

Explaining the flow of elastic liquids

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Complex fluids

- ▶ What & where? tooth paste, soup, ketchup, synthetic fibres, plastic bags, anti-splat ink-jet printing, oil well drilling muds
- ▶ Why & when? micron microstructure: nano reacts in 10^{-9} s, time \propto volume, so micron in 1s
- ▶ Which today? not shear-thinning (easy physics of thinning and easy effect on flow)
- ▶ Now have:
 - ▶ reproducible experiments - standard well-characterised fluids
 - ▶ numerics consistent - 5 benchmark problems
 - ▶ Time to ask: **what is underlying reason for effects**

More than: Viscous + Elastic

- ▶ **Viscous:**
Bernoulli, lift, added mass, waves, boundary layers, stability, turbulence
- ▶ **Elastic:**
structures, FE, waves, crack, composites
- ▶ **Visco-elastic is more**
Not halfway between Viscous & Elastic – strange flows to explain

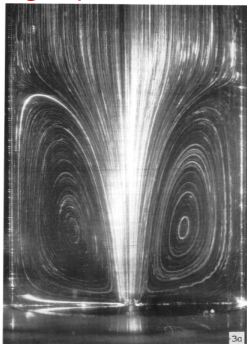
- ▶ Observations to explain
- ▶ How well does Oldroyd-B do?
 - half correct
- ▶ The FENE modification
 - anisotropy & stress boundary layers
- ▶ Conclusions – the reasons why

Flows to explain

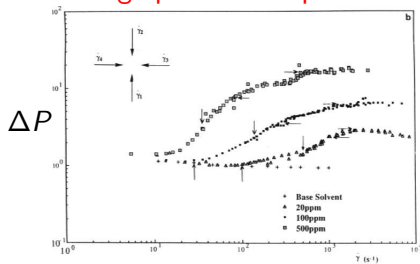
- ▶ **Contraction flow**
large upstream vortices, large pressure drop
- ▶ **Flow past a sphere**
long wake, increased drag
- ▶ **M1 project on extensional viscosity**
large stresses but confusion for value of viscosity
- ▶ **Capillary squeezing of a liquid filament**
very slow to break

Contraction flow

large upstream vortex



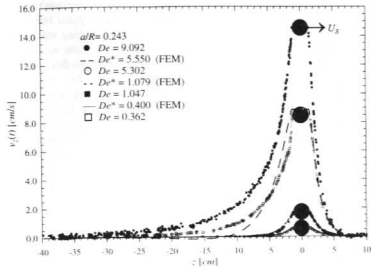
large pressure drops



Cartalos & Piau 1992 JNNFM 92

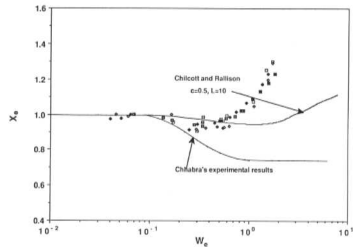
Flow past a sphere

long wake



Arigo, Rajagopalan, Shapley & McKinley 1995 JNNFM

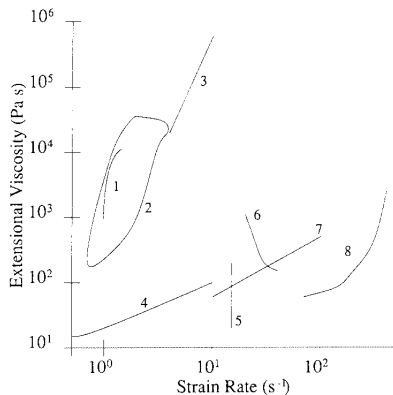
increased drag



Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

also negative wakes!

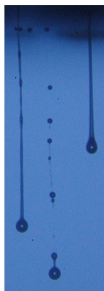
M1 project



Keiller 1992 JNNFM

no simple extensional viscosity

Effect of polymer of Drop-on-Demand printer



©2007 Steve Hoath, Ian Hutchings & Graham Martin

Too much polymer for jet to break up into drops.

Flows to explain

- ▶ **Contraction flow**
large upstream vortices, large pressure drop
- ▶ **Flow past a sphere**
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- ▶ **M1 project on extensional viscosity**
large stresses but confusion for value of viscosity
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Governing equations

Mass $\nabla \cdot \mathbf{u} = 0$

Momentum $\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot \sigma$

Constitutive $\sigma(\nabla \mathbf{u})$ Not known

Start with simplest

Oldroyd-B model fluid simplest viscous + elastic

$$\sigma = -p\mathbf{I} + 2\mu_0\mathbf{E} + G\mathbf{A}$$

stress viscous elastic
 μ_0 viscosity G elastic modulus

with \mathbf{A} microstructure.

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla\mathbf{u} + \nabla\mathbf{u}^T \cdot \mathbf{A} - \frac{1}{\tau}(\mathbf{A} - \mathbf{I})$$

deform with flow relaxes
 τ relaxation time

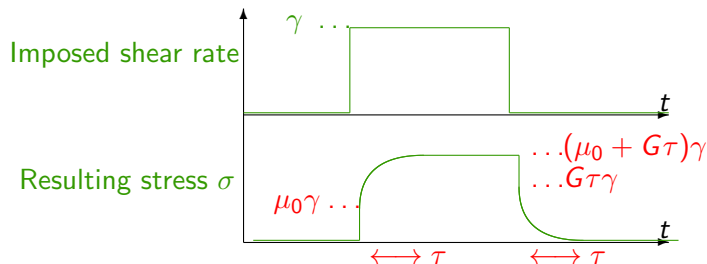
Does this model work/fail?

Investigating Oldroyd-B

1. Steady & weak $\frac{D}{Dt}, \nabla \mathbf{u} \ll 1/\tau$
2. Unsteady & weak $\nabla \mathbf{u} \ll 1/\tau$
– linear viscoelasticity
3. Slightly nonlinear $\nabla \mathbf{u} \lesssim 1/\tau$
– 2nd order fluid
4. Very Fast $\nabla \mathbf{u} \gg 1/\tau$
5. Strongly elastic $2\mu_0 E \ll GA$

But will suppress detailed maths and numerics.

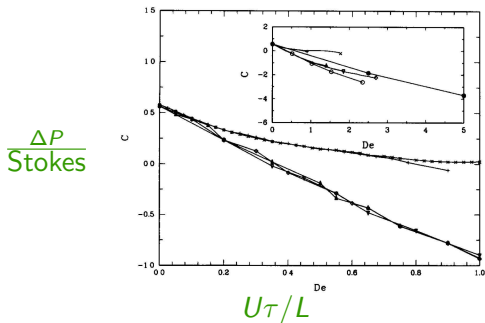
Linear visco-elasticity – common to all fluid models



- ▶ Early viscosity μ_0
- ▶ Steady state viscosity $\mu_0 + G\tau$
- ▶ Takes τ to build up to steady state;
steady deformation = shear rate $\gamma \times$ memory time τ

Stress relaxation is a special property of non-Newtonian fluids, which is not in elastic solids nor viscous liquids

Numerical Oldroyd-B



Debbaut, Marchal & Crochet
1988 JNNFM

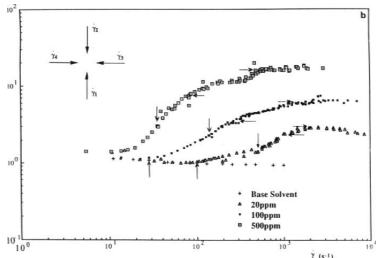
Coates, Armstrong & Brown
1992 JNNFM

Δp scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

But lower drop by early-time viscosity μ_0 if flow fast

Experiments

$$\frac{\Delta P}{\text{Stokes}}$$



Cartalos & Piau 1992 JNNFM 92

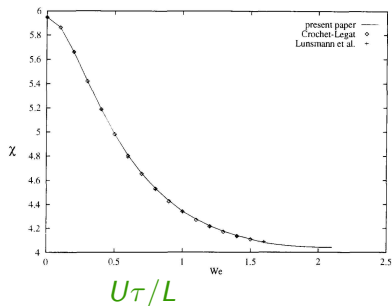
$$U_T/L$$

Experiments have a tiny decrease in pressure drop!

Oldroyd-B has no big increase in Δp , and no big upstream vortices

Numerical Oldroyd-B

Drag
Stokes



Yurun & Crochet 1995 JNNFM

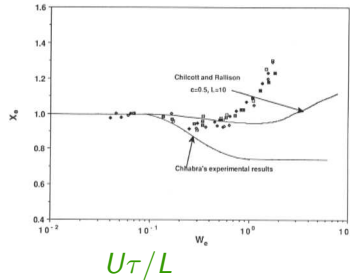
Drag scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

Lower drag by early-time viscosity μ_0 if flow fast

... flow past a sphere

Experiments

Drag
Stokes



Tirtaatmadja, Uhlherr & Sridhar

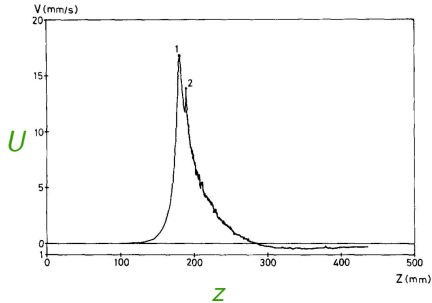
1990 JNNFM

Experiments have a tiny decrease in drag!

Oldroyd-B has no big increase in drag, and no big wake

... and negative wakes

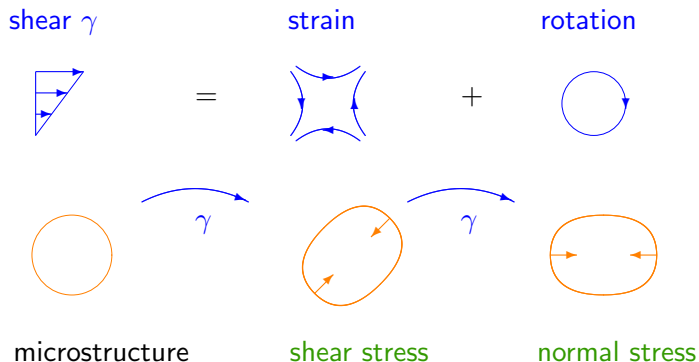
Experiment



Bisgaard 1983 JNNFM

Driven by unrelaxed elastic stress in wake.

Tension in streamlines – slightly nonlinear effect



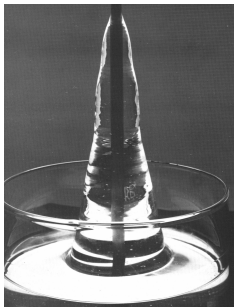
Shear stress = $G \times (\text{rate} = \gamma) \times (\text{memory time} = \tau)$

Normal stress (tension in streamlines) = shear stress $\times \gamma\tau$.

Tension in streamlines

- ▶ Rod climbing
- ▶ Secondary circulation
- ▶ Migration into chains
- ▶ Migration to centre of pipe
- ▶ Falling rods align with gravity
- ▶ Stabilisation of jets
- ▶ Co-extrusion instability
- ▶ Taylor-Couette instability

Rod climbing

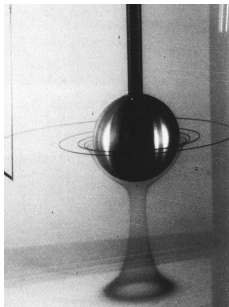


Bird, Armstrong & Hassager

1987, Vol 1 (2nd ed) pg 62

Tension in streamlines \rightarrow hoop stress
 \rightarrow squeeze fluid in & up.

Secondary flow



Bird, Armstrong & Hassager

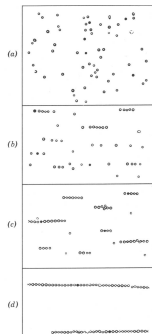
1987, Vol 1 (2nd ed) pg 70

Tension in streamlines \rightarrow hoop stress
 \rightarrow squeeze fluid in.

Non-Newtonian effects opposite sign to inertial

Migration into chains

shear γ



Bird, Armstrong & Hassager

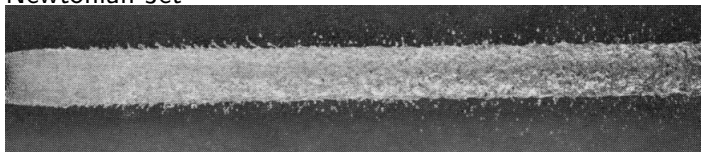
1987, Vol 1 (2nd ed) pg 87

Tension in streamlines \longrightarrow hoop stress
 \longrightarrow squeeze particles together

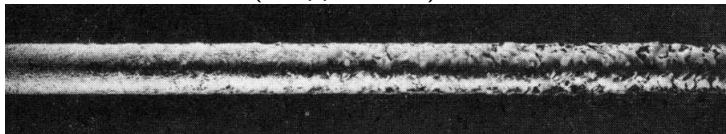
Also migration to centre of a tube,
and alignment with gravity of sedimenting rods.

Stabilisation of jets

Newtonian Jet



Non-Newtonian Jet (200ppm PEO)



Hoyt & Taylor 1977 JFM

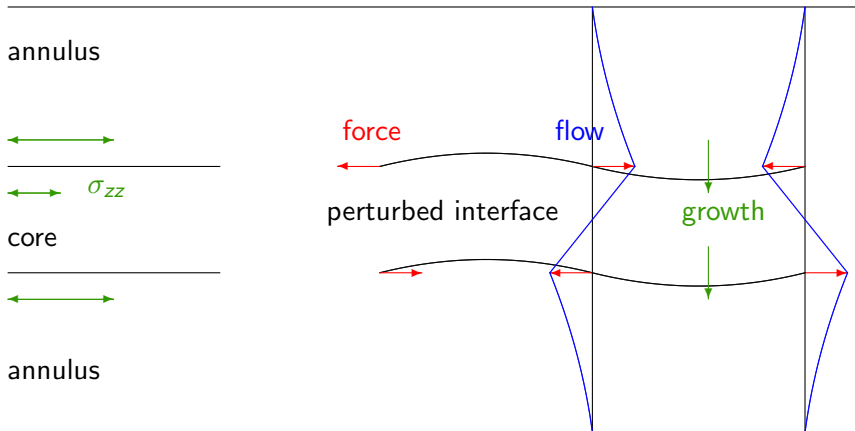
Tension in streamlines in surface shear layer

For fire hoses, and reduce explosive mist

Co-extrusion instability

If core less elastic, then jump in tension in streamlines

Jump OK is interface unperturbed



Tension in streamlines – when very fast

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \mathbf{A} - \frac{1}{\tau} (\mathbf{A} - \mathbf{I})$$

Fast: no time to relax: deforms where **speeds up** (steady flow)

$$\mathbf{A} = g(\psi) \mathbf{u} \mathbf{u} \quad \text{tensioned streamlines again}$$

g from matching to slower (relaxing) region

Momentum $\nabla \cdot \sigma = 0$, purely elastic $\sigma = G\mathbf{A}$

$$0 = -\nabla p + Gg^{1/2} \mathbf{u} \cdot \nabla g^{1/2} \mathbf{u}$$

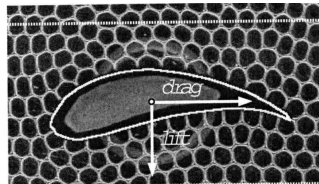
Euler equation!

... very fast

$$0 = -\nabla p + Gg^{1/2} \mathbf{u} \cdot \nabla g^{1/2} \mathbf{u}$$

Anti-Bernoulli

$$p - \frac{1}{2} Ggu^2 = \text{const}$$



Dollet, Aubouy & Graner 2005 PRL

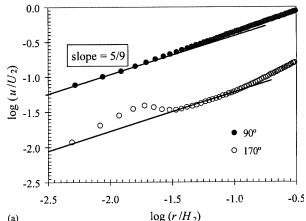
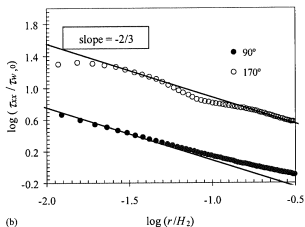
... very fast

Potential flows $g^{1/2} \mathbf{u} = \nabla \phi$

Flow around sharp 270° corner:

Hinch 1995 JNNFM

$$\phi = r^{2/3} \cos \frac{2}{3}\theta, \quad \sigma \propto r^{-2/3} \quad \psi = r^{14/9} \sin^{7/3} \frac{2}{3}\theta$$



Alves, Oliveira & Pinho 2003 JNNFM

Capillary squeezing – controlled by relaxation



Mass $\dot{a} = -\frac{1}{2} E a$

Momentum $\frac{\chi}{a} = 3\mu_0 E + G(A_{zz} - A_{rr})$

Microstructure $\dot{A}_{zz} = 2E A_{zz} - \frac{1}{\tau} (A_{zz} - 1)$

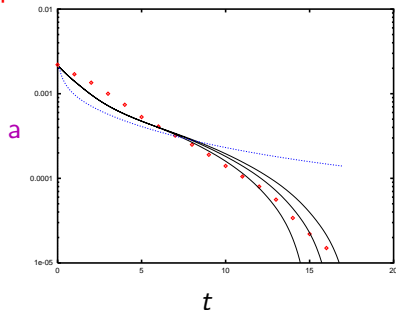
Solution $a(t) = a(0) e^{-t/3\tau}$

Need **slow** $E = 1/3\tau$ to stop A_{zz} relaxing from χ/Ga

... capillary squeezing

Oldroyd-B $a(t) = a(0)e^{-t/3\tau}$ does not break

Experiments S1 fluid

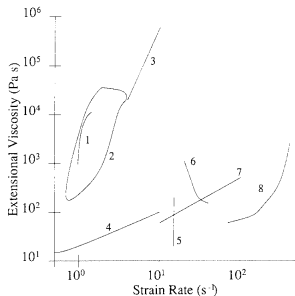


Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

but filament eventually breaks in experiments

no simple extensional viscosity



1. Open syphon
2. Spin line
3. Contraction
4. Opposing Jet
5. Falling drop
6. Falling bob
7. Contraction
8. Contraction

Keiller 1992 JNNFM

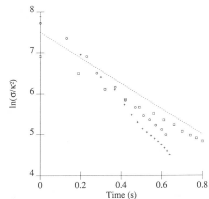
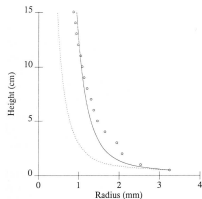
really elastic responses

... M1 project

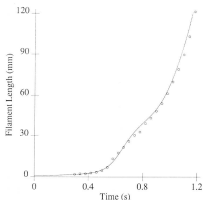
Fit data with Oldroyd-B: $\mu_0 = 5$, $G = 3.5$, $\tau = 0.3$ from shear

Keiller 1992 JNNFM

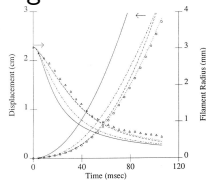
1. Open syphon Binding 1990 2. Spin line Oliver 1992



5. Falling drop Jones 1990



6. Falling bob Matta 1990



Oldroyd B: Successes & Failures

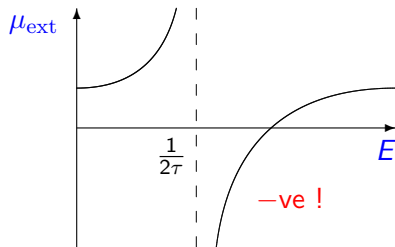
Simplest viscosity μ_0 + elasticity G + relaxation τ

- ▶ M1 Project
- ▶ Tension in streamlines
- ▶ Contraction:
 - ▶ Δp small decrease,
 - ▶ no big increase, no large vortices
- ▶ Sphere:
 - ▶ Force small decrease,
 - ▶ no increase, no long wake
- ▶ Capillary squeezing:
 - ▶ long time-scale,
 - ▶ no break

Also difficult numerically at $\frac{U\tau}{L} > 1$

Disaster in Oldroyd-B

Steady extensional flow



Microstructure deforms without limit if $E > \frac{1}{2\tau}$: $A = e^{(2E - \frac{1}{\tau})t}$

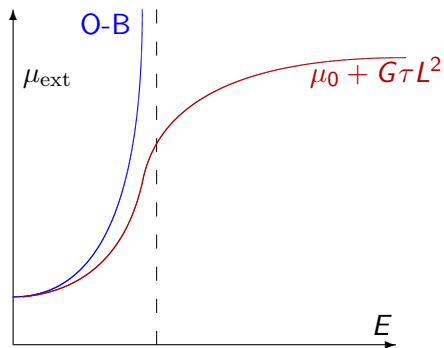
Need to limit deformation of microstructure

Finite Extension Nonlinear Elasticity

$$\frac{DA}{Dt} = A \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot A - \frac{f}{\tau} (A - \mathbf{I})$$

$$\sigma = -p\mathbf{I} + 2\mu_0 E + GfA$$

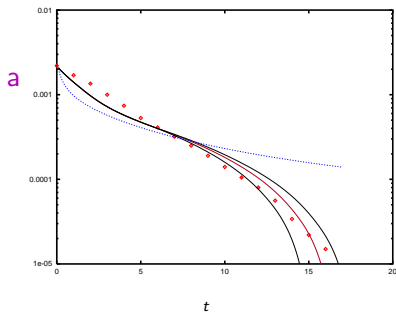
$$f = \frac{L^2}{L^2 - \text{trace } A} \quad \text{keeps } A < L^2$$



Large extensional viscosity $\mu_0 + G\tau L^2$, but small shear viscosity μ_0

FENE capillary squeezing

Filament breaks in with FENE $L = 20$

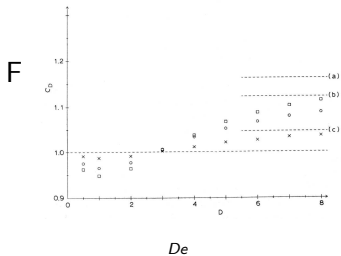


Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

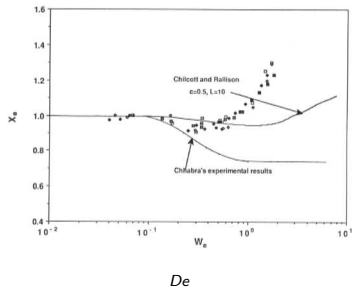
FENE flow past a sphere

FENE



Chilcott & Rallison 1988 JNNFM

Experiments M1

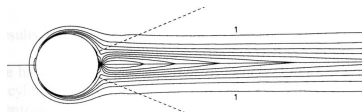


Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

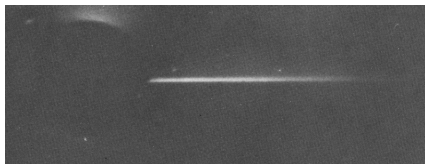
FENE gives drag increase

... FENE flow past sphere

FENE drag increase from long wake of high stress



Chilcott & Rallison 1988 JNNFM

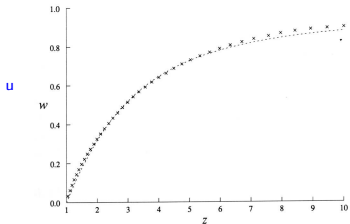
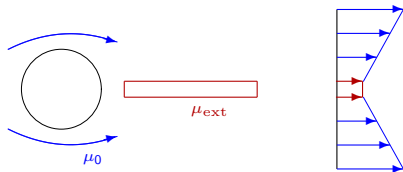


Cressely & Hocquart 1980 Opt Act

“Birefringent strand”

... birefringent strands

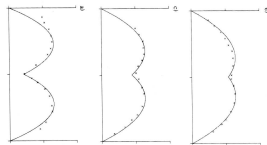
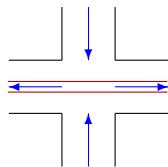
Boundary layers of high stress: μ_{ext} in wake, μ_0 elsewhere.



Harlen, Rallison & Chilcott 1990 JNNFM

... birefringent strands

Can apply to all flows with stagnation points, e.g.

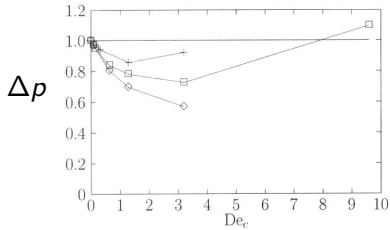


Harlen, Rallison & Chilcott 1990 JNNFM

Also cusps at rear stagnation point of bubbles.

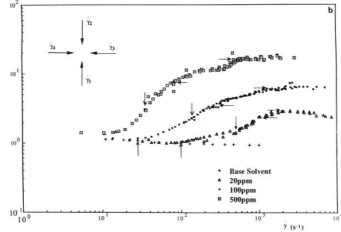
FENE contraction flow

FENE $L = 5$



Szabo, Rallison & Hinch 1997 JNNFM

Experiments



Cartalos & Piau 1992 JNNFM

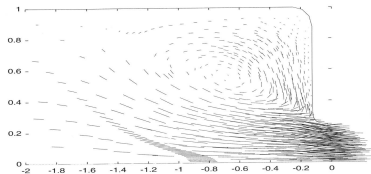
FENE gives increase in pressure drop

... FENE contraction flow

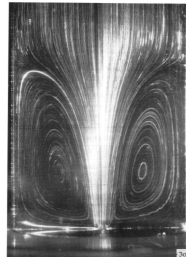
Increase in pressure drop from long upstream vortex

Experiments

FENE $L = 5$

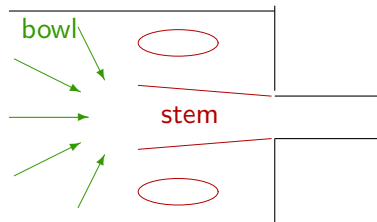


Szabo, Rallison & Hinch 1997 JNNFM



Cartalos & Piau 1992 JNNFM

... a champagne-glass model



Bowl: point sink flow, full stretch if $De > L^{3/2}$.

Stem: balance $\mu_{\text{ext}} \frac{\partial^2 u}{\partial r^2} = \mu_{\text{shear}} \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}$

if small cone angle $\Delta\theta = \sqrt{\frac{\mu_{\text{shear}}}{\mu_{\text{ext}}}}$

Flow anisotropy from material anisotropy: $\mu_{\text{ext}} \gg \mu_{\text{shear}}$ TDR

Conclusions for FENE modification

- ▶ **Contraction:** Δp increases, large upstream vortex
- ▶ **Sphere:** drag increase, long wake
- ▶ **Capillary squeezing:** filament breaks
- ▶ **Numerically safe**

Failures of Oldroyd-B corrected.

But sometimes need small L to fit experiments.

Understanding flow of elastic liquids?

In Oldroyd B

- ▶ Tension in streamlines
- ▶ Stress relaxation – transients see $\mu_0 < \mu_{\text{steady}}$
- ▶ Flows controlled by relaxation – E to stop relaxation, very slow

In FENE – deformation of microstructure limited

- ▶ μ_{ext} large – increase Δp & drag
- ▶ $\mu_{\text{ext}} \gg \mu_{\text{shear}}$ – flow anisotropy

independent of details of model?

More than viscous + elastic