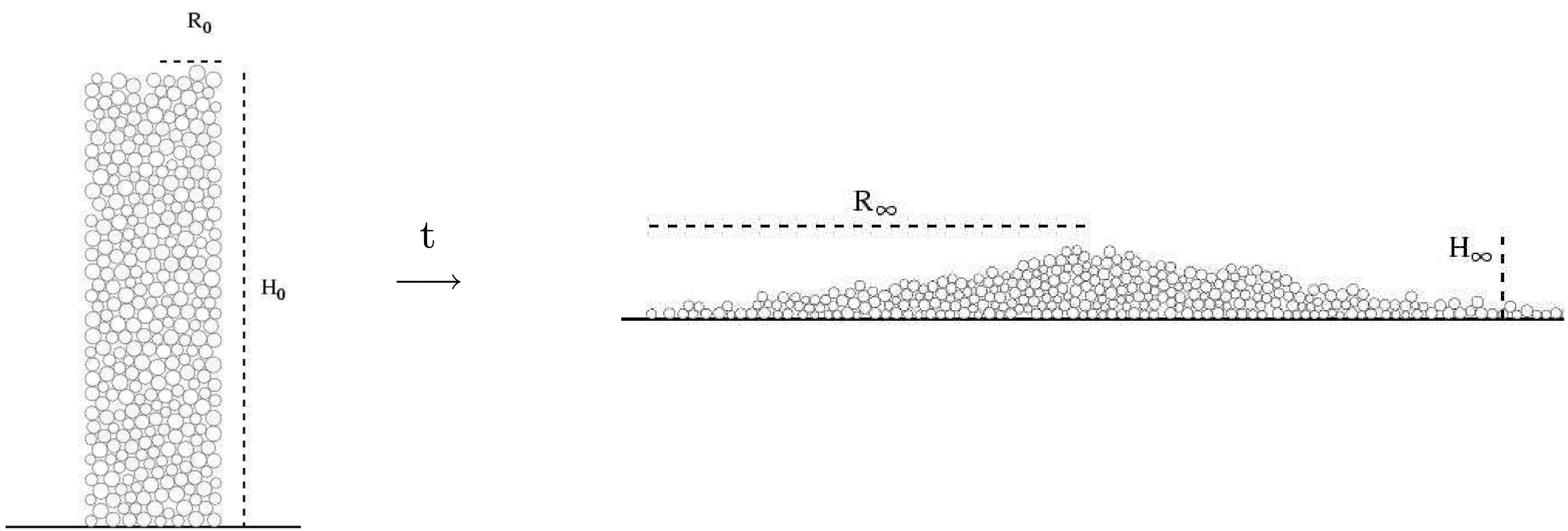
Geophysical events

Hope, British Columbia, 1965
$4.6\,10^7\,\mathrm{m}^3$



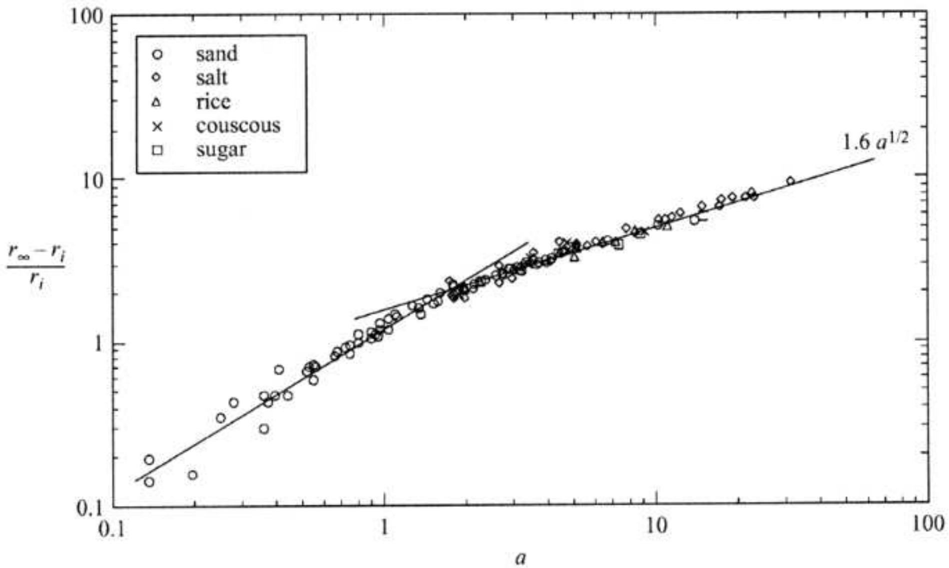
Venezuela

Problem



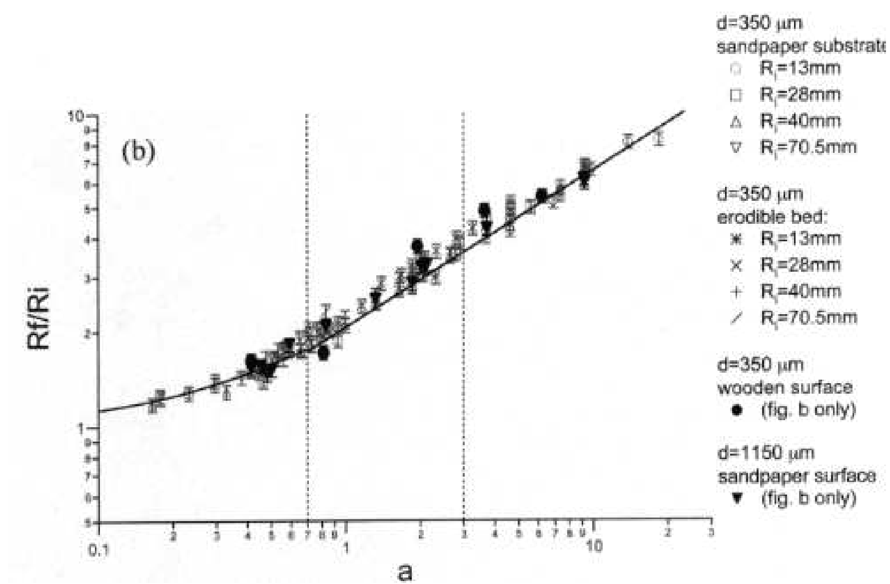
What determines the runout distance R_∞?

Experiments

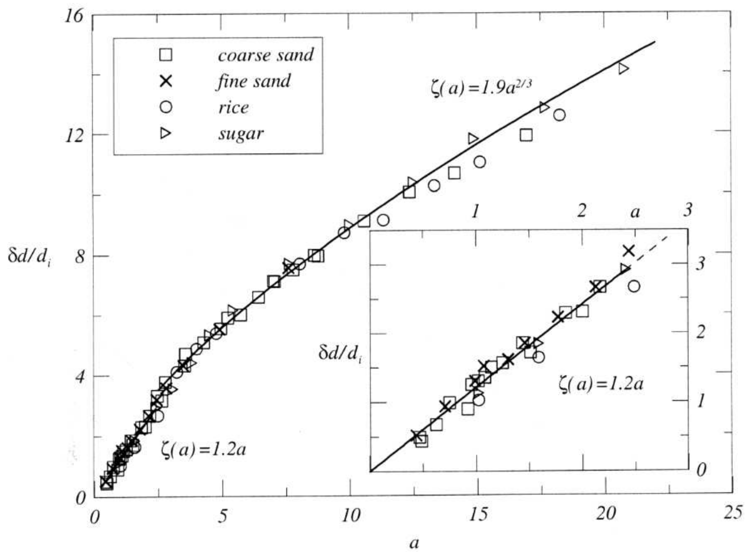


Lube, Huppert, Sparks & Hallworth 2004 JFM

Lajeunesse, Mangeney-Castelnau & Vilotte 2004 PoF

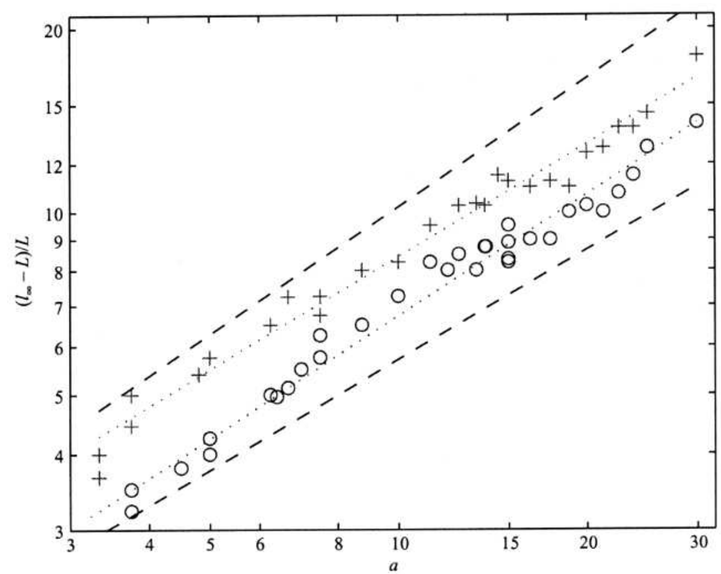


Experiments in 2D channel



Lube, Huppert, Sparks & Freundt 2005 PRE

Balmforth & Kerswell 2005 JFM



Runout in experiments

By Huppert *et al*, by Lajeunesse *et al*, by Balmforth *et al*

Independent of type, size & number of particles

Axisymmetric:

$$R_\infty/R_0 = 1 + 1.8a^{1/2}$$

2D:

$$R_\infty/R_0 = 1 + 2a^{2/3}$$

if $a > 2$, where aspect ratio $a = H_0/R_0$.

Simple laws, difficult to explain

Doomed theories

Initial potential energy $\rho g H_0$

becomes vertical kinetic energy $\frac{1}{2}\rho w^2 = \rho g H_0$

becomes horizontal kinetic energy $\frac{1}{2}\rho u^2 = \frac{1}{2}\rho w^2$

Sliding mass M resisted by solid (Coulomb) friction $\mu M g$

Runout: $R_\infty = H_0/\mu$ $10\times$ too large, wrong power-law

Numerical simulations of grains flowing

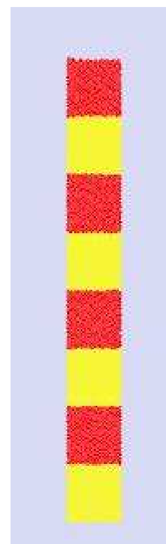
1. Event-driven: for dilute granular gases, jump t to next collision, but condensate

2. Soft-particle: cannot resolve real deformation of $1\,\mathrm{nm}$ on $1\,\mu\mathrm{s}$, so artificially very soft, by 10^{-6}, plus artificial dash-pots for dissipation

3. Hard-particle: repulsive force to stop overlap, Coulombic friction in sliding contacts, underdetermined

Numerical simulations by Lydie

DEM method, 2D, hard spheres (discs),
Coulombic friction, 5000 particles, polydisperse sizes.



Robust numerical simulations

Results independent of

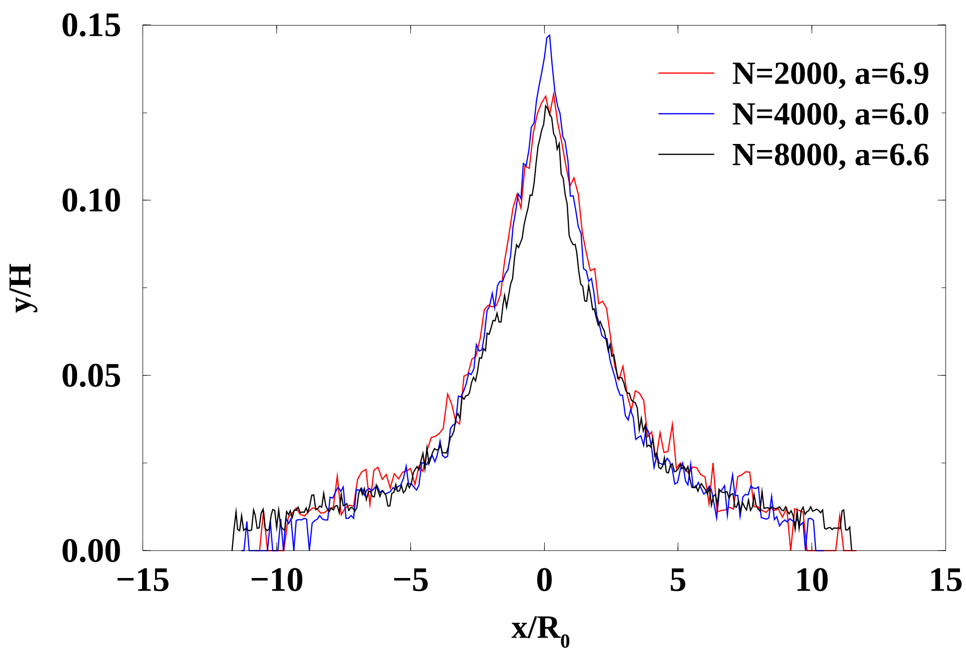
- number of particles
- polydispersity in size of particles
- value of coefficient of Coulombic friction
- value of coefficient of restitution

except for extreme cases.

Consider final deposits for different parameters

Independent of number of grains

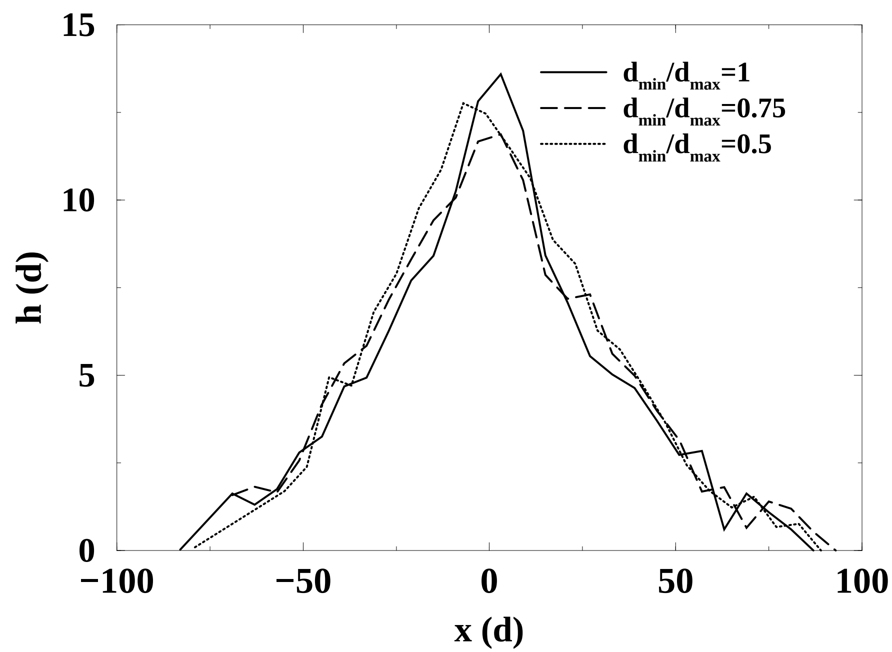
Final deposit:



Independent of polydispersity

Uniform distribution of radii between in $[d_{\min}, d_{\max}]$, with $d_{\min}/d_{\max} = 1, 0.75, 0.5$

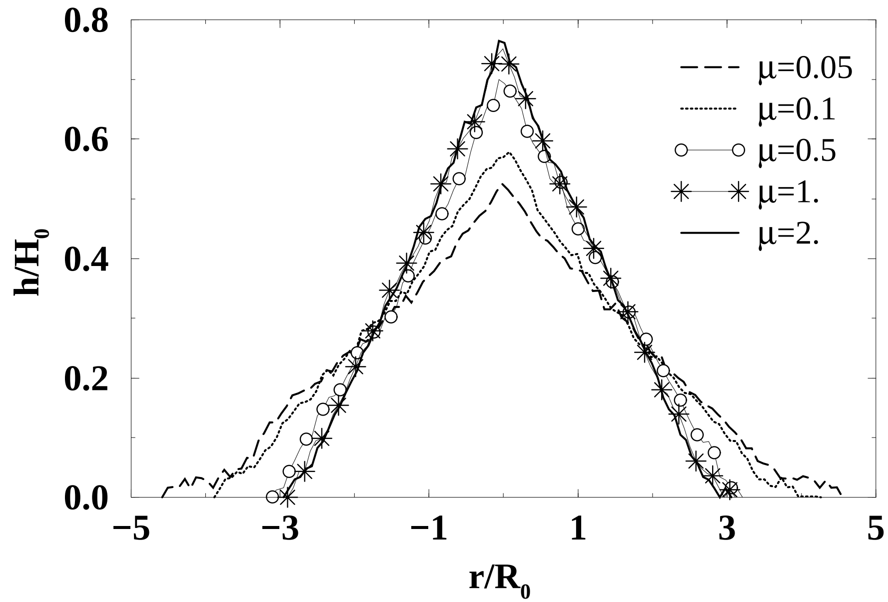
Final deposit:



Small fines would segregate and fall to bottom

Independent of inter-grain friction

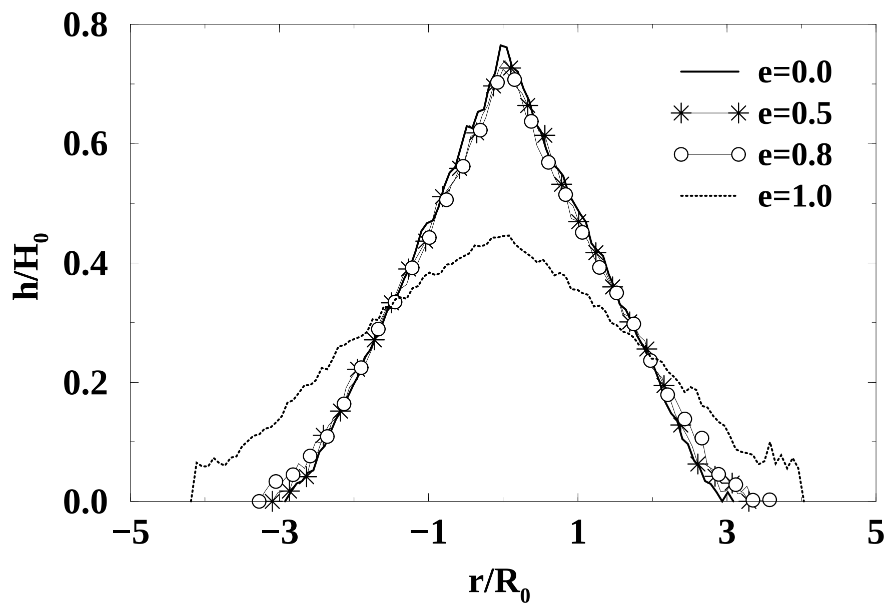
Final deposit for $\mu = 0.05, 0.1, 0.5, 1, 2$



Different only if very slippery

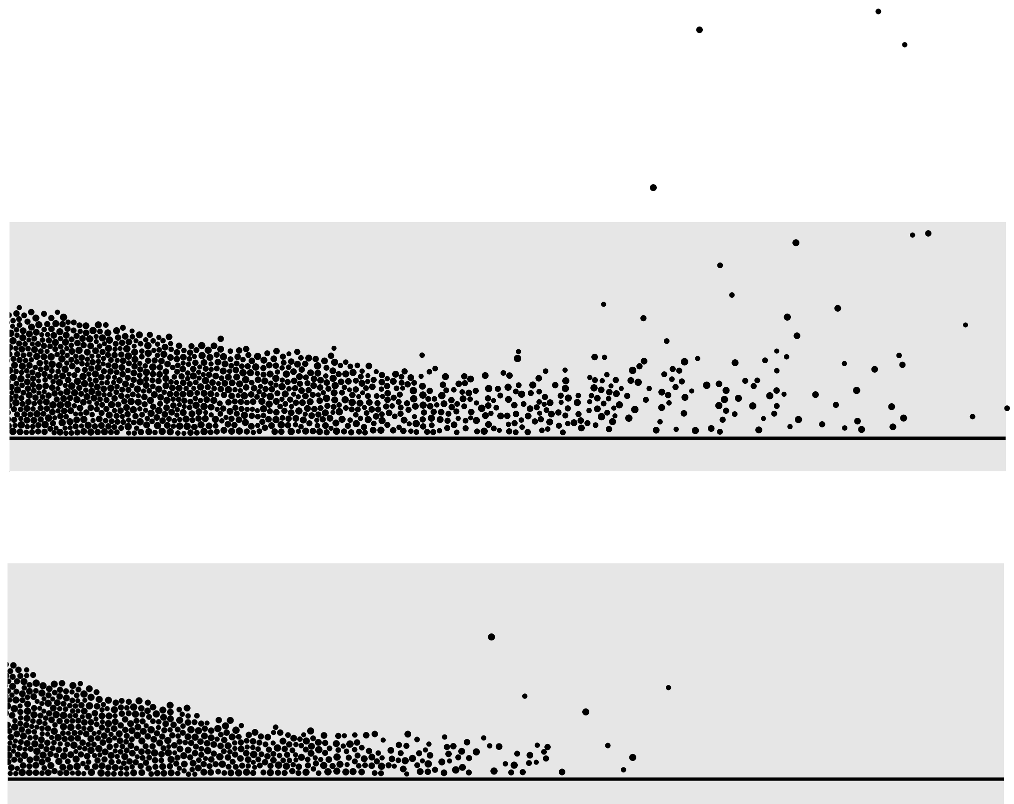
Independent of restitution

Final deposit for $e = 0, 0.5.0.8, 1.0$



Different only if e very close to 1

Effect of restitution



$e = 1.0$

$e = 0.8$

Too bouncy if $e = 1$.

Numerical simulations

Results independent of

- number of particles
- polydispersity in size of particles
- value of coefficient of Coulombic friction
- value of coefficient of restitution

except for extreme cases.

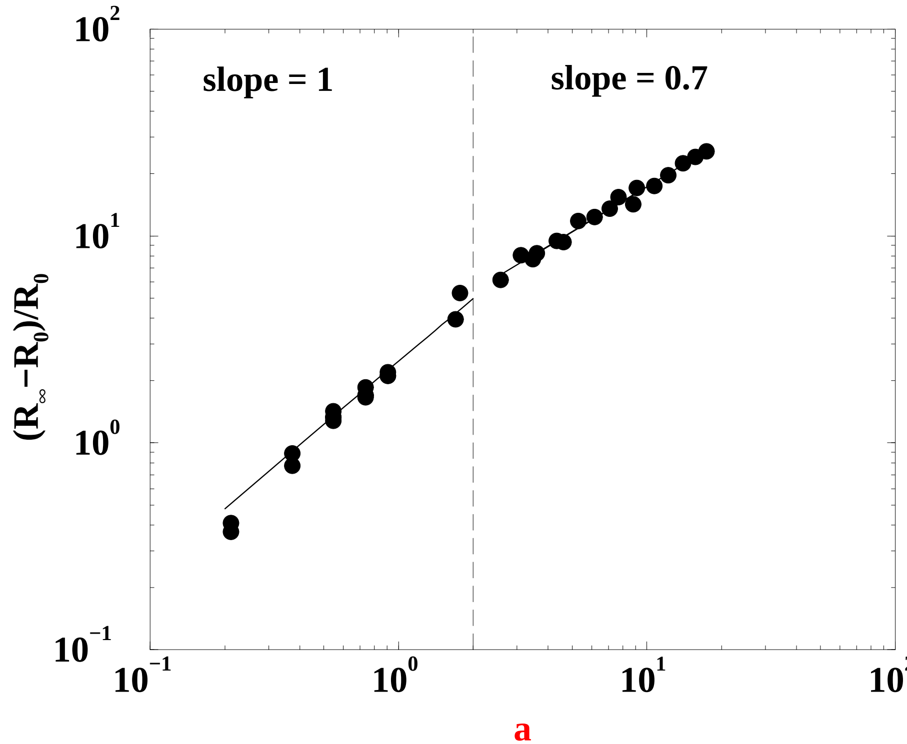
Like independent of type, size & number of particles in experiments.

Results of simulations

Runout (in 2D):

If $a > 2$

$$R_\infty/R_0 = 1 + 3.5a^{0.7}$$



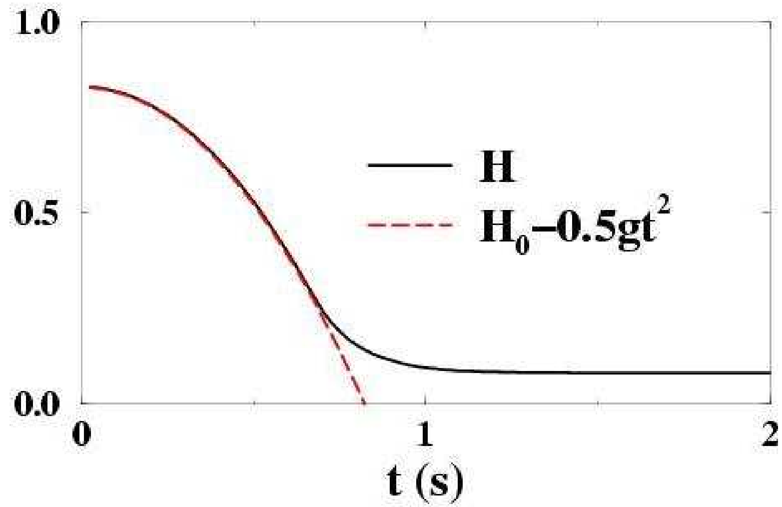
Simulations 3.5 vs experiments 2.

Dissipation low for discs?

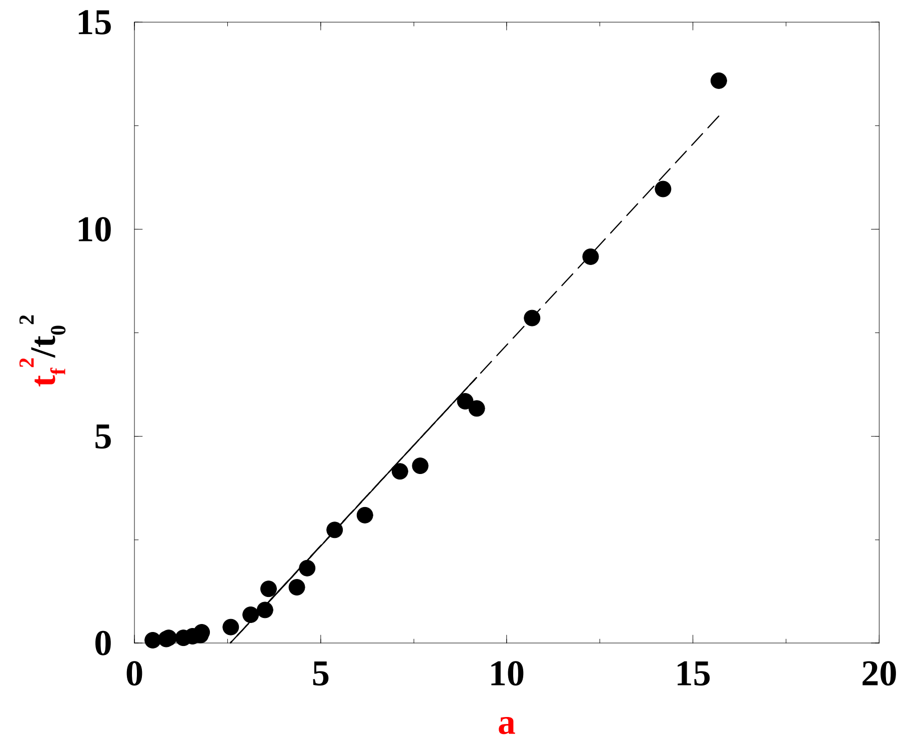
. . . more details

- Free fall of column while $h(t) > 2.5R_0$
- Duration of flow $T_\infty = 2.25\sqrt{2H_0/g}$
- Universal position of front as function of time, normalised
- Dissipation in horizontal flow

Free fall

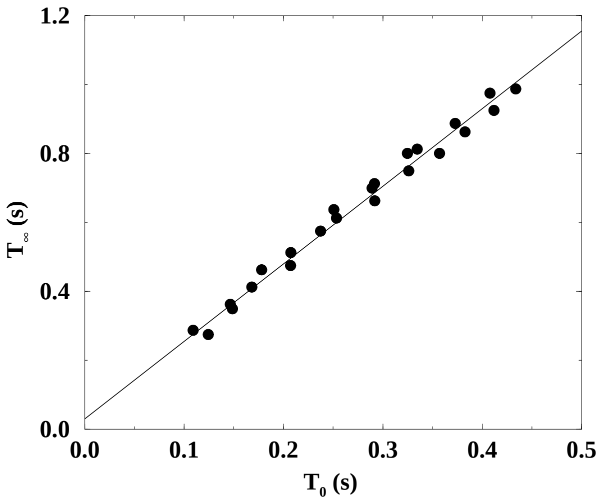


Column in free fall for
$t_f = \sqrt{2(H_0 - 2.5R_0)/g}$



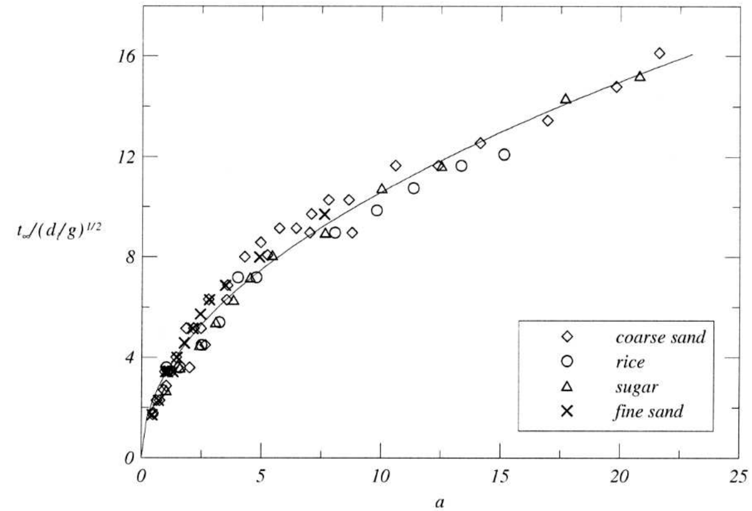
Duration of flow, T_∞

Simulations



$$T_\infty = 3.2\sqrt{H_0/g}$$

Experiments



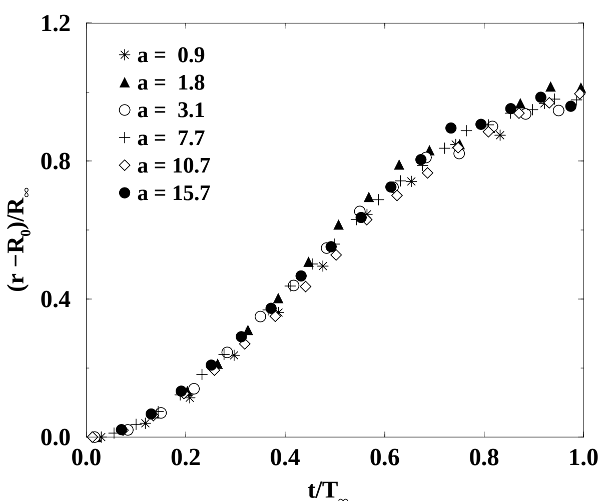
Lube, Huppert, Sparks & Freundt 2005 PRE

$$T_\infty = 3.3\sqrt{H_0/g}$$

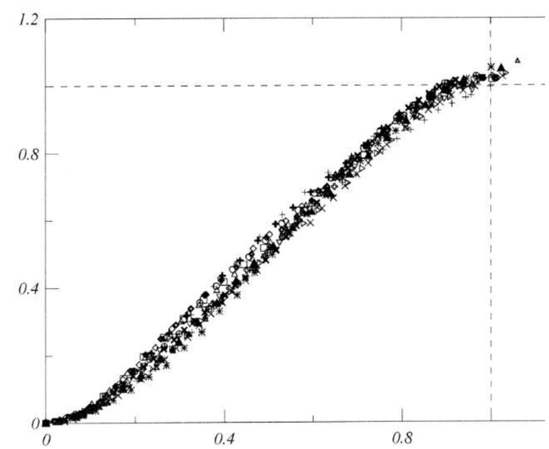
Moving front

Front $r(t)$: $(r - R_0)/(R_\infty - R_0)$ vs t/T_∞

Simulations



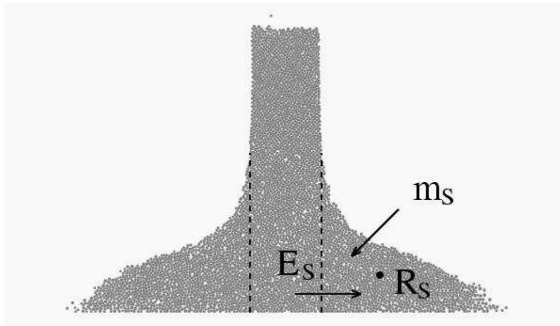
Experiments



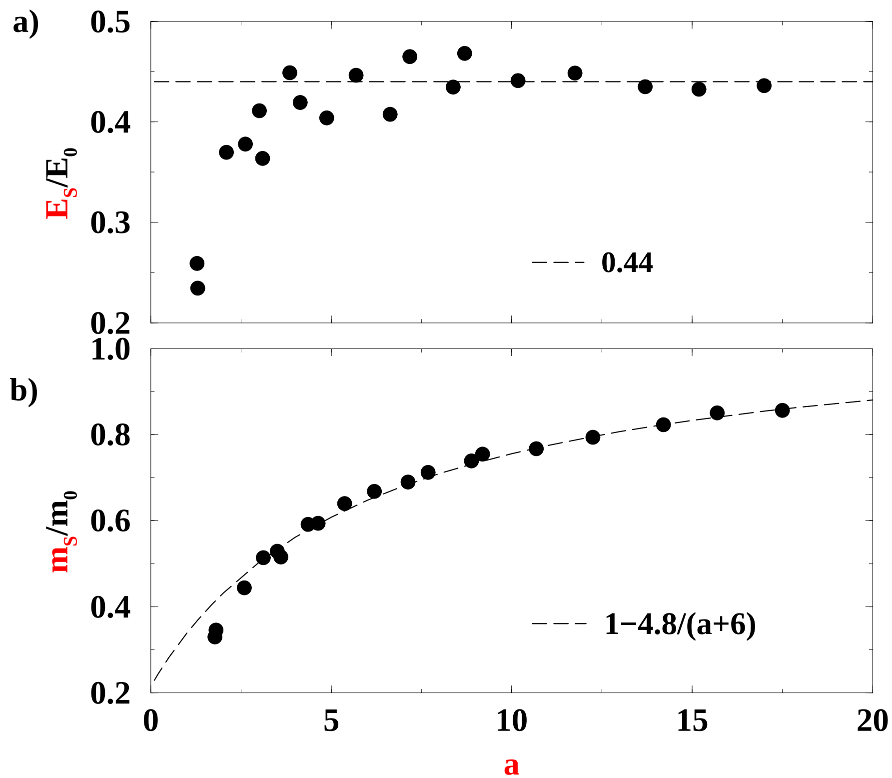
Lube, Huppert, Sparks & Freundt 2005 PRE

Intermediate times at nearly constant velocity $\sqrt{2gR_0}$

The horizontal flow

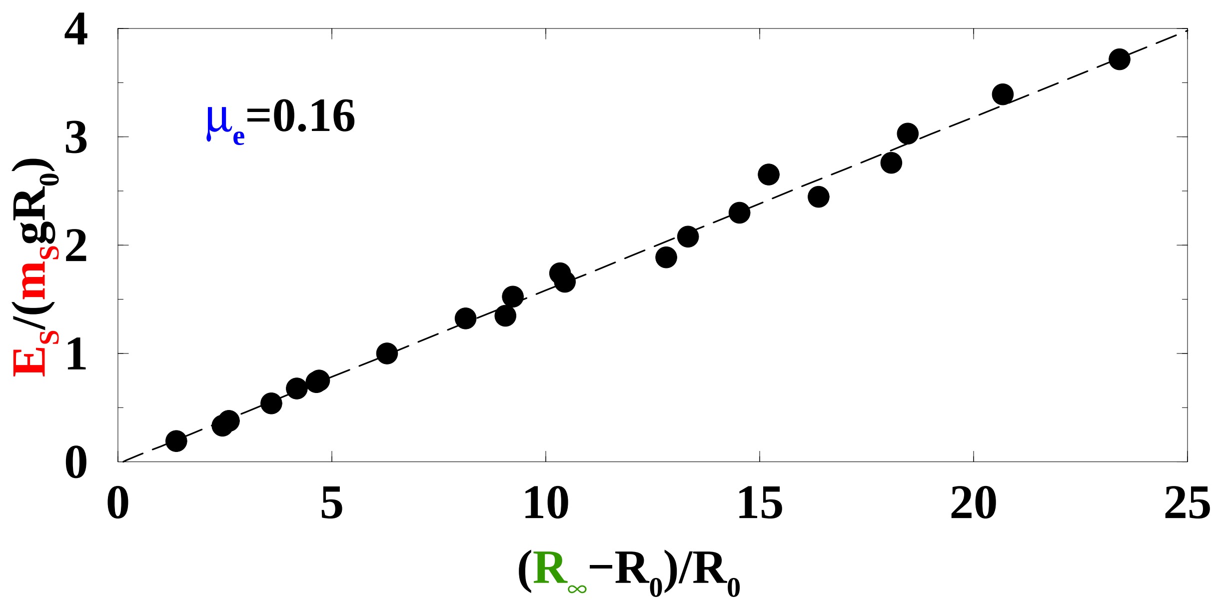


Mass flowing m_S and associated energy E_S as function of aspect ratio a



Dissipation of horizontal flow

Flowing mass m_S with energy E_s has runout R_∞



$$E_S = \mu_e m_S g (R_\infty - R_0)$$

with simple effective friction $\mu_e = 0.16$ independent of a.

$\mu_e = 0.47$ for centre of mass

A shallow-water model

For runout in a thin layer

Depth-averaged horizontal velocity $\overline{u}$

Depth-integrated horizontal momentum:

$$\frac{\partial(h\overline{u})}{\partial t} + \beta \frac{\partial(h\overline{u}^2)}{\partial r} = -K g h \frac{\partial h}{\partial r} - \mu g h$$

with

- β velocity profile factor
- K ‘Earth coefficient’
- μ basal Coulomb friction coefficient

Velocity profile factor β

Mass flux to momentum flux correction:

$$\beta = h \int_0^h u^2 \, dz \Big/ \left(\int_0^h u \, dz \right)^2$$

value depends on velocity profile in vertical

$$\beta = \begin{cases} 1 & \text{plug-flow} \\ \frac{4}{3} & \text{linear} \\ \frac{6}{5} & \text{parabolic} \end{cases}$$

Earth coefficient K

'Hydrostatic' balance in vertical

$$\sigma_{zz} = -\rho g(h(x) - z)$$

Plastic yielding

$$\sigma_{xx} = K\sigma_{zz} \quad \text{with} \quad K = \frac{1 + \sin\delta}{1 - \sin\delta}$$

Horizontal 'pressure gradient'

$$\frac{\partial \sigma_{xx}}{\partial x} = -K\rho \frac{\partial h}{\partial x}$$

Now

$$K = \begin{cases} 1/3 & \text{in 'passive failure'} \quad u_x > 0 \\ 3 & \text{in 'active failure'} \quad u_x < 0 \end{cases}$$

BUT best $K = 1$ Pouliquen & Forterre 2002 JFM

Basal friction μ

Take $\mu = 0.43$ to fit runout in simulations.

$$\frac{\partial(h\overline{u})}{\partial t} + \beta \frac{\partial(h\overline{u}^2)}{\partial r} = -Kgh\frac{\partial h}{\partial r} - \mu g h$$

Also take $\beta = 1$ for simplicity, and $K = 1$ as in previous studies.

Speculate: no change in qualitative behaviour for different values of coefficients

‘Raining’ into shallow water

Initial tall column is not shallow,
but known to free-fall, so velocity at base is gt

Add mass to thin horizontal layer as rain from the tall column

$$\frac{\partial h}{\partial t} + \frac{\partial (\overline{u} h)}{\partial r} = q$$

where

$$q(r,t) = \begin{cases} gt & 0 \le r \le R_0 \\ 0 & R_0 < r \end{cases} \qquad \text{for} \quad 0 < t < \sqrt{2H_0/g}$$

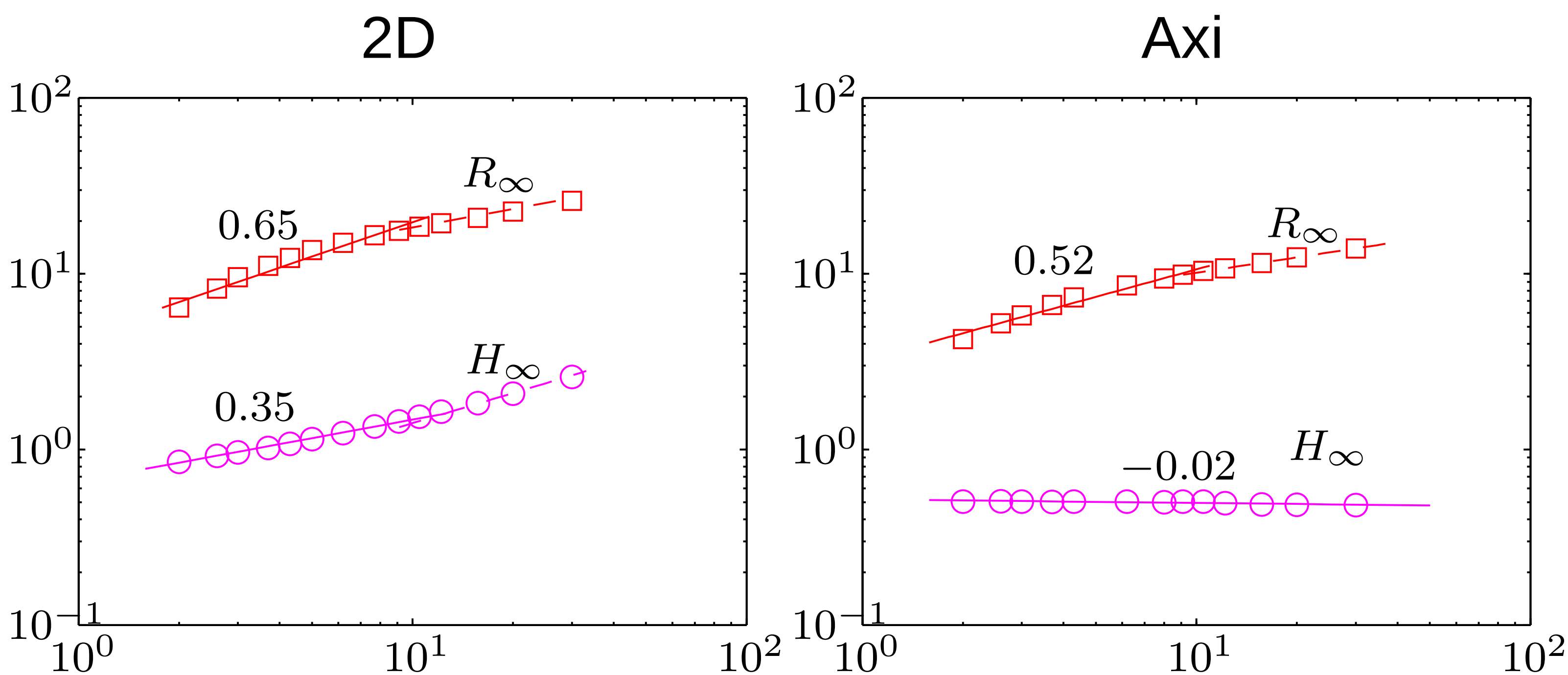
No change to momentum equation, as adding mass with zero horizontal momentum.

Numerical method

- Conservative, shock-capturing, Roe solver
- Pre-layer 10^{-7}, initial column height 10^{-1}
- Validation: dam-break ($\mu = 0$) in 2D
- Alternative Lagrangian method for 2D

Results from shallow-water model

Runout

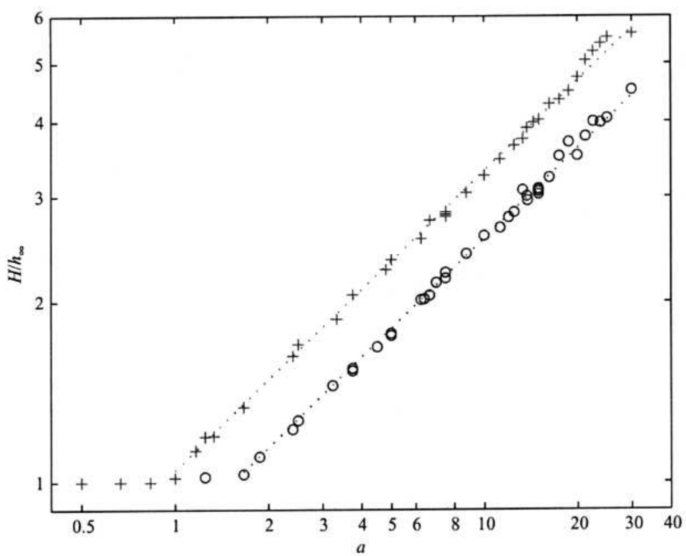


$$R_\infty/R_0 = \begin{cases} 1 + 4.4a^{0.65} & \text{2D} \quad 4.4 \longrightarrow 2 \text{ in experiments} \\ 1 + 3.2a^{0.52} & \text{Axi} \quad 3.2 \longrightarrow 1.8 \text{ in experiments} \end{cases}$$

Height of deposit

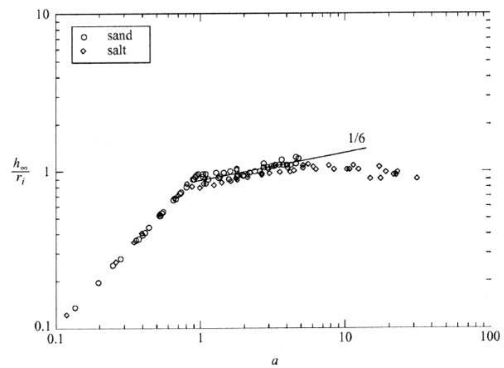
$$H_\infty/R_0 = \begin{cases} 0.66a^{0.35} & \text{2D} \quad 0.66 \longrightarrow 1 \quad \text{in experiments} \\ 0.52a^{-0.02} & \text{Axi} \quad 0.52 \longrightarrow 1 \quad \text{in experiments} \end{cases}$$

2D

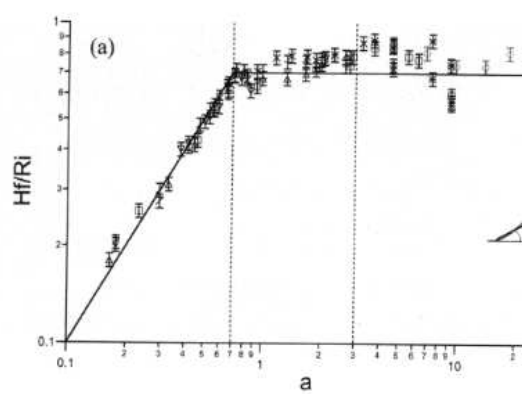


Balmforth & Kerswell 2005 JFM

Axi



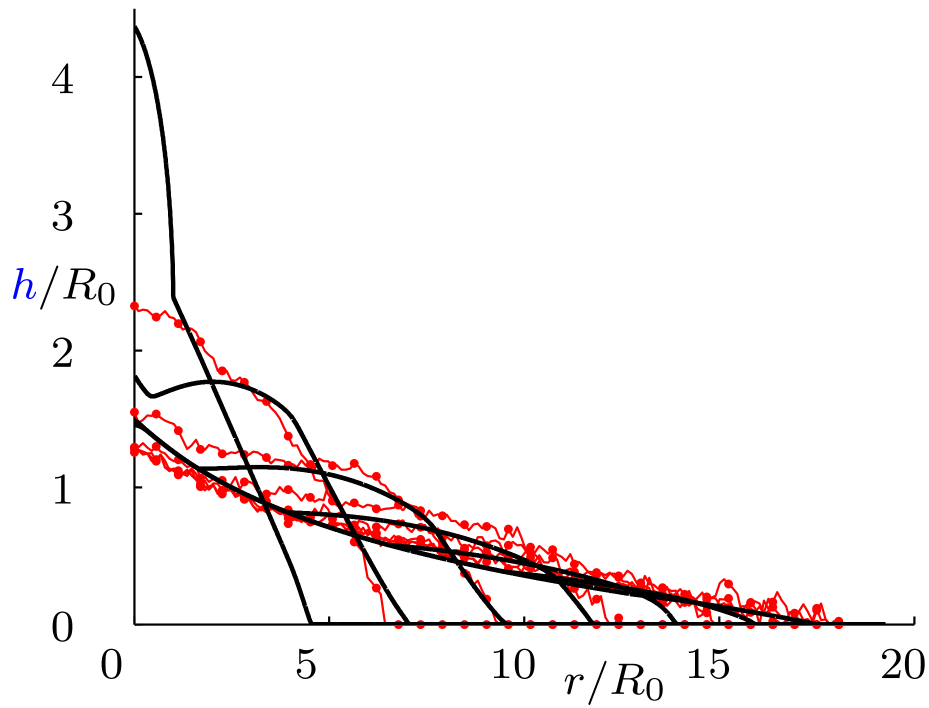
Lube, Huppert, Sparks & Hallworth 2004 JFM



Lajeunesse, Mangeney-Castelnau & Vilotte 2004 PoF

Evolution of deposit

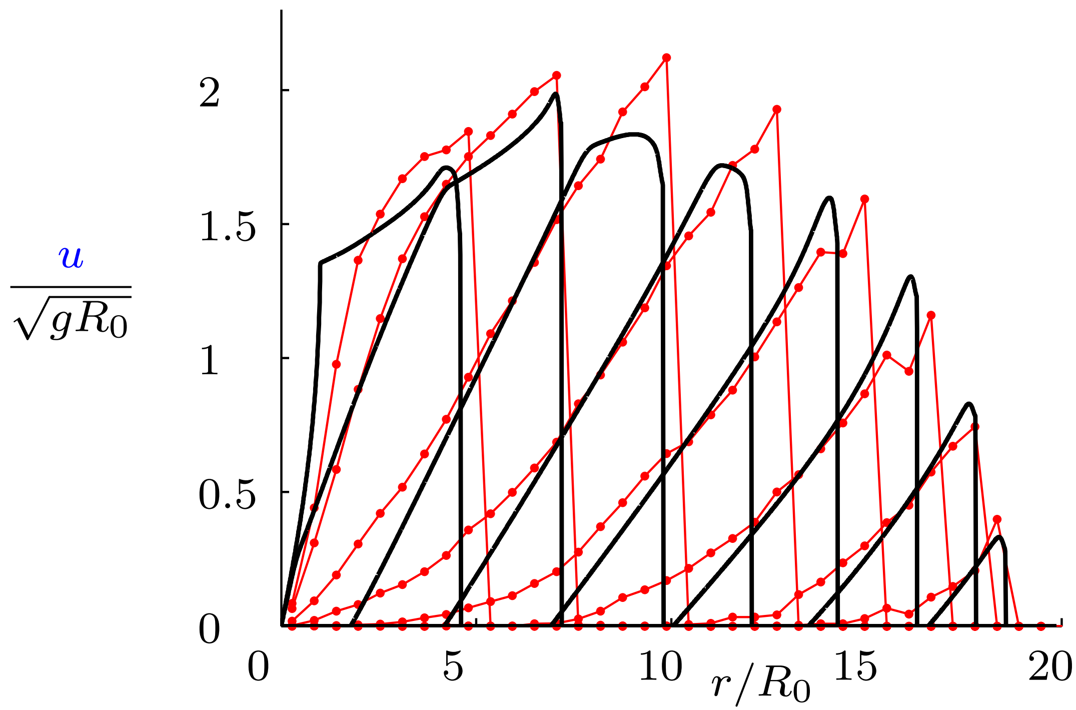
Shallow-water vs Simulations



$t = 0.3\,(0.1)\,1.0$, $a = 9.1$

Velocity profiles

Shallow-water vs Simulations



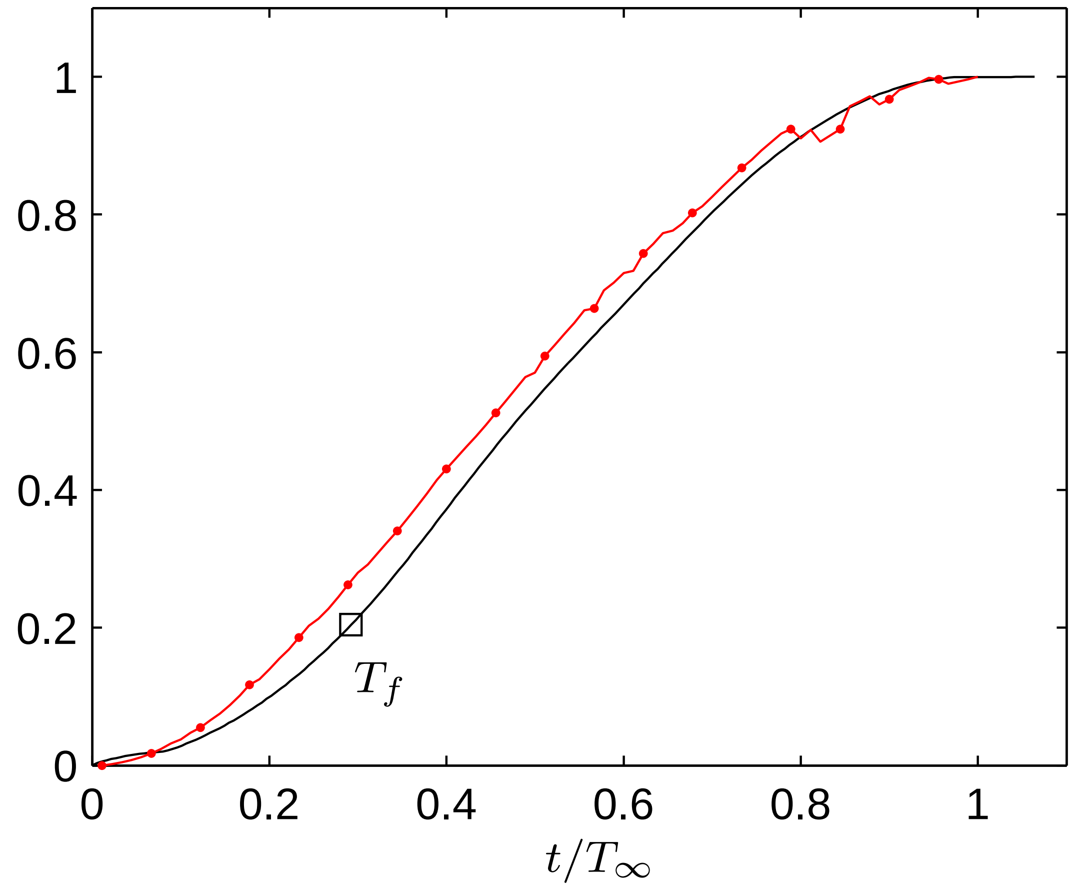
$t = 0.3\,(0.1)\,1.0$, $a = 9.1$

Stops from the centre, triangle wave propagates

Moving front

Shallow-water vs Simulations

$$\frac{r(t) - R_0}{R_\infty - R_0}$$



A little explanation?

Why 2D different from axisymmetric?

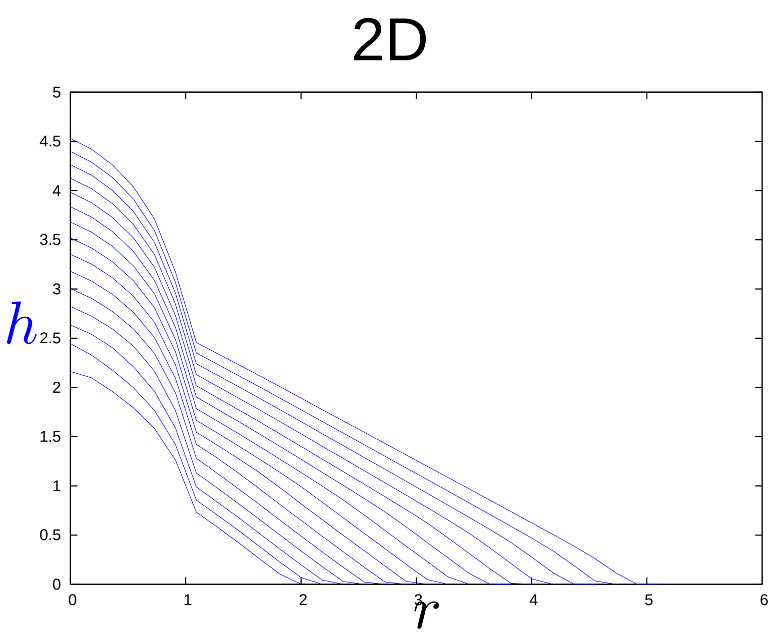
Simple power-laws?

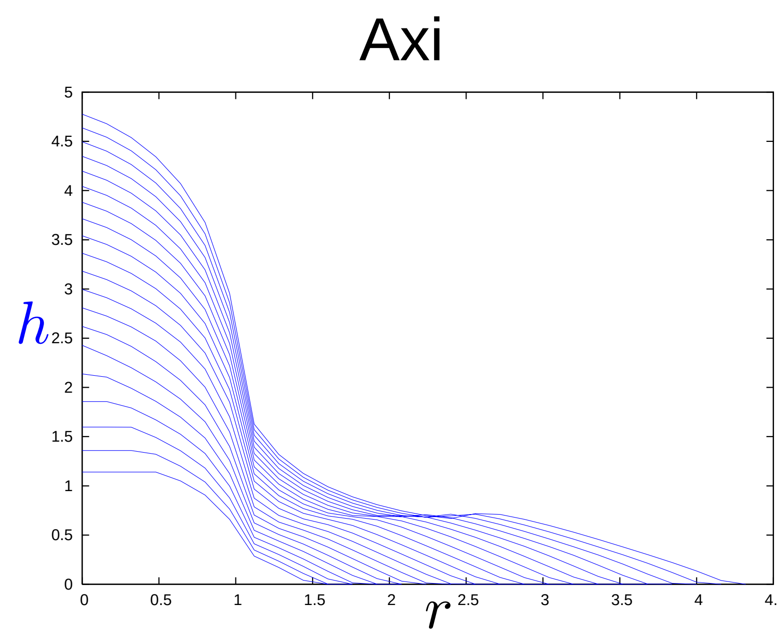
Three phases

- Leaving the base
- Propagating wave
- Deceleration

Leaving the base

During the rain $1.5 < t < 4.2 = \sqrt{2a}$, $a = 9.1$

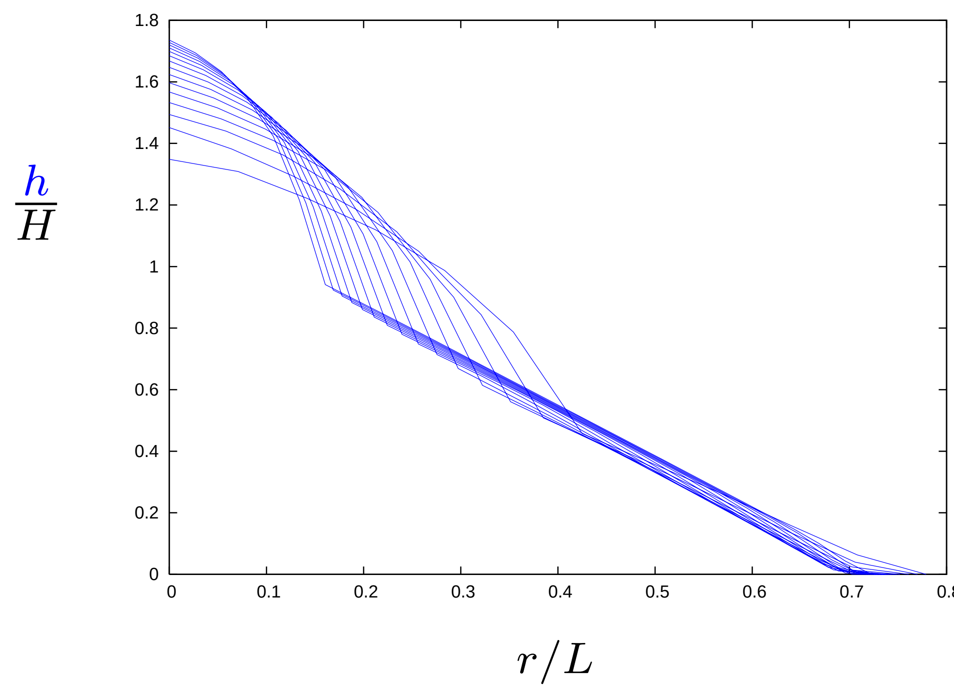




Leaving the base of the column, 2D

- Height $H(t)$ length $L(t)$
- Mass in 2D : $HL = gt^2 R_0$
- Acceleration by slope: $L/t^2 = gH/L$

$$L(t) = (gt^2)^{2/3} R_0^{1/3}$$
$$H(t) = (gt^2)^{1/3} R_0^{2/3}$$

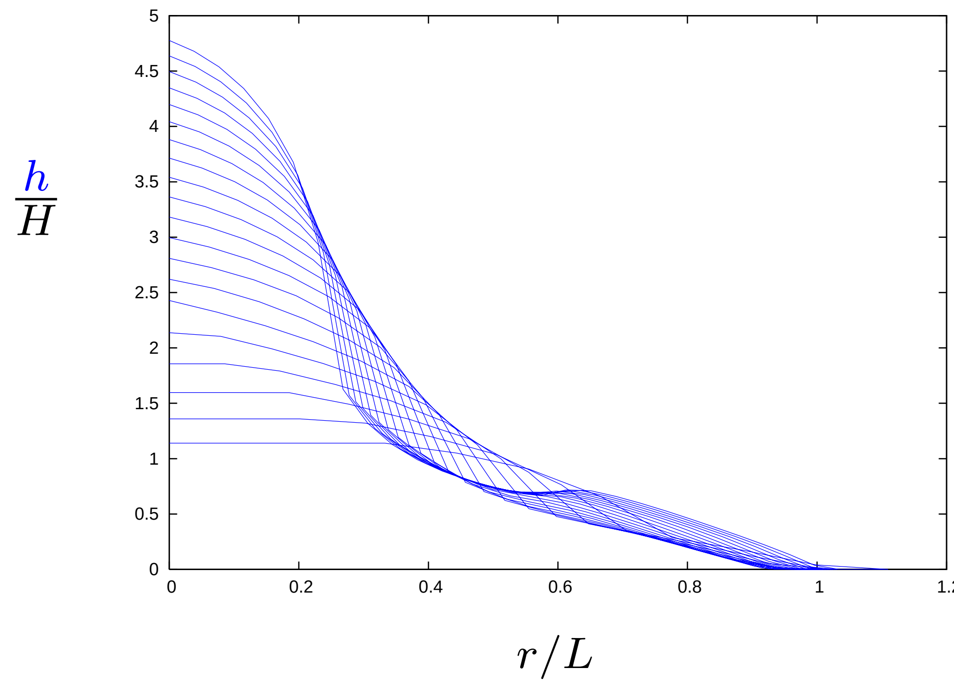


At the end of the rain $L = 1.1 R_0 a^{2/3}$

Leaving the base of the column, Axi

- Height $H(t)$ et Length $L(t)$
- Mass in Axi : $HL^2 = gt^2 R_0^2$
- Acceleration by slope: $L/t^2 = gH/L$

$$L(t) = (gt^2)^{1/2} R_0^{1/2}$$
$$H(t) = R_0$$



At the end of the rain $L = 1.4 R_0 a^{1/2}$

Difference between 2D and Axi

Axisymmetric geometry has more area to store grains, so shorter runout and lower height

$$L = \begin{cases} 1.4a^{1/2} & \text{Axi} \\ 1.1a^{2/3} & \text{2D} \end{cases}$$

at the end of the rain $t = \sqrt{2gH_0}$.

Final runout c3 times greater, but moving at $2\sqrt{gR_0}$.

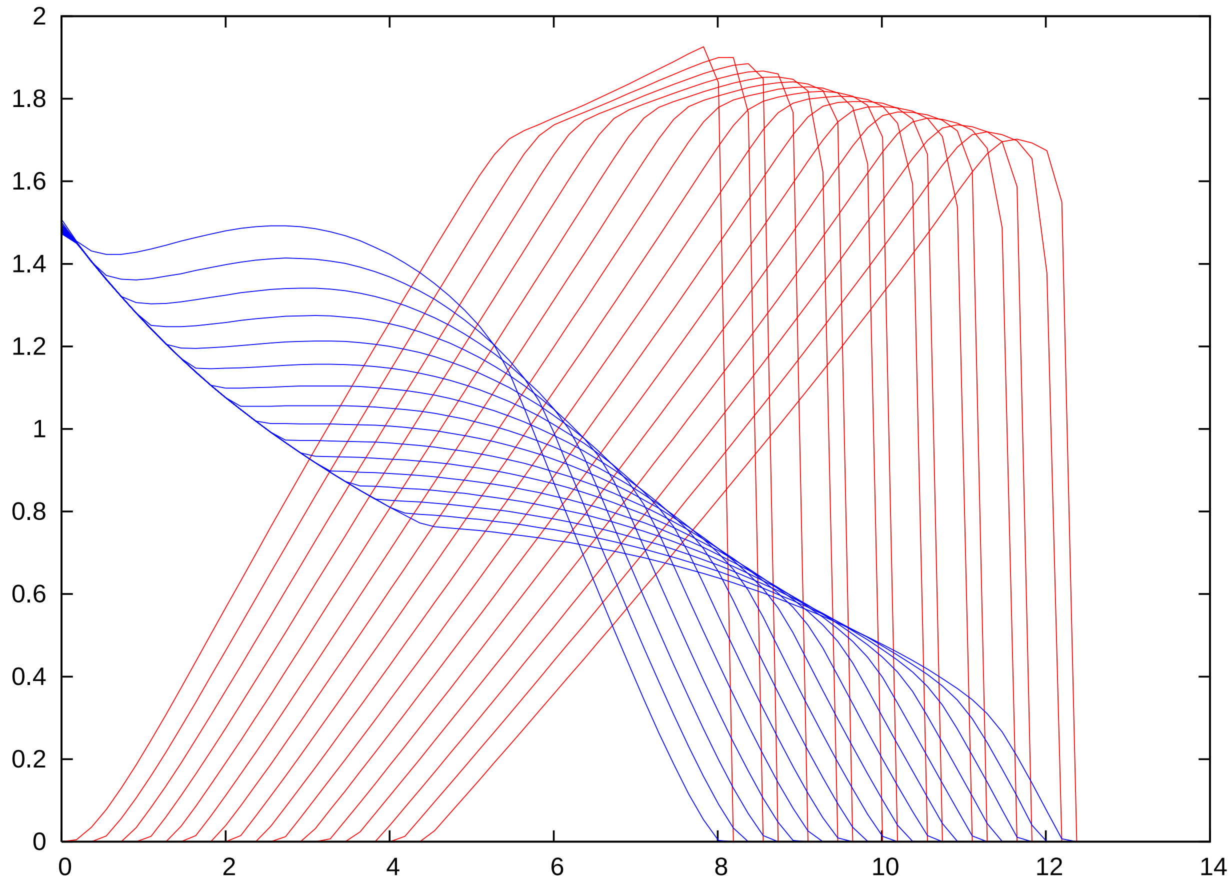
However deceleration of $2\sqrt{gR_0}$ at μg would only double runout. Need further.

Propagation of a wave, 2D

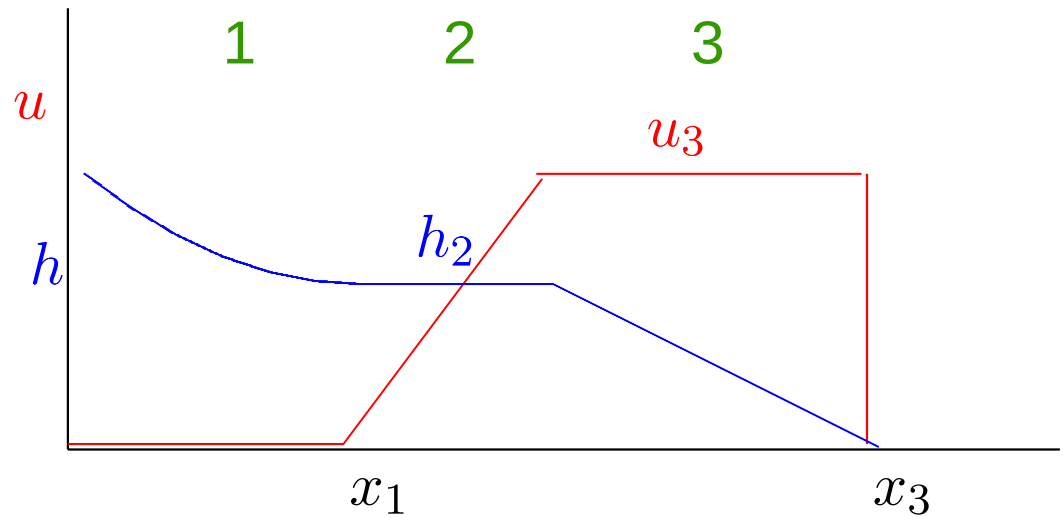
Extends runout before deceleration.

$5.8 < t < 8.1, \qquad a = 9.1$

$h(x,t)$
$u(x,t)$



A trapezoidal wave?



1: stopped $u(x,t) = 0$, $h(x,t) = h(x,\infty)$

2: flat $h(x,t) = h_2(t)$, $u(x,t) = \alpha(t)(x - x_1(t))$

3: constant velocity $u(x,t) = u_3$,

at angle of repose $h(x,t) = \mu(x_3(t) - x)$

Region 2

Flat $h(x,t) = h_2(t)$, so decelerate with μg

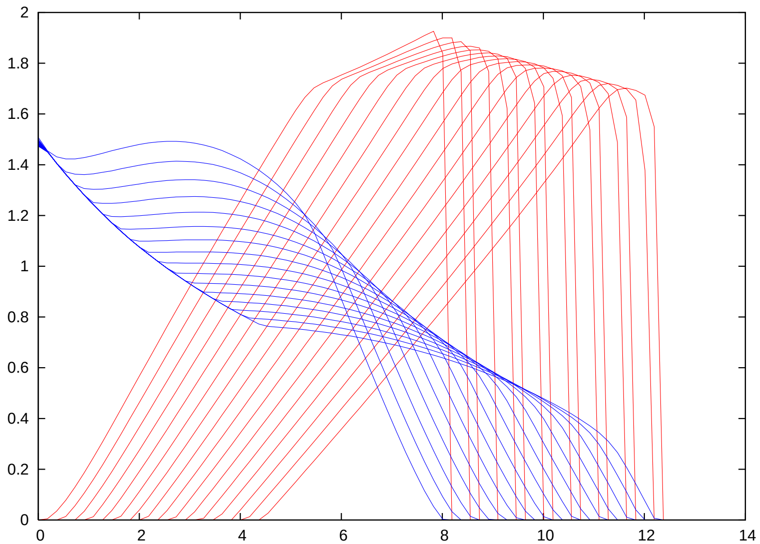
Linear velocity $u(x,t) = \alpha(x - x_1)$ is constant deceleration if

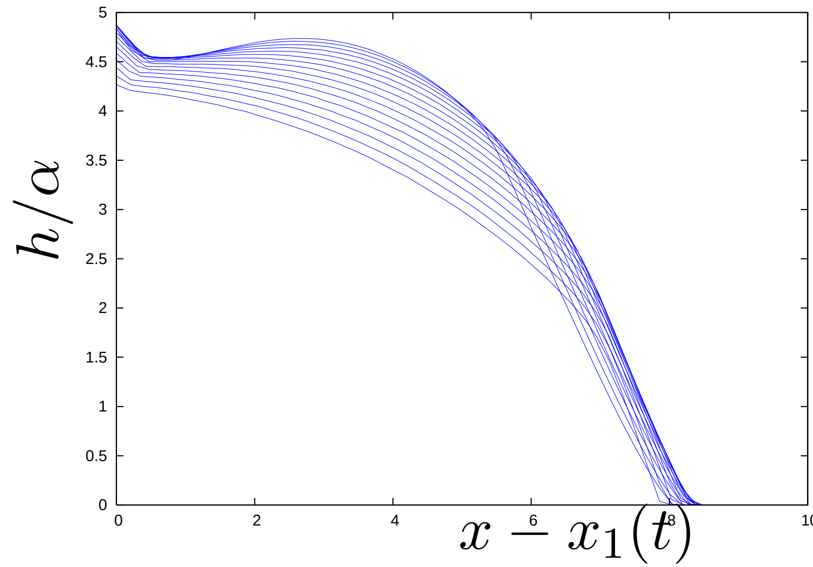
$$\alpha(t) = \frac{1}{t - t_0} \quad \text{and} \quad x_1(t) = x_0 + \tfrac{1}{2}\mu g (t - t_0)^2$$

And height decrease as $h_2(t) = h_0 \alpha(t)$

Test by plotting $h(x,t)/\alpha$ and $u(x,t)/\alpha$ vs $x - x_1$

Test solution for h in region 2

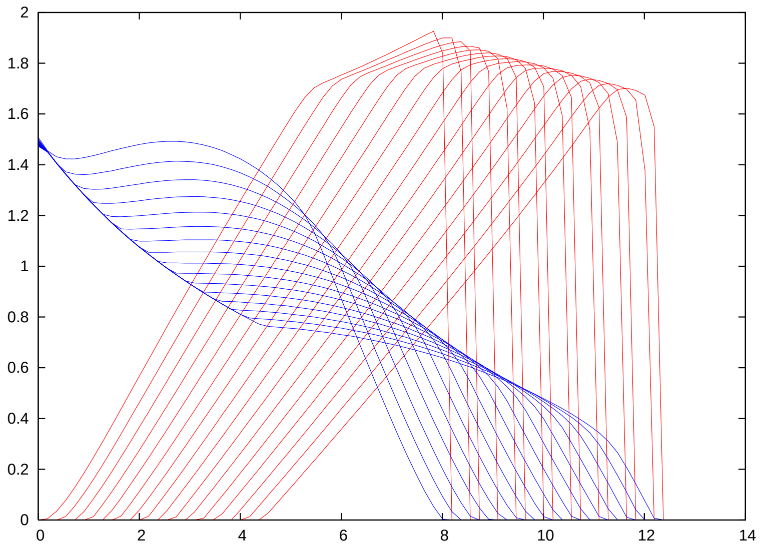


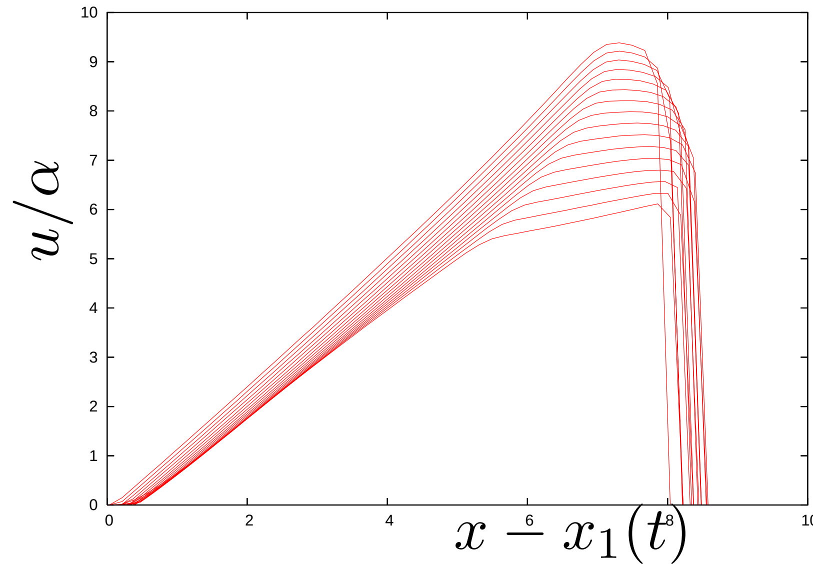


$$\alpha(t) = \frac{1}{t - t_0} \quad \text{and} \quad x_1(t) = x_0 + \tfrac{1}{2}\mu g(t - t_0)^2$$

Deceleration at μg

Test solution for u in region 2





$$\alpha(t) = \frac{1}{t - t_0} \quad \text{and} \quad x_1(t) = x_0 + \tfrac{1}{2}\mu g(t - t_0)^2$$

Deceleration at μg

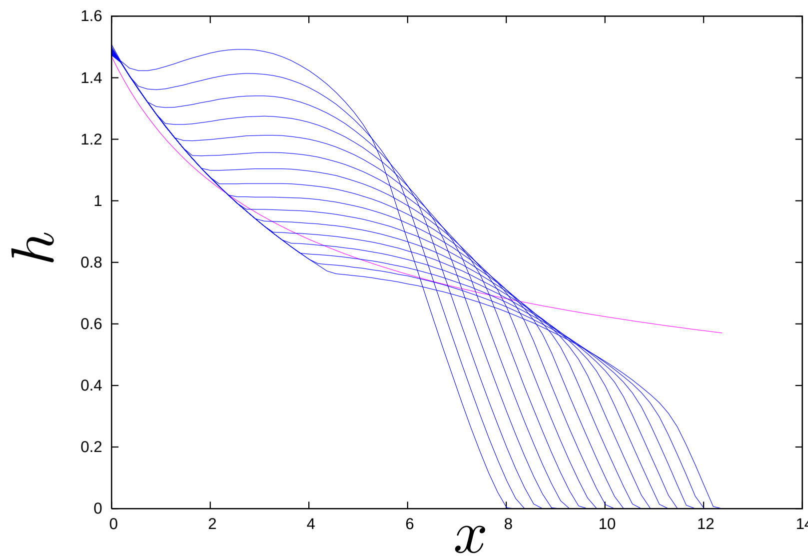
Shape of final deposit, region 1

Final deposit

$$h(x,\infty) = h_2(t) \quad \text{at} \quad x = x_1(t)$$

So

$$h(x,\infty) = \frac{h_0}{\sqrt{2(x-x_0)/\mu}}$$



Extension of runout during wave propagation

Initial length $L = 1.1 R_0 a^{2/3}$ of region 3 where $u = u_3 = 2\sqrt{g R_0} a^{1/6}$

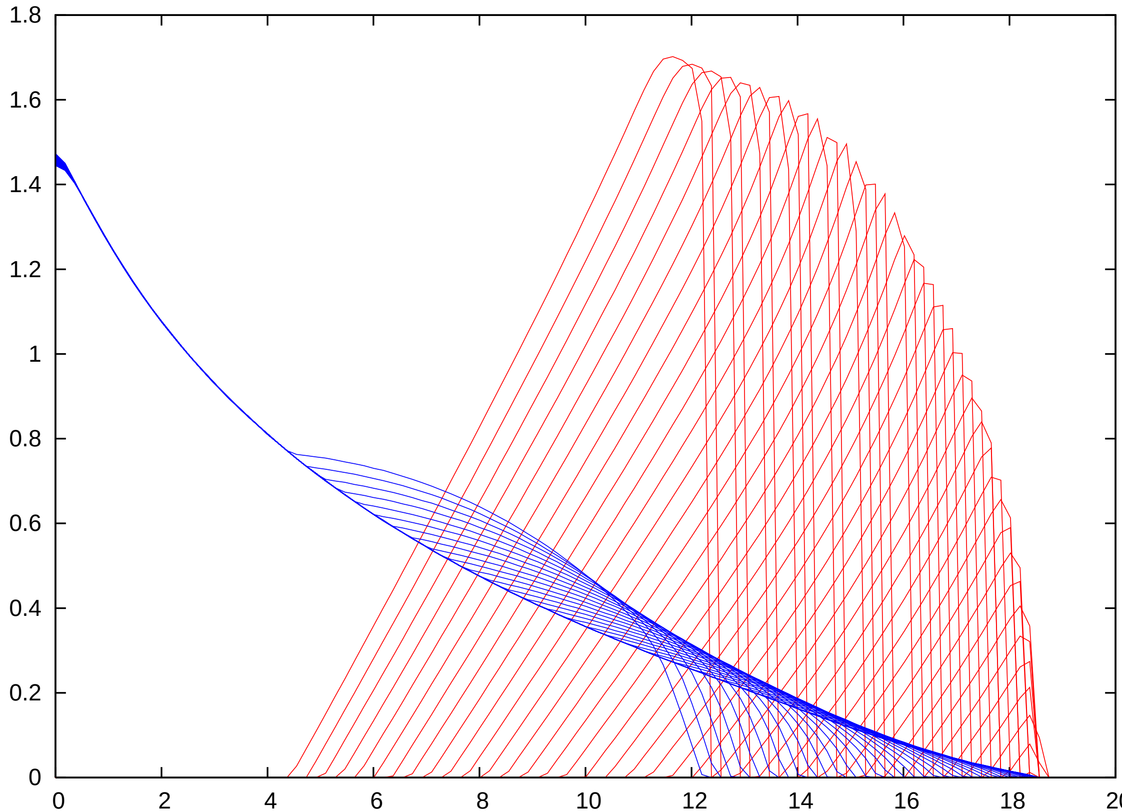
Accelerate at μg through L in time $\sqrt{\frac{2L}{\mu g}}$

Distance travelled at u_3 is $2.2 R_0 a^{1/2}$

Deceleration, 2D

$8.3 < t < 14, \qquad a = 9.1$

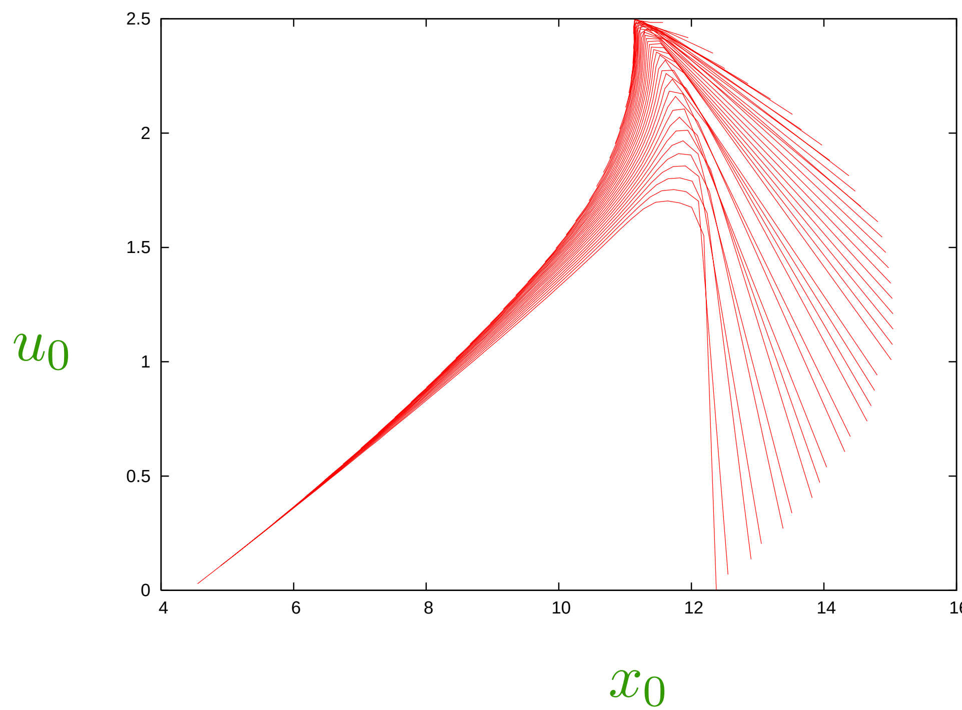
$h(x,t)$
$u(x,t)$



Deceleration:

Small slope, so initial conditions decelerate with μg:

$$u = \quad u_0(x_0) - \mu g(t - t_0)$$
$$x = \quad x_0 + u_0(x_0)(t - t_0) - \tfrac{1}{2}\mu g(t - t_0)^2$$



Extension of runout during deceleration

Decelerate from $u = u_3 = 2\sqrt{gR_0}a^{1/6}$

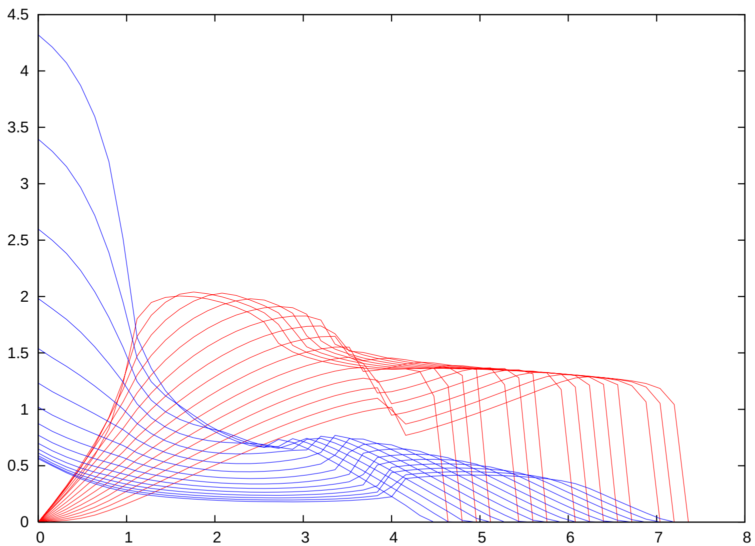
at μg

in distance $3.5a^{1/3}$

Axisymmetric similar

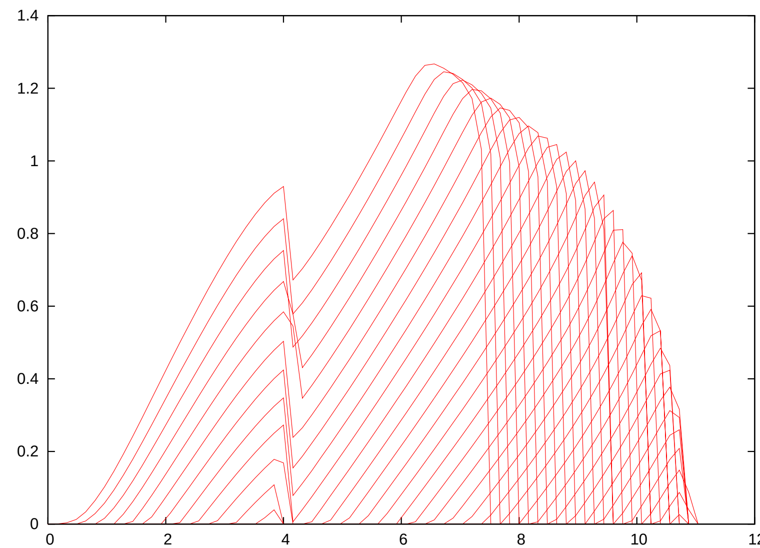
Propagating wave

$4.3 < t < 6.3 \qquad a = 9.1$



Deceleration

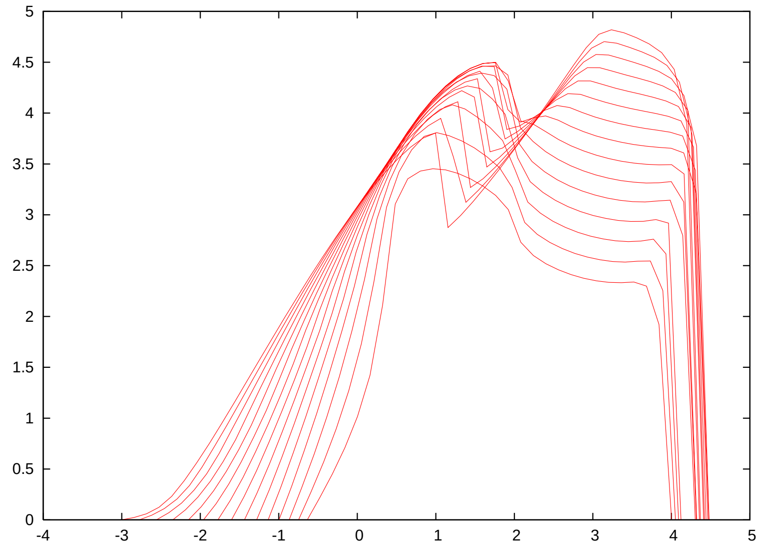
$6.5 < t < 11 \qquad a = 9.1$



Axisymmetric similar

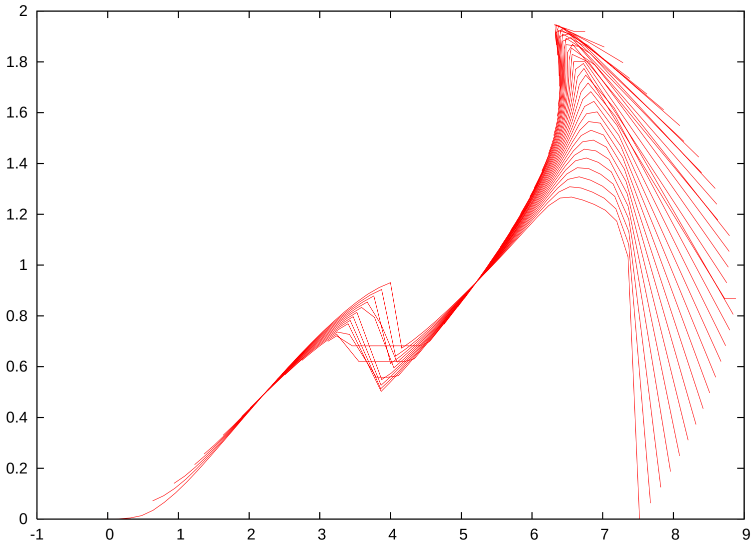
Propagating wave

$u\alpha(t)$



Deceleration

$u_0(x_0)$



A little explanation?

Three phases

- Leaving the base

  Runout $1.1R_0 a^{2/3}$ (2D) $1.5R_0 a^{1/2}$ (Axi)

- Propagating wave

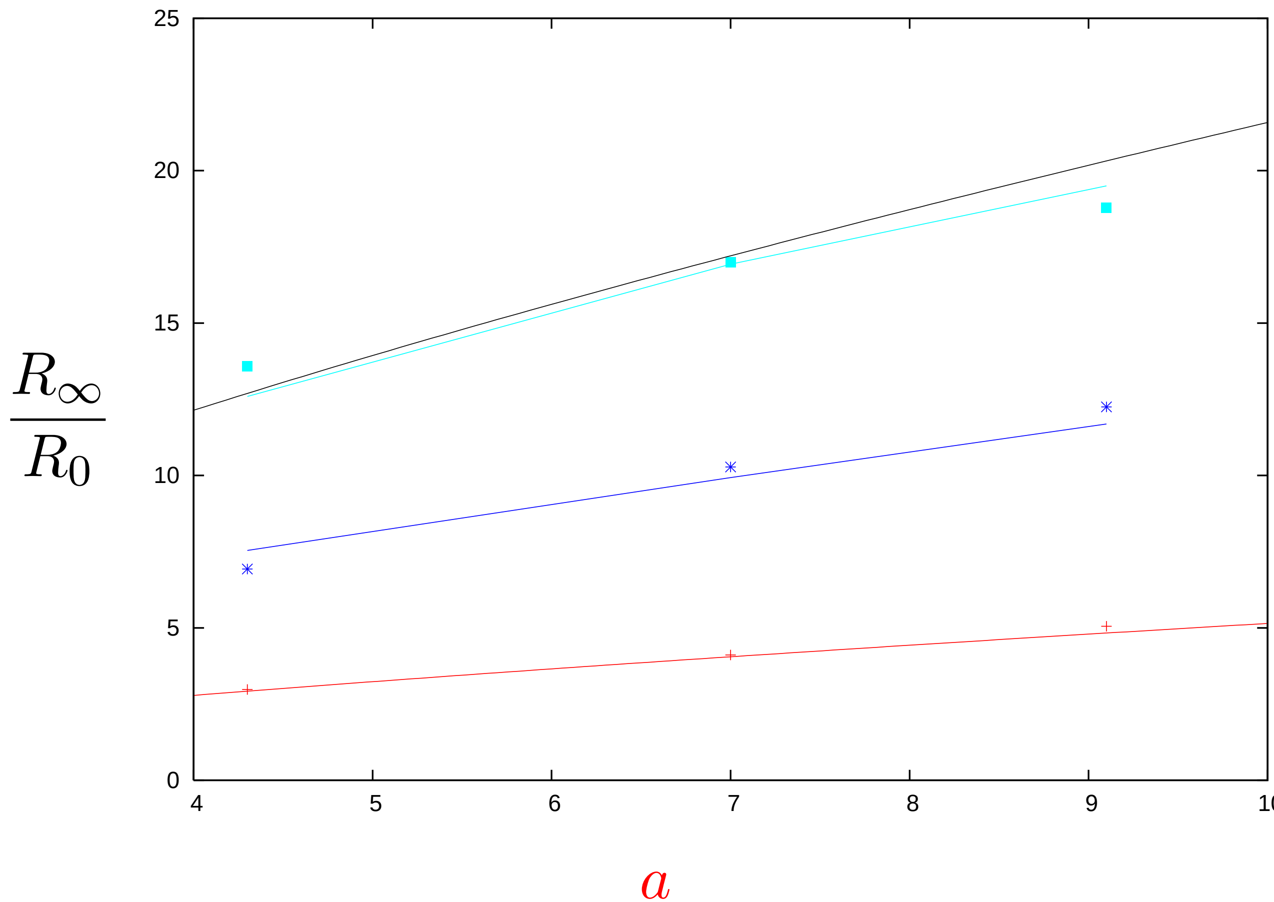
  $+2.2R_0 a^{1/2}$ $+1.6R_0 a^{1/4}$

- Deceleration

  $+3.5R_0 a^{1/3}$ $+3.7R_0$

A little explanation?

Three phases, 2D



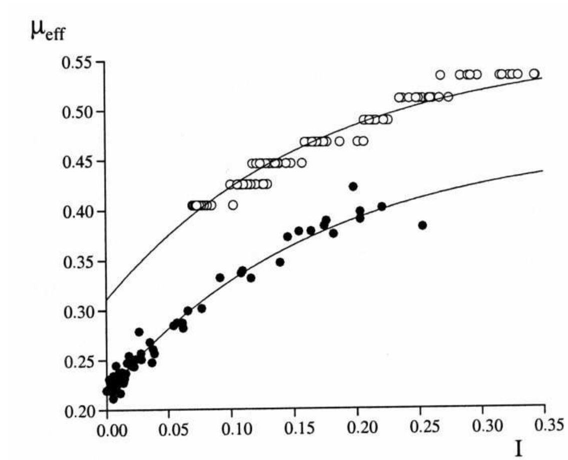
Next step?

A continuum description of dense granular flows? Plastic?

$$\sigma = -p\mathbf{I} + 2\mu p\mathbf{e}/\left|e\right|$$

Incompressible, with Coulombic friction for deviatoric stress.

$$\mu(I) = \frac{\mu_0 I_0 + \mu_\infty I}{I_0 + I}$$

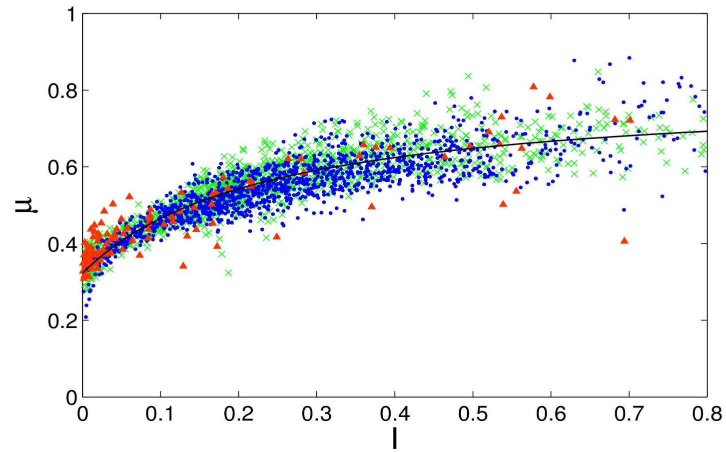


GdR MiDi 2004 EPJ-E

‘Inertia’ or Bagnold number

$$I = \dot\gamma d/\sqrt{p/\rho} = \text{deformation rate} \times \text{Relaxtion time}$$

Rheology from 3D-DEM of column collapse

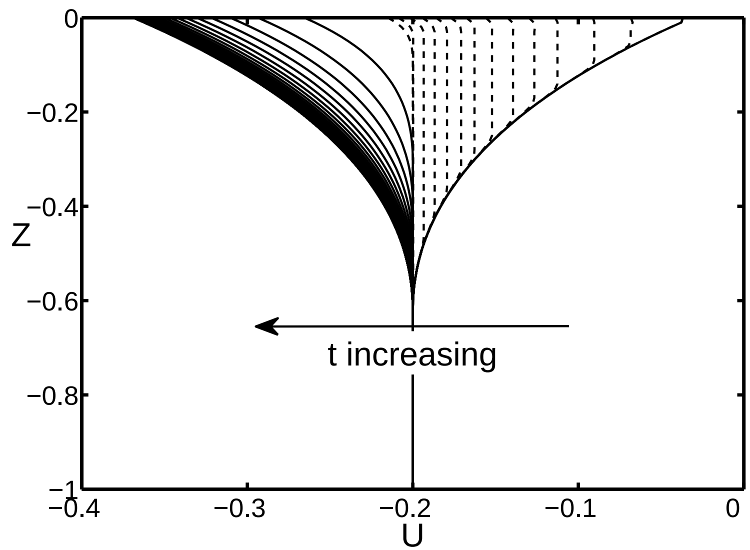


Laurant Lacaze & Rich Kerwell 2008/9?

How to compute with this rheology?

Reversed dragged plate

Chris Cawthorne

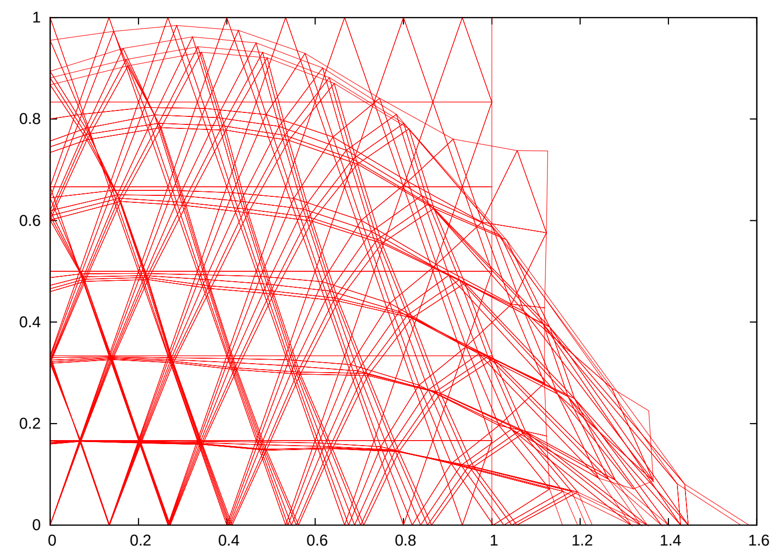


- Finite depth
- No smooth maximum – rigid blocks

Lagrangian Finite Element

Chris Cawthorne

$$\sigma = -p\mathbf{I} + 2\mu p\mathbf{e}/\left|\mathbf{e}\right|$$



Problems: $\mathbf{e}/\sqrt{\epsilon^2 + |e|^2}$
Spurious pressure modes – smooth
Highly deformed mesh – swap diagonals