Fluctuations in the velocities of sedimenting particles

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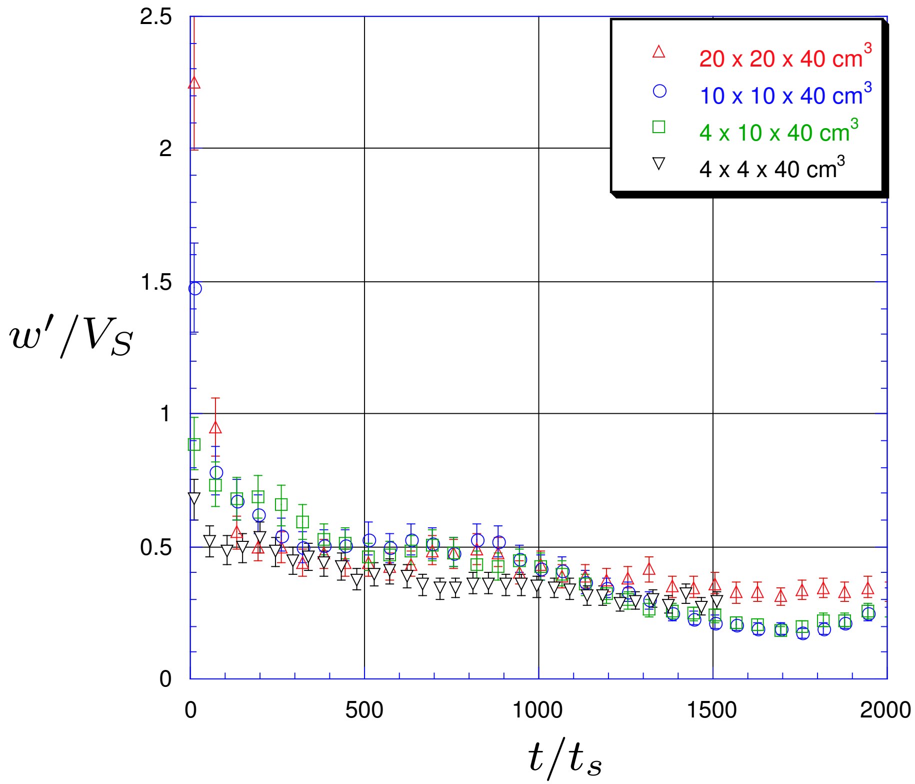
Old paradox: velocity fluctuations

- Theory: depend on size L of box $w' = V_S \sqrt{\phi \frac{L}{a}}$
- Experiments: no such dependence

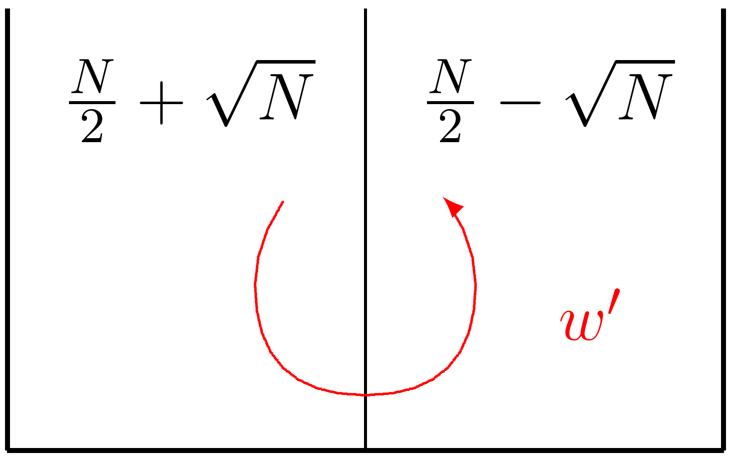


Scaling

- Theory (Caflisch & Luke 1985). Dilute: pair separated by r have $w' \sim V_S \frac{a}{r}$, $p \sim n$ (constant), so averaging

$$\int w'^2 p \, dV \quad \text{diverges like} \quad \phi \frac{L}{a}$$

- Explanation (Hinch 1988)



$$w' = \frac{\sqrt{N} mg}{6\pi\mu L} = V_S \sqrt{\phi \frac{L}{a}}$$

Computer simulations

Solve Stokes equations in fluid and force balance on each particle.

Good to Poor:

- Boundary integral (Acrivos) Singular subtractions, iterate from last δt, adapt grid.
- FEM (Joseph) or FD or Lattice Boltzmann (Ladd) with $\mathbf{f}(\mathbf{x})$ inside particles so rigid.
- Stokesian Dynamics (Brady) pairwise approx near, with multi-particle far re-summed.
- Today's a few Fourier modes.

Numerical Method

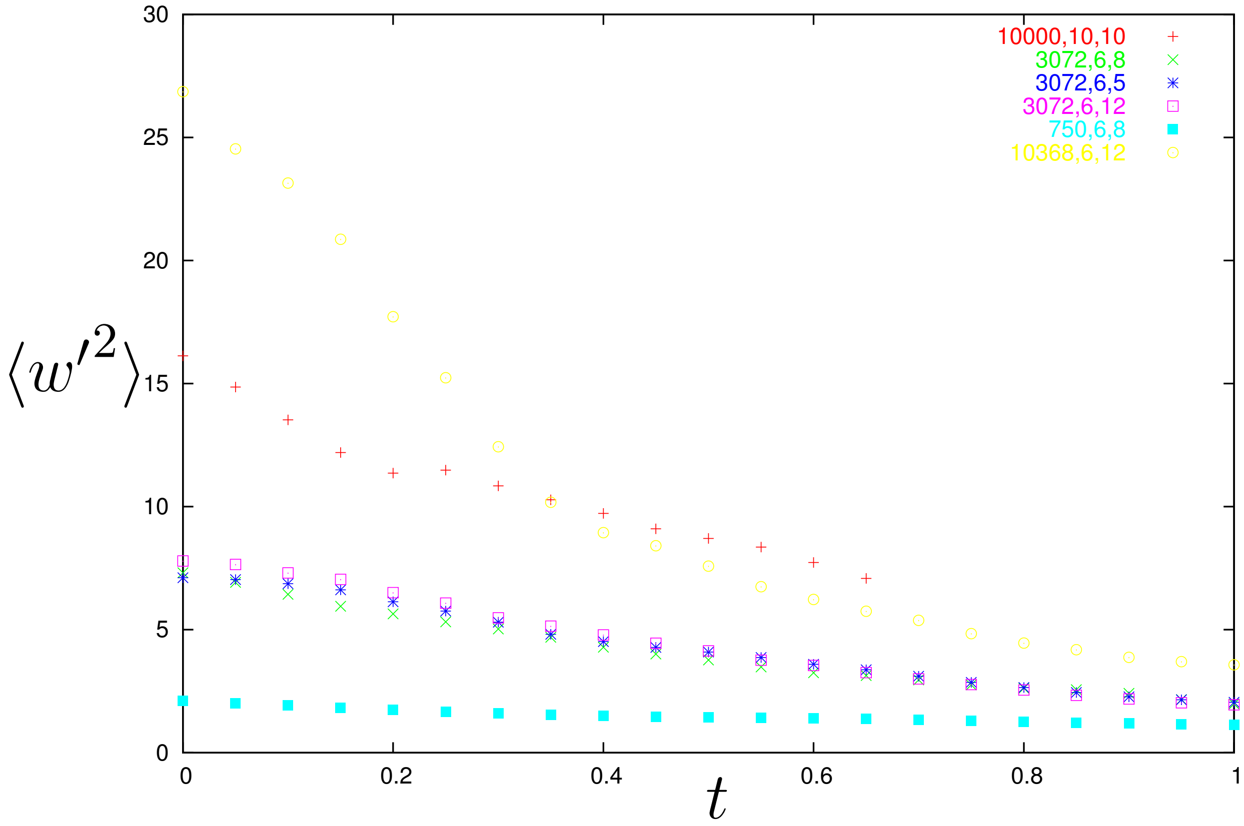
- Point-force particles (dilute), mono-disperse.
- Stokes flow by Fourier modes:

$$\mathbf{u} = \sum_{klm} U_{klm} \left(k \sin kx \cos ly \cos mz, \ldots, \ldots \right)$$

- Take number of Fourier modes $\simeq$ number of particles.

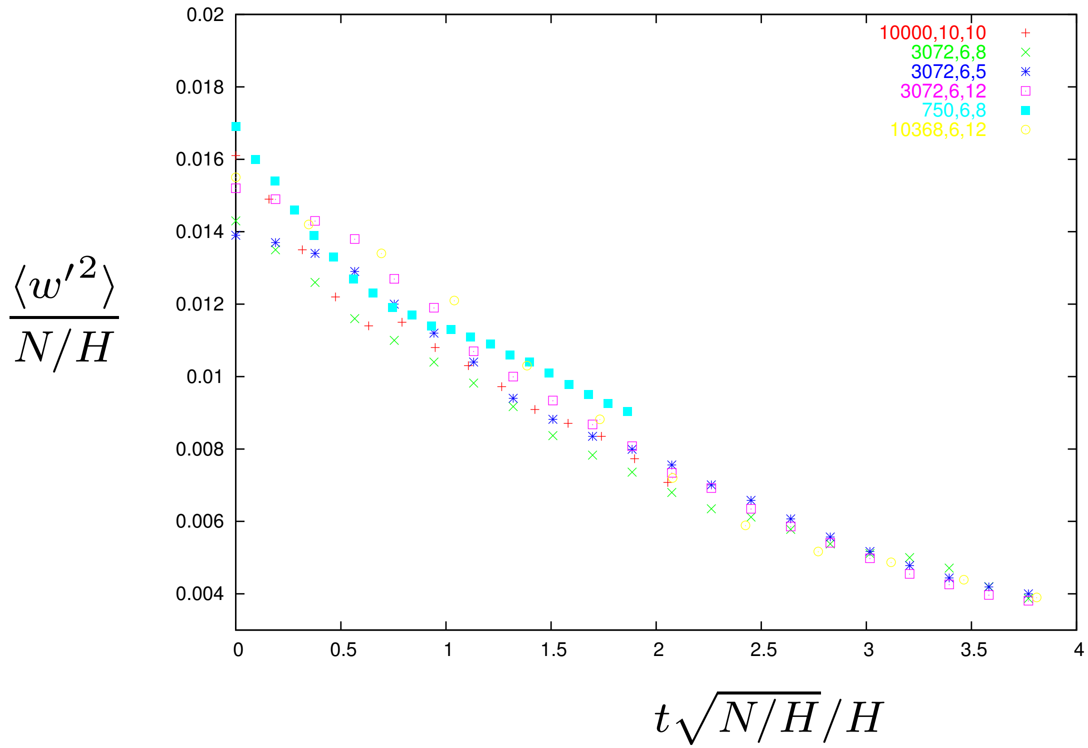
  Hence resolve flow-scales $\geq$ inter-particle separation.

Velocity fluctuations



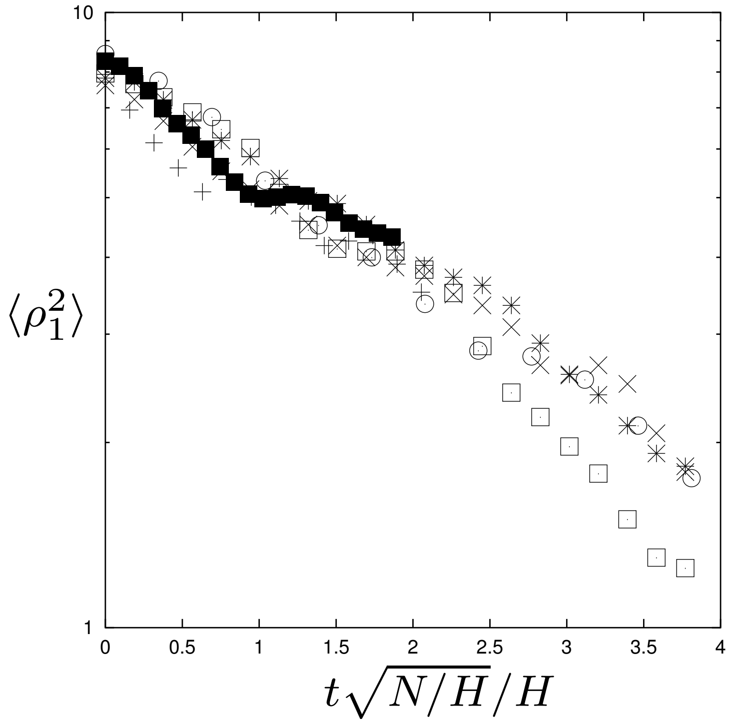
Different numbers of particles, height of cell, and Fourier modes.

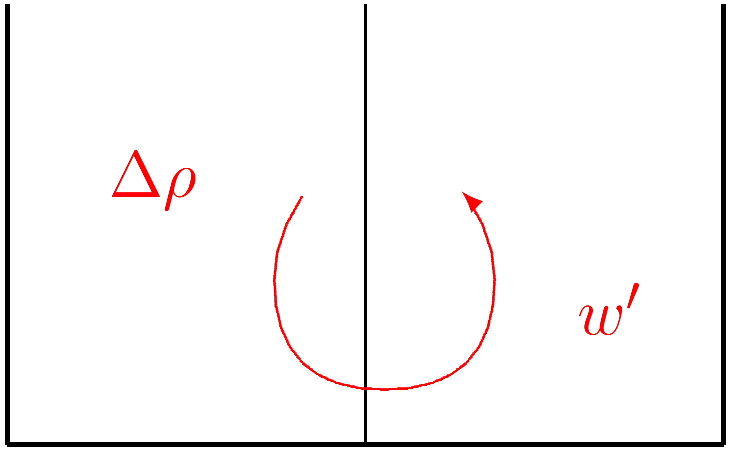
Velocity fluctuations replotted



- Hence initially $w' = 1.1 V_S \sqrt{\phi L/a}$.
- Decays on time to fall H at w'.

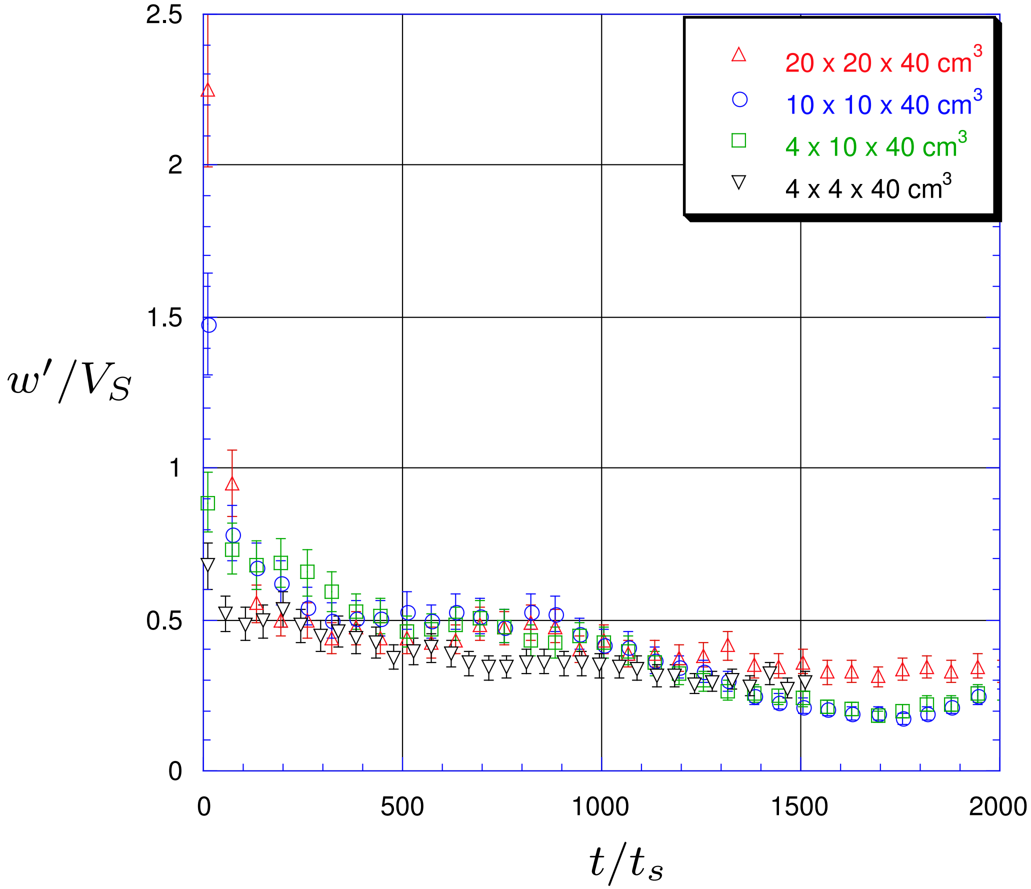
Density fluctuations also decay





- Velocity fluctuations are convection due to horizontal density variations.

But experiments. . .



. . . do not decay after initial adjustment.

Initial values $1.2\sqrt{\phi L/a}$?

Resolution: stratification? (Luke 2000)

- Consider blob of size ℓ, number density n.
- Number in blob $N = n\ell^3$, fluctuation $\sqrt{N}$.
  Variation due to stratification $\ell \frac{\partial}{\partial z}\left(n\ell^3\right)$.
- Hence stratification wins when

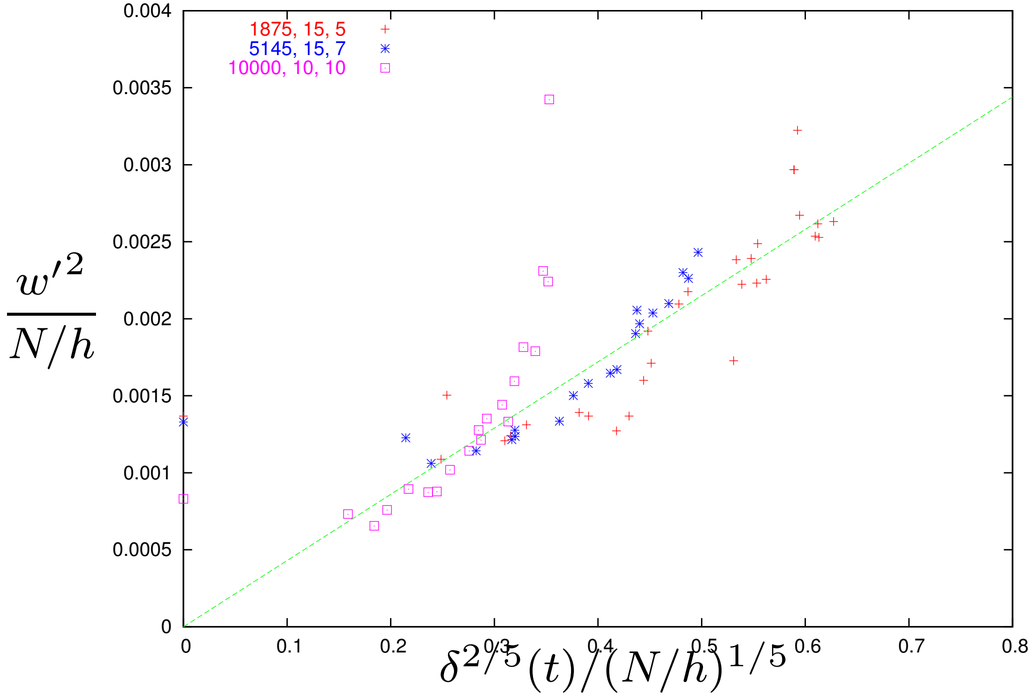
$$\ell > \ell_* = \left(n^{1/2} \Big/ -\frac{\partial n}{\partial z}\right)^{2/5}$$

- Then velocity fluctuation, indpt of box size if $\ell_* < L$.

$$w'_* = \frac{{N_*}^{1/2} mg}{6\pi\mu\ell_*} = V_S\left(\phi\frac{\ell_*}{a}\right)^{1/2} = V_S\frac{\phi^{3/5}}{\left(-a\frac{\partial\phi}{\partial z}\right)^{1/5}}$$

- But what determines $\frac{\partial\phi}{\partial z}$ in experiments?

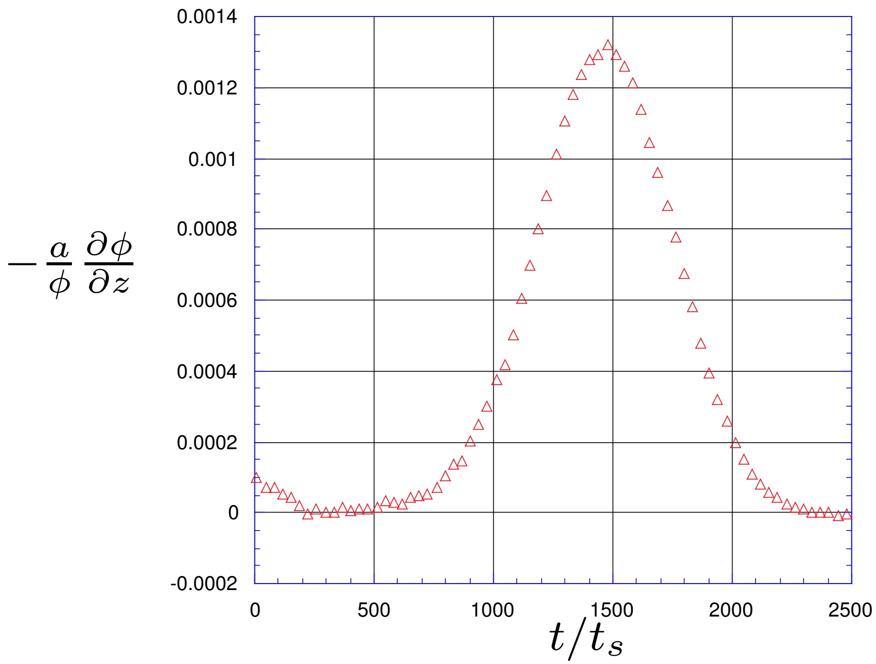
Numerical test of stratification

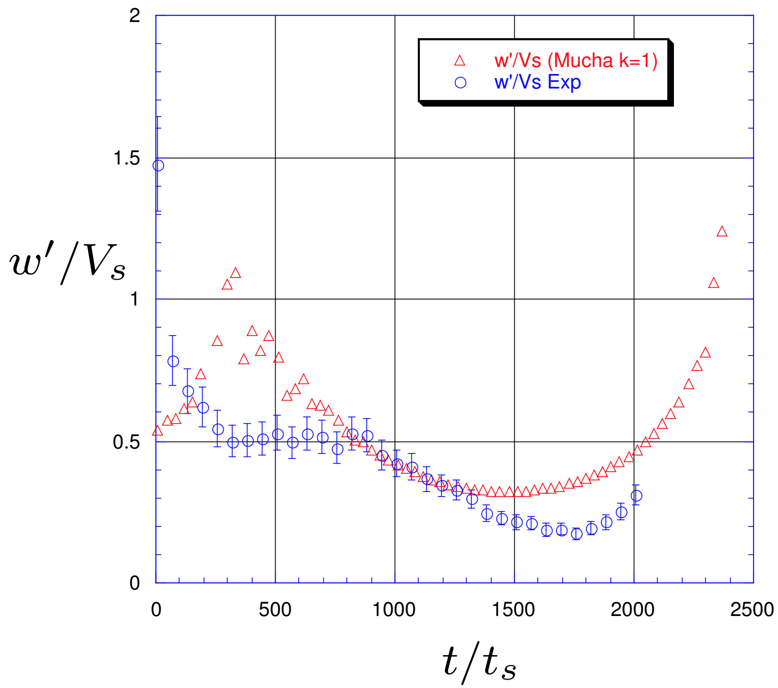


- Hence $w' = 0.49 V_s \phi^{3/5} \left(-a\partial\phi/\partial z\right)^{-1/5}$

Experimental test of stratification

- Measured stratification
- Velocity fluctuation





- Stratification only partial explanation for experiments

Stratification from diffuse front

(Mucha & Brenner 2003/4)

- Front between top of suspension and clear fluid above diffuses $\rightarrow$ stratified region.

- Self-diffusivity $D = w'_* \ell_* = 2.75 V_s a \phi^{4/5} \left(-a\partial\phi/\partial z\right)^{-3/5}$

- Nonlinear diffusion equation

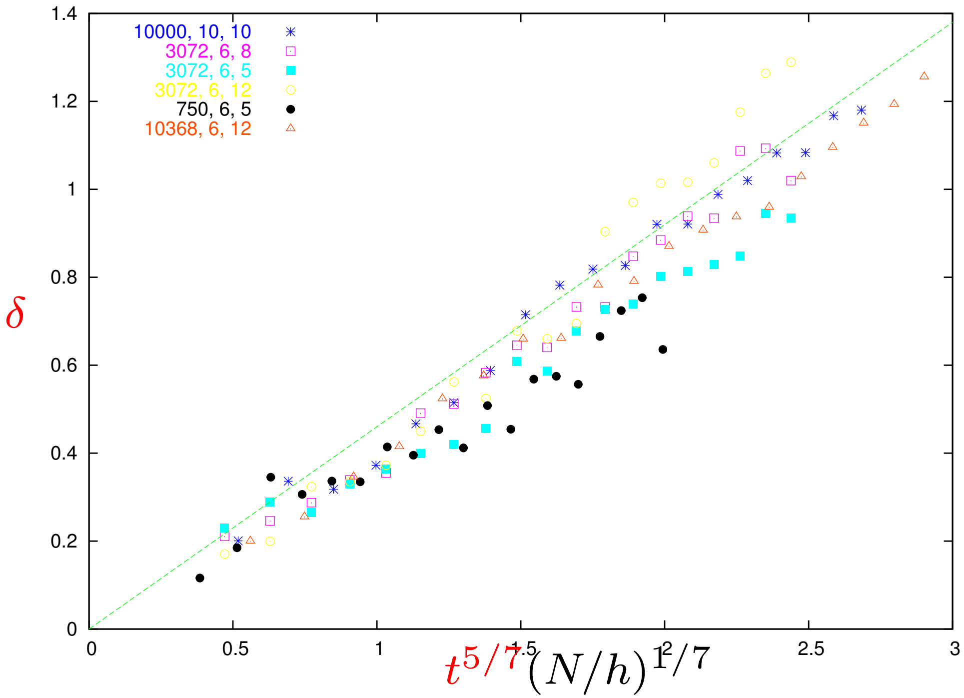
$$\frac{\partial\phi}{\partial t} - \frac{\partial(V_s\phi)}{\partial z} = \frac{\partial}{\partial z}\left(2.75 V_s a^{2/5} \phi^{4/5} \left(-\frac{\partial\phi}{\partial z}\right)^{2/5}\right)$$

- Numerical value 2.75 of diffusivity from . . .

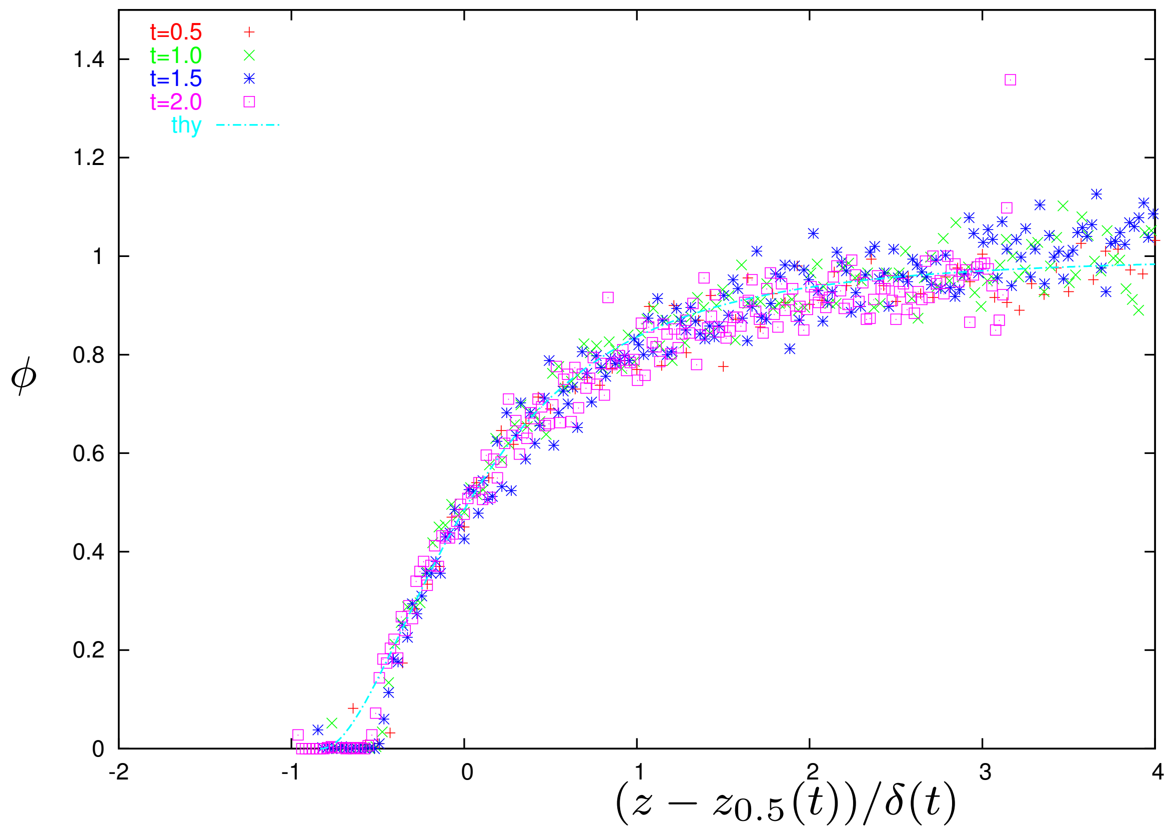
Diffusivity

- Diffusivity 2.75 from thickness of front

$$\delta = 3.07 a \phi^{1/7} (V_s t/a)^{5/7}$$



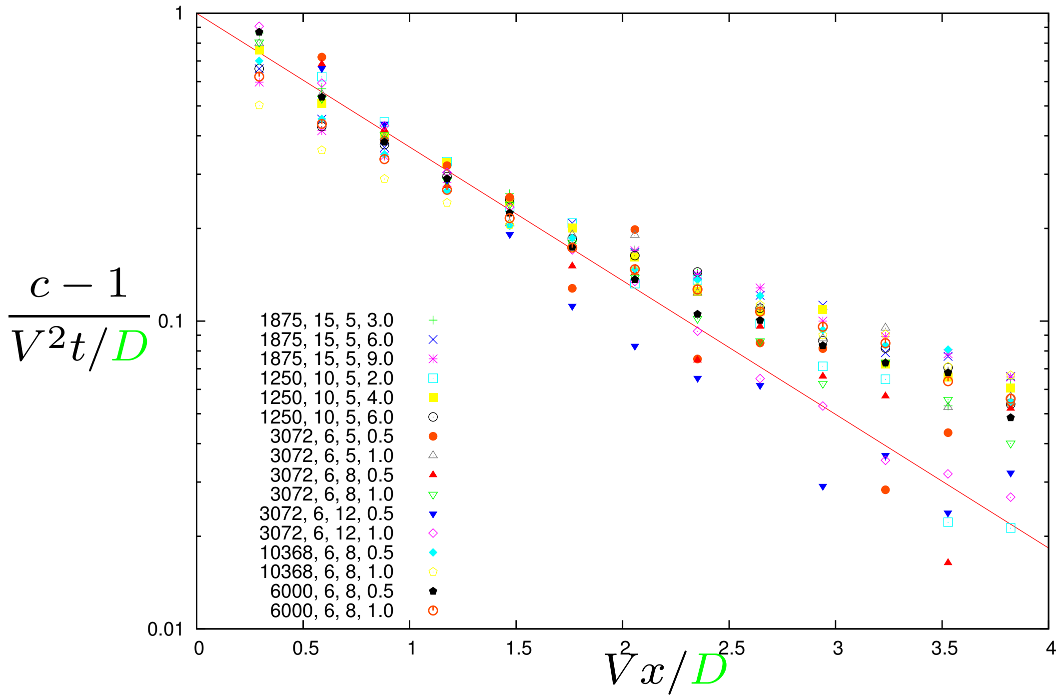
Front: self-similar profile



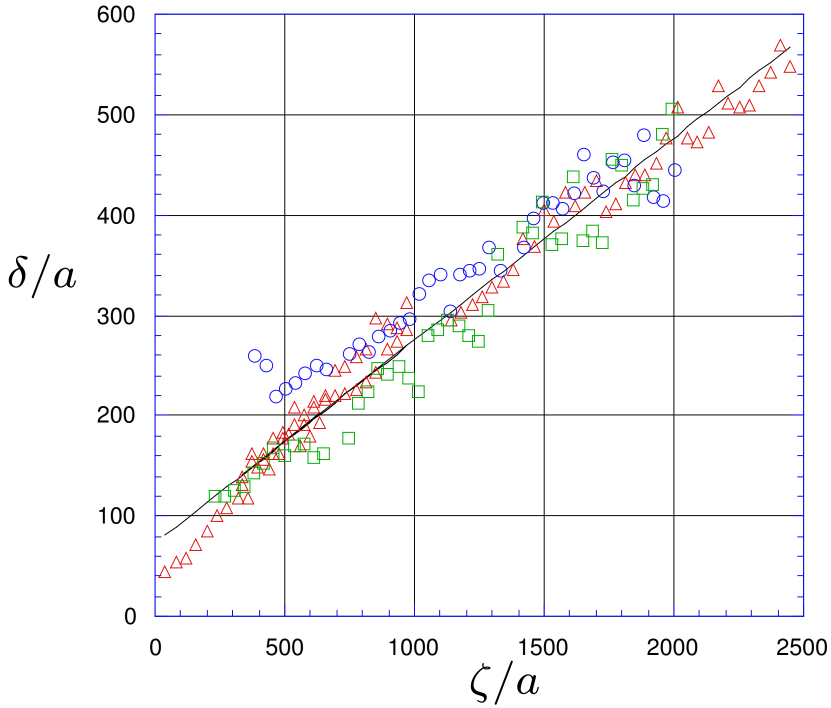
- Nonlinear diffusion equation predicts concentration profile in diffusing front at top of suspension

Sediment: self-similar profile

- The nonlinear diffusion equation has $c - 1 \propto t^{2/3}$ over a layer $\delta \propto t^{1/3}$.
- But find sediment of constant diffusivity $D = 4.7 U_s a$
  $c \sim 1 + (V^2 t/D) e^{-Vx/D}$



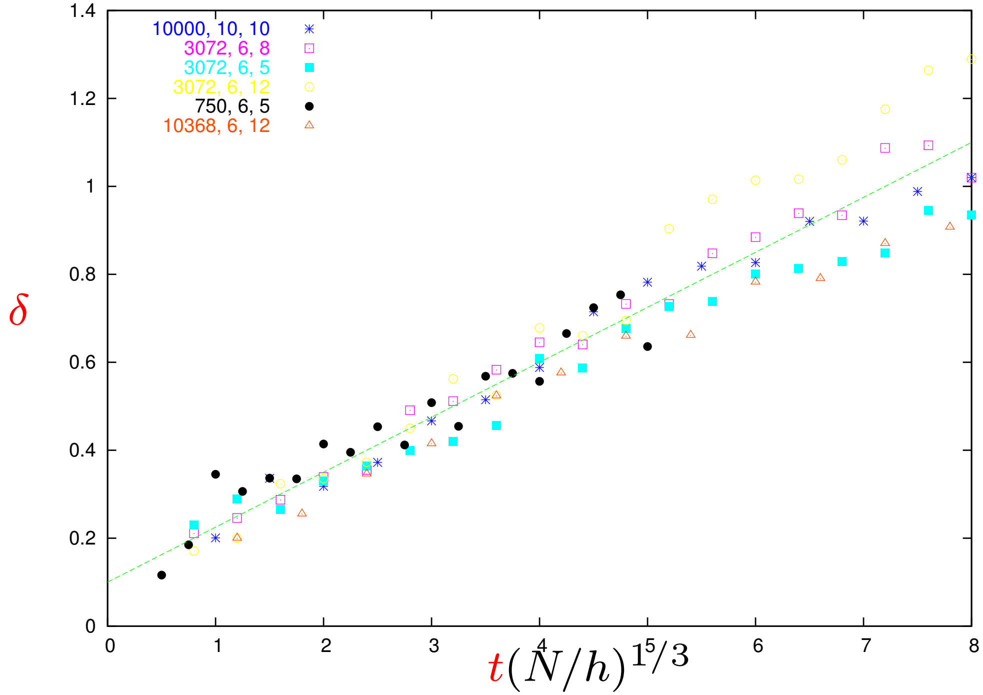
Front spreading in experiments



- Thickness $\delta = 0.2 V_s t$
- Linear growth mostly due to polydisperse particles

Linear spreading in simulations?

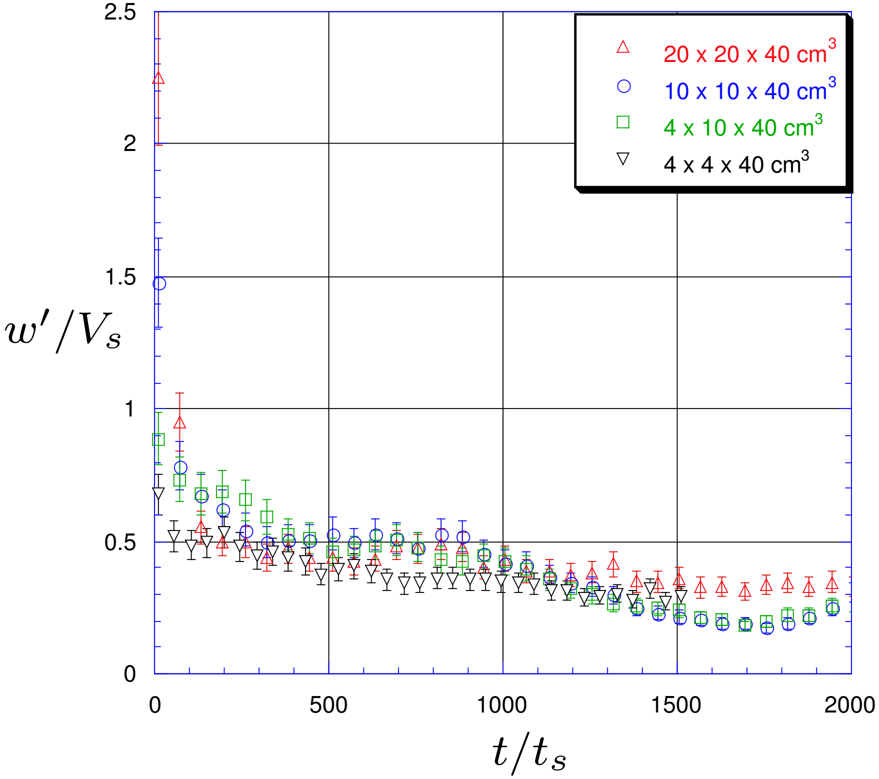
- Simulations are monodisperse



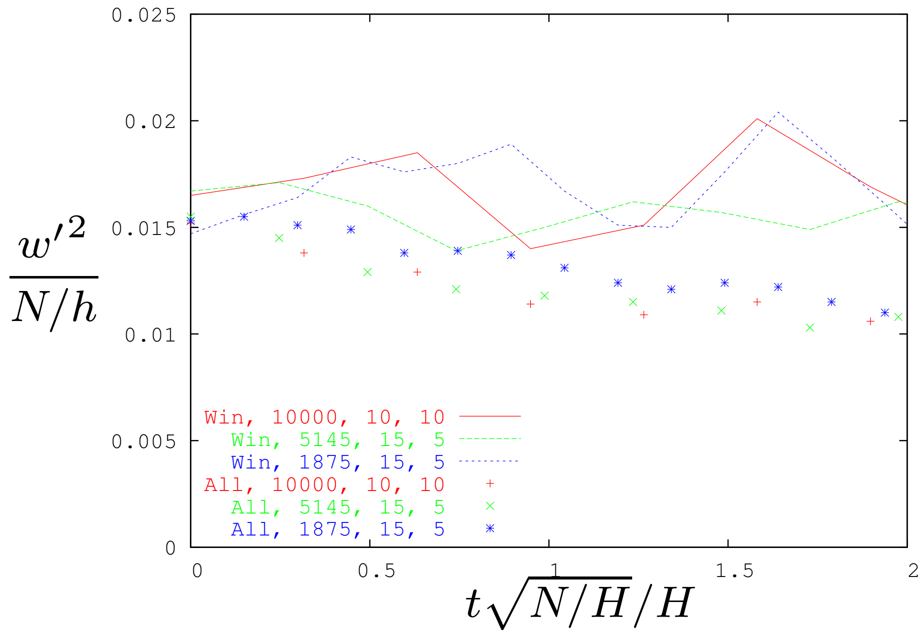
- Thickness $\delta = 1.5\phi^{1/3}V_s t$, but was also $t^{5/7}$!

Constant fluctuations outside front

- Experiments



- Simulations



Win: $3 < z < 6$, All: $0 < z < h$.

Conclusions

- Stratification inhibits velocity fluctuations in spreading front.
- No effect on interior. Slow to diffuse there?
- Mixing in experiments produces some stratification? and also non-random concentration fluctuations?
- Paradox not entirely resolved