Explaining the flow of elastic liquids

Penner Lecture, Winter 2006

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Complex fluids

- What & where?
- Why & when?
- Which today?
- 20 years to review

More than Viscous + Elastic

- Viscous:
  Bernoulli, lift, added mass, waves, boundary layers, stability, turbulence
- Elastic:
  structures, FE, waves, crack, composites
- Visco-elastic is more
  Not halfway between Viscous & Elastic – strange flows to explain

Outline

- Observations to explain
- How well does Oldroyd-B do?
  – half correct
- The FENE modification
  – anisotropy & stress boundary layers
- Conclusions – the reasons why

Flows to explain

A Contraction flow

large upstream vorticies, large pressure drop

B Flow past a sphere

long wake, increased drag

C M1 project on extensional viscosity

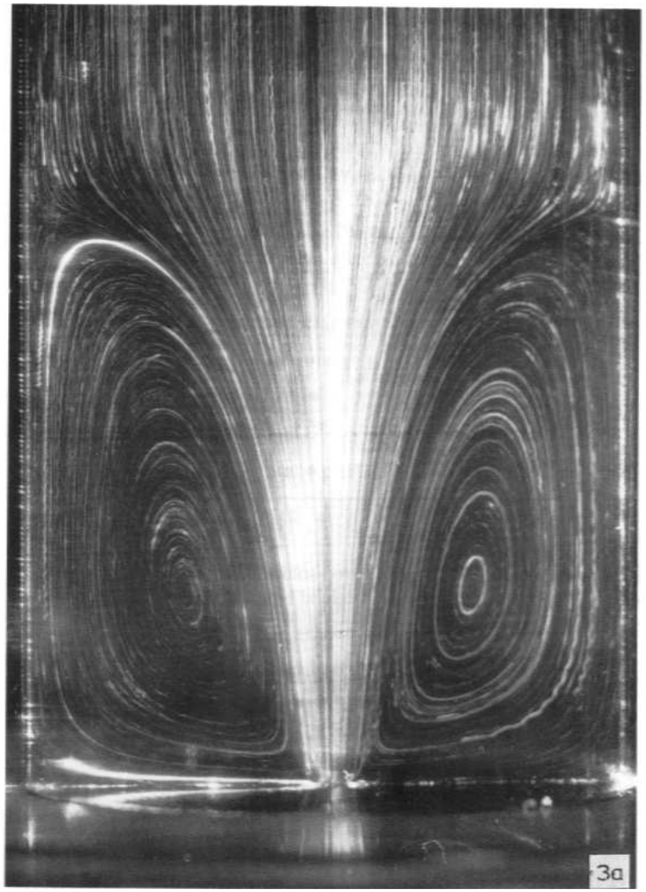
large stresses but confusion for value of viscosity

D Capillary squeezing of a liquid filament

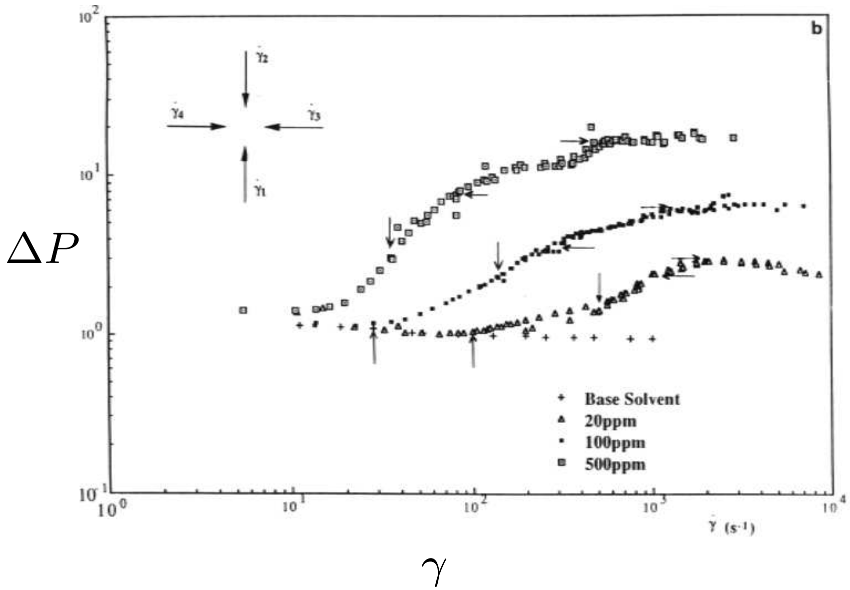
very slow to break

A. Contraction flow

large upstream vortex



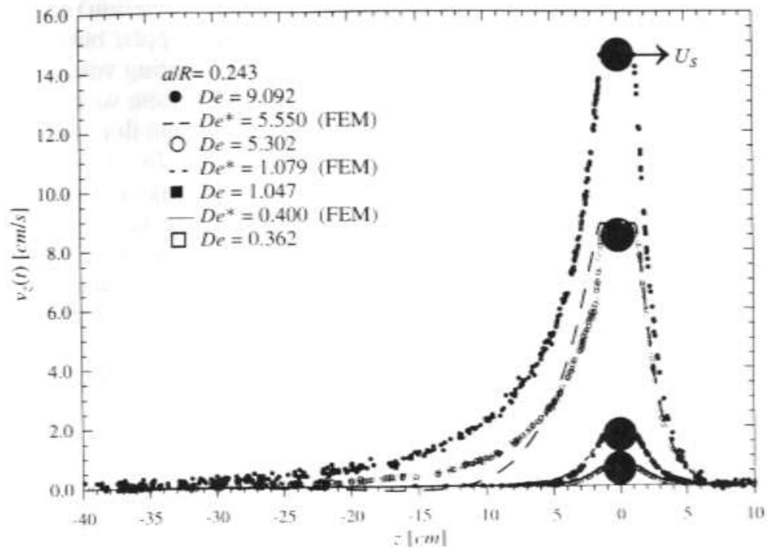
large pressure drops



Cartalos & Piau 1992 JNNFM 92

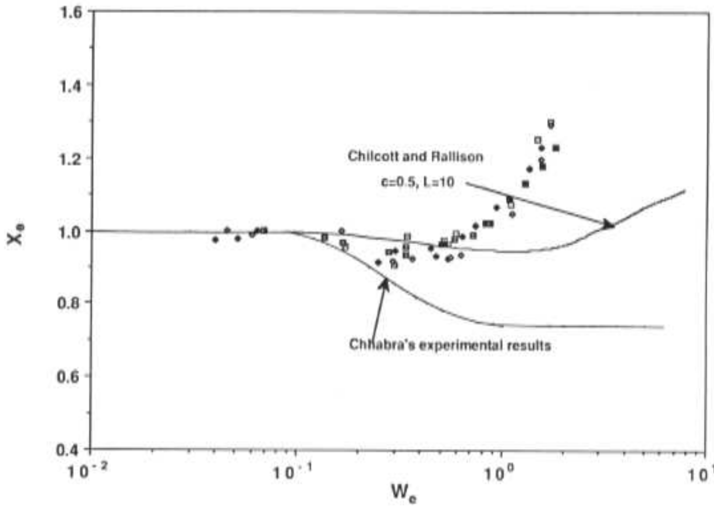
B. Flow past a sphere

long wake



Arigo, Rajagopalan, Shapley & McKinley 1995 JNNFM

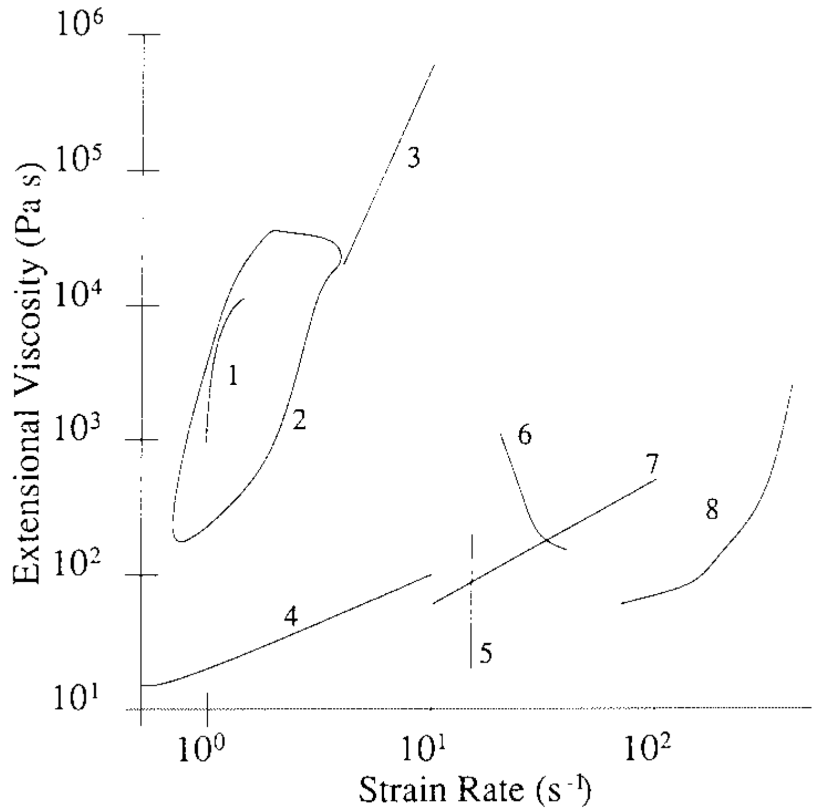
increased drag



Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

also negative wakes!

C. M1 project



Keiller 1992 JNNFM

no simple extensional viscosity

Flows to explain

A Contraction flow

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C M1 project on extensional viscosity

large stresses but confusion for value of viscosity

D Capillary squeezing of a liquid filament

very slow to break

. . . and more.

Governing equations

Mass $\nabla \cdot \mathbf{u} = 0$

Momentum $\rho \dfrac{D\mathbf{u}}{Dt} = \nabla \cdot \sigma$

Constitutive $\sigma(\nabla \mathbf{u})$ Not known

Start with simplest

– viscous part $+$ elastic part, elastic part can relax.

What can be learnt from simplest – useful?

– any behaviour independent of details of model?

Oldroyd-B model fluid simplest viscous + elastic

$$\sigma \quad = \quad -p\mathbf{I} \quad +2\mu_0\mathbf{E} \quad +G\mathbf{A}$$

stress viscous elastic

μ_0 viscosity G elastic modulus

with microstructure $\mathbf{A}$

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A} \quad -\frac{1}{\tau}(\mathbf{A}-\mathbf{I})$$

deform with fluid relaxes

τ relaxation time

Deforming with the fluid

Fluid line element $\delta\ell$ deforms as

$$\frac{d\delta\ell}{dt} = \delta\ell \cdot \nabla\mathbf{u}$$

Hence the second order tensor (stress)

$$\mathbf{A} = \delta\ell\,\delta\ell$$

will deform as

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla\mathbf{u} + \nabla\mathbf{u}^T \cdot \mathbf{A}$$

Oldroyd-B model fluid simplest viscous + elastic

$$\sigma = -p\mathbf{I} + 2\mu_0\mathbf{E} + G\mathbf{A}$$

stress viscous elastic

μ_0 viscosity G elastic modulus

with $\mathbf{A}$ microstructure.

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A}\cdot\nabla\mathbf{u} + \nabla\mathbf{u}^T\cdot\mathbf{A} - \frac{1}{\tau}(\mathbf{A}-\mathbf{I})$$

deform with fluid relaxes

τ relaxation time

Does this model work/fail?

Deborah/Weissenberg number

Fluid relaxation time τ gives nondimensional group

$$De = \frac{U\tau}{L} = \frac{\text{fluid time } \tau}{\text{flow time } L/U}$$

$De \ll 1$: fluid relaxed $\Longrightarrow$ liquid like

$De \gg 1$: little relaxed $\Longrightarrow$ solid like

Investigating Oldroyd-B

1. Steady & weak $\frac{D}{Dt}, \nabla\mathbf{u} \ll 1/\tau$
2. Unsteady & weak $\nabla\mathbf{u} \ll 1/\tau$

   – linear viscoelasticity
3. Slightly nonlinear $\nabla\mathbf{u} \lesssim 1/\tau$

   – 2nd order fluid
4. Very Fast $\nabla\mathbf{u} \gg 1/\tau$
5. Strongly elastic $2\mu_0 E \ll GA$

1. Steady & weak $\frac{D}{Dt}, \nabla \mathbf{u} \ll 1/\tau \ (De \ll 1)$

Microstructure

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \mathbf{A} - \frac{1}{\tau}(\mathbf{A} - \mathbf{I})$$

$$\therefore \mathbf{A} \sim \mathbf{I} + 2\mathbf{E}\tau$$

i.e. $\mathbf{A} = \mathbf{I}$ is deformed by flow $\mathbf{E}$ until $t = \tau$, when it relaxes/forgets.

Stress

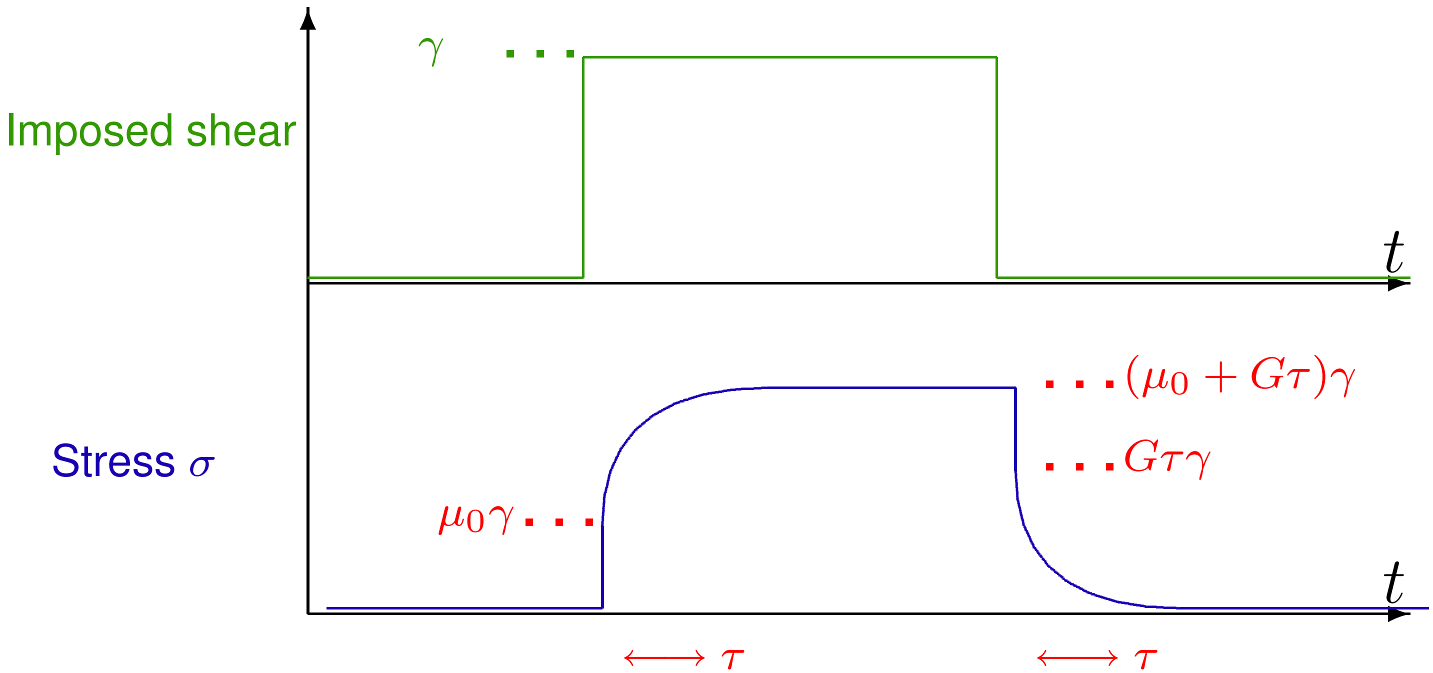
$$\begin{aligned} \sigma &= -p\mathbf{I} + 2\mu_0 \mathbf{E} + G\mathbf{A} \\ &= -p\mathbf{I} + 2(\mu_0 + G\tau)\mathbf{E} \end{aligned}$$

effective viscosity

So Newtonian. Use $\mu_0 + G\tau$ is comparisons of Δp and Drag.

2. Unsteady & weak $\nabla \mathbf{u} \ll 1/\tau$

$$\frac{D\mathbf{A}}{Dt} \approx 2E + \frac{1}{\tau}(\mathbf{A} - \mathbf{I})$$



Takes τ to build up to steady state

Stress relaxation – in all CE

Startup

Instantaneous viscous stress $\mu_0\gamma$.

Deformation at rate γ for memory time τ gives

deformation $\gamma\tau$.

So elastic stress $G\gamma\tau$.

Hence steady state viscosity $\mu_0 + G\tau$, but only after time τ.

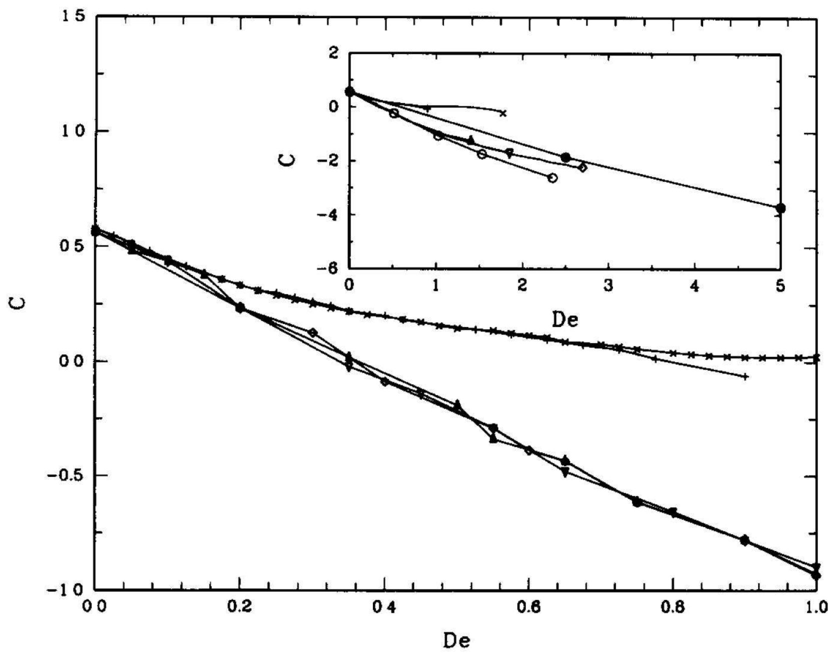
Stress relaxation is a special property of non-Newtonian fluids, which is not in elastic solids nor viscous liquids

NB steady flows are unsteady Lagrangian.

A. Contraction flow Lagrangian unsteady

Numerical Oldroyd-B

$\frac{\Delta P}{\text{Stokes}}$



$De = U\tau/L$

Debbaut, Marchal & Crochet 1988 JNNFM

Coates, Armstrong & Brown 1992 JNNFM

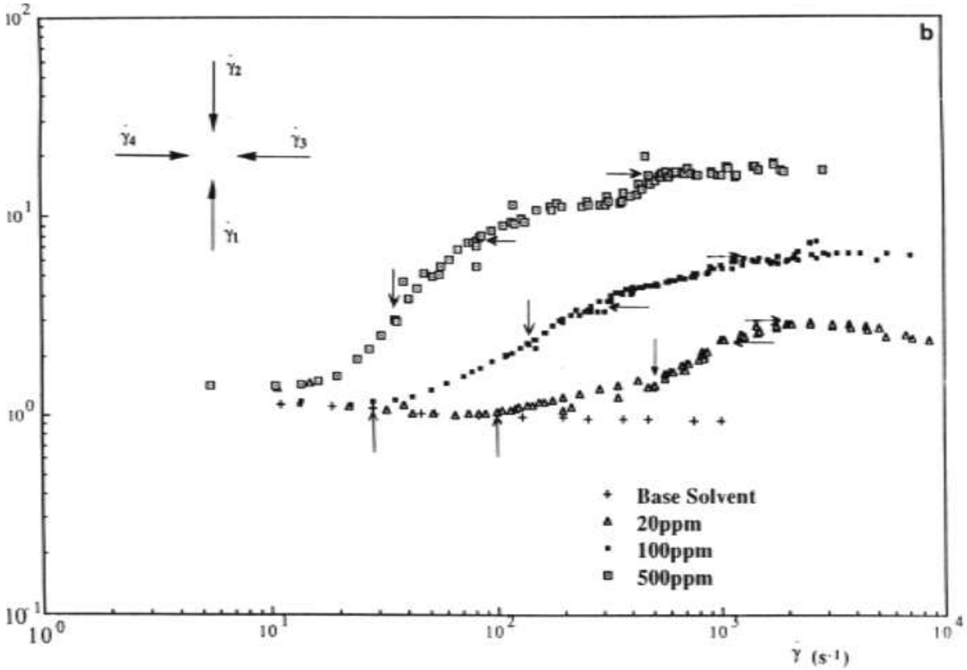
Δp scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

But lower drop by early-time viscosity μ_0 if flow fast

…contraction flow

Experiments

$\frac{\Delta P}{\text{Stokes}}$



$De = U\tau/L$

Cartalos & Piau 1992

JNNFM 92

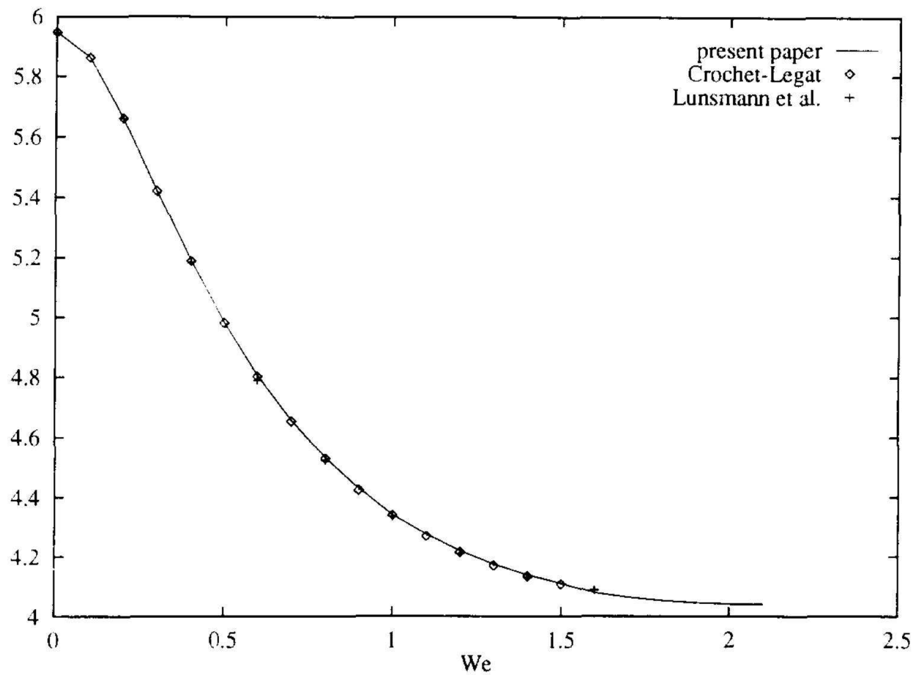
Experiments have a tiny decrease in pressure drop!

Oldroyd-B has no big increase is Δp, and no big upstream vorticies

B. Flow past a sphere Lagrangian unsteady

Numerical Oldroyd-B

Drag/Stokes χ



$De = U\tau/L$

Yurun & Crochet 1995

JNNFM

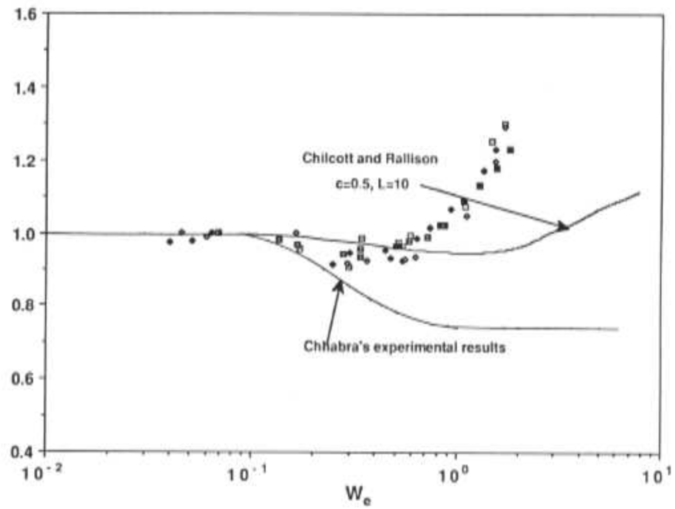
Drag scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

But lower drag by early-time viscosity μ_0 if flow fast

. . . flow past a sphere

Experiments

Drag / Stokes



$De = U\tau/L$

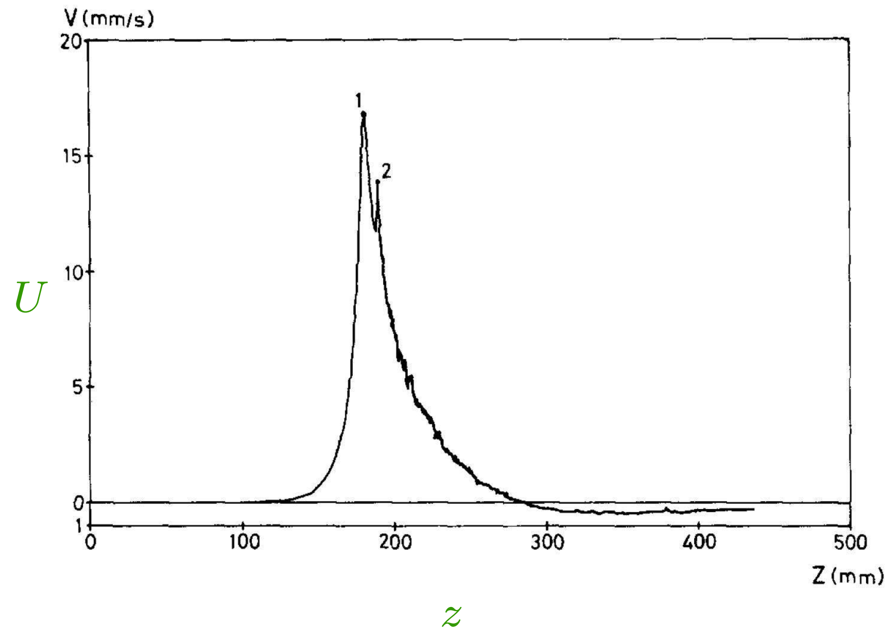
Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

Experiments have a tiny decrease in drag!

Oldroyd-B has no big increase in drag, and no big wake

…and negative wakes

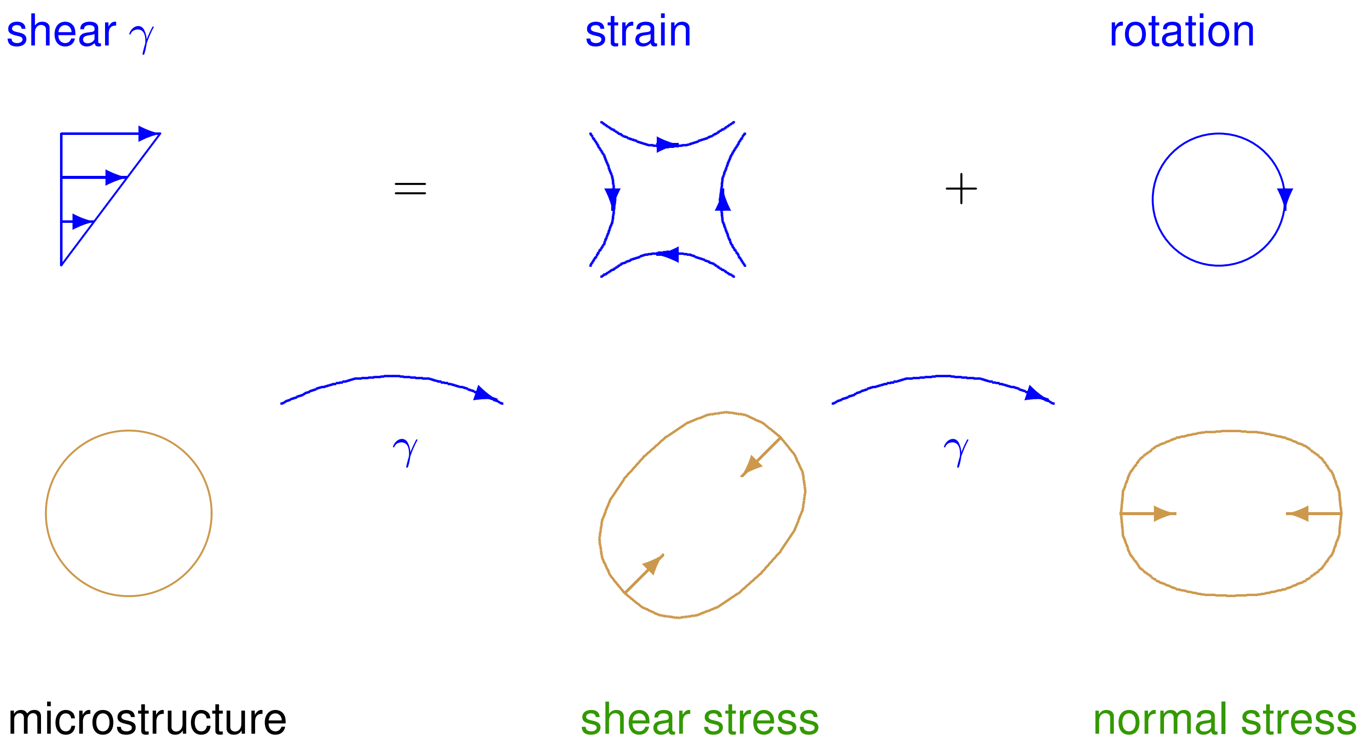
Experiment



Bisgaard 1983 JNNFM

Driven by unrelaxed elastic stress in wake.

3. Slightly nonlinear



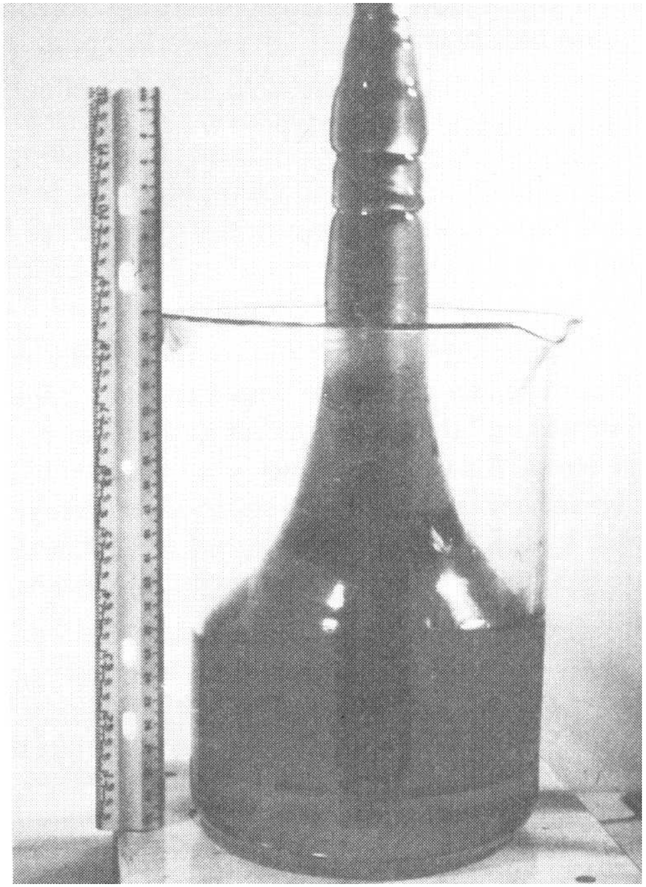
Shear stress $= G \times$ (rate $= \gamma$) $\times$ (memory time $= \tau$)

Normal stress (tension in streamlines) = shear stress $\times \gamma\tau$.

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability

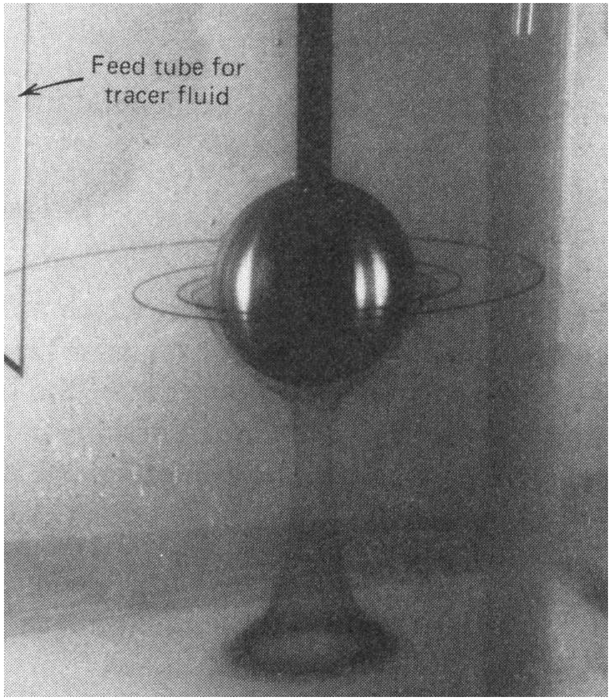
Rod climbing



Bird, Armstrong & Hassager 1987, Vol 1 (2nd ed) pg 62

Tension in streamlines $\longrightarrow$ hoop stress $\longrightarrow$ squeeze fluid in & up.

Secondary flow



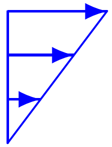
Bird, Armstrong & Hassager 1987, Vol 1 (2nd ed) pg 70

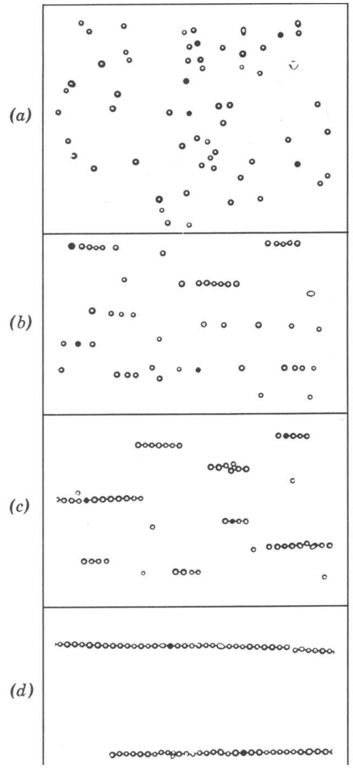
Tension in streamlines $\longrightarrow$ hoop stress $\longrightarrow$ squeeze fluid in.

Non-Newtonian effects opposite sign to inertial

Migration into chains

shear γ



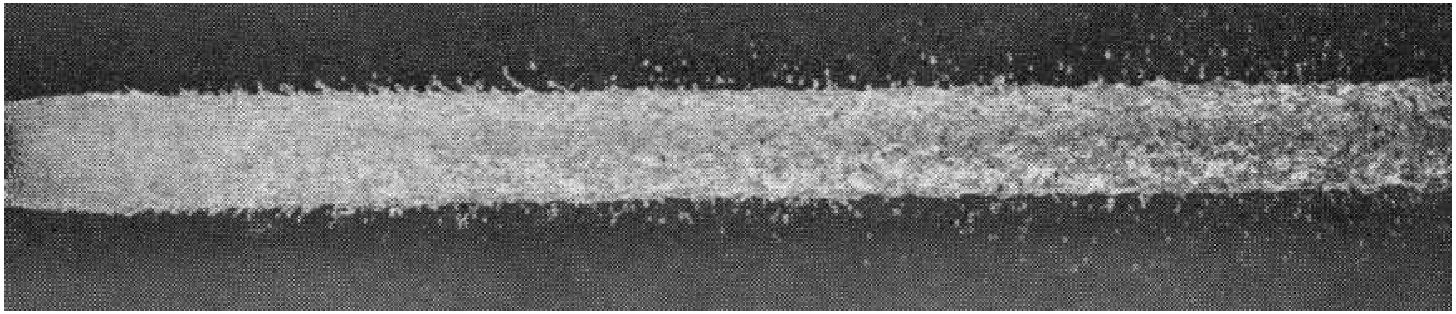


Bird, Armstrong & Hassager 1987, Vol 1 (2nd ed) pg 87

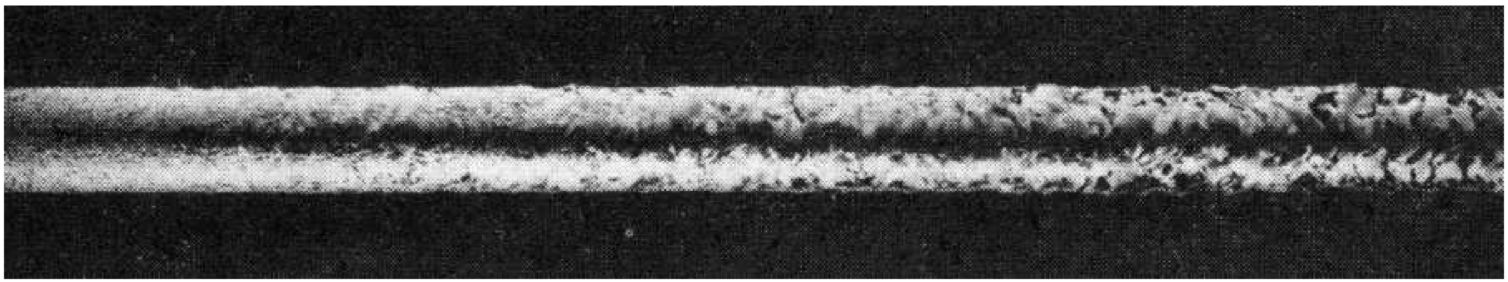
Tension in streamlines $\longrightarrow$ hoop stress
$\longrightarrow$ squeeze particles together

Stabilisation of jets

Newtonian Jet



Non-Newtonian Jet (200ppm PEO)



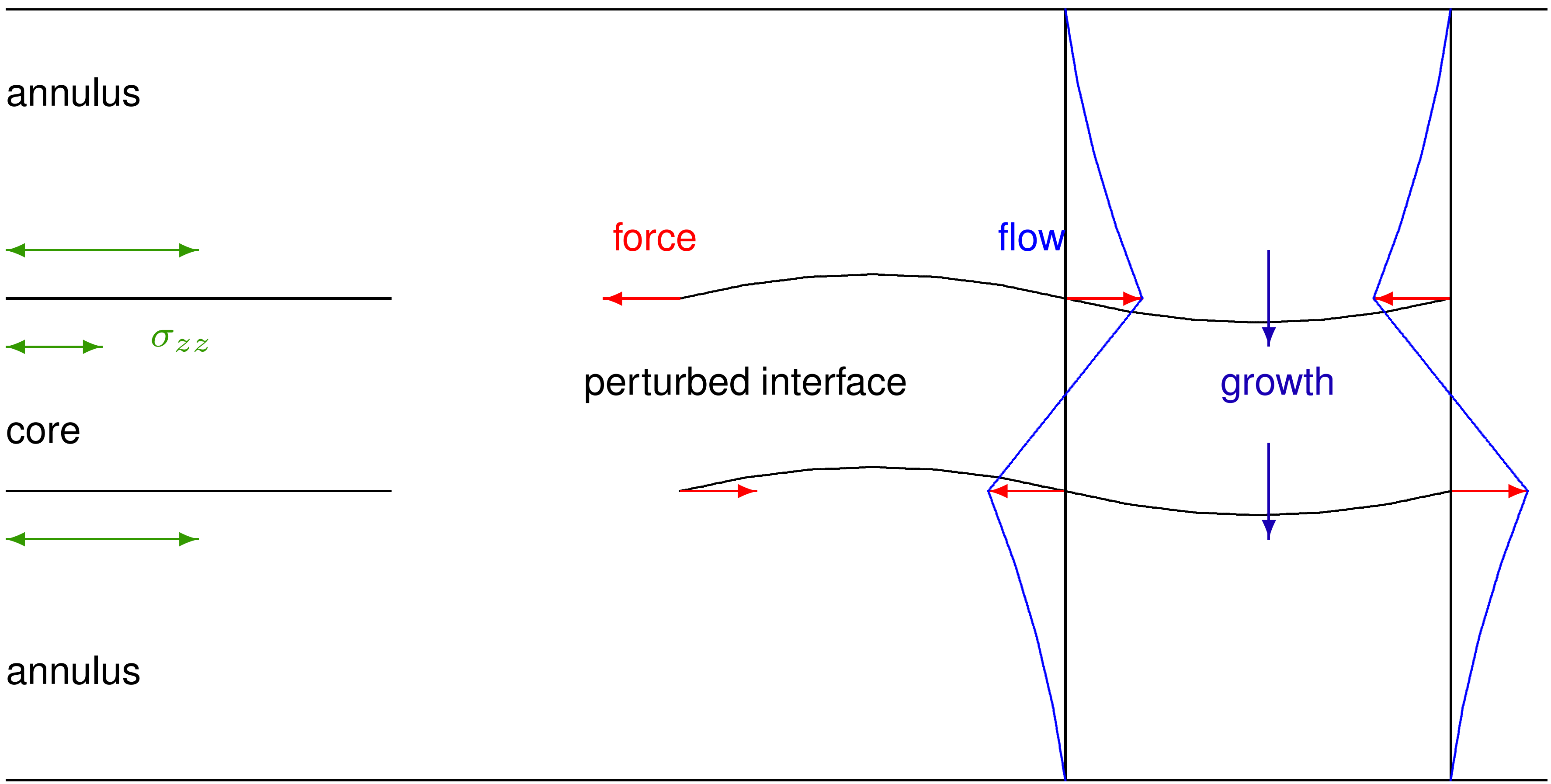
Hoyt & Taylor 1977 JFM

Tension in streamlines in surface shear layer

Co-extrusion instability

If core less elastic, then jump in tension in streamlines

Jump OK is interface unperturbed



Hinch, Harris & Rallison 1992 JNNFM

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability

4. Very Fast, $De \gg 1$

$$\frac{D\mathbf{A}}{Dt} = \mathbf{A} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \mathbf{A} - \frac{1}{\tau}(\mathbf{A} - \mathbf{I})$$

Fast: no time to relax: deforms where speeds up (steady flow)

$$\mathbf{A} = g(\psi)\mathbf{u}\mathbf{u}$$ tensioned streamlines again

g from matching to slower (relaxing) region

Momentum $\nabla \cdot \sigma = 0$, purely elastic $\sigma = -p\mathbf{I} + G\mathbf{A}$

$$0 = -\nabla p + G g^{1/2}\mathbf{u} \cdot \nabla g^{1/2}\mathbf{u}$$

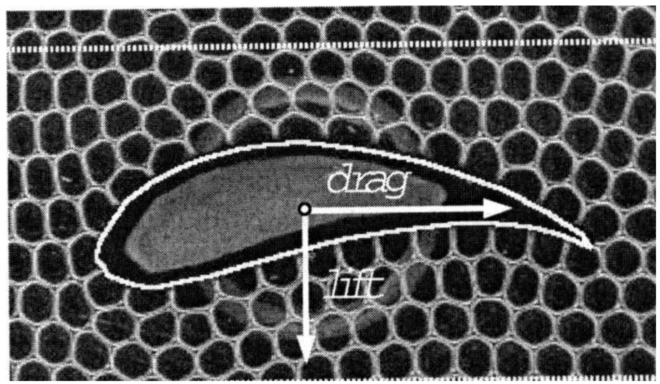
Euler equation

. . . very fast

$$0 = -\nabla p + G g^{1/2} \mathbf{u} \cdot \nabla g^{1/2} \mathbf{u}$$

Anti-Bernoulli

$$p - \tfrac{1}{2} G g u^2 = \text{const}$$



Dollet, Aubouy & Graner 2005 PRL

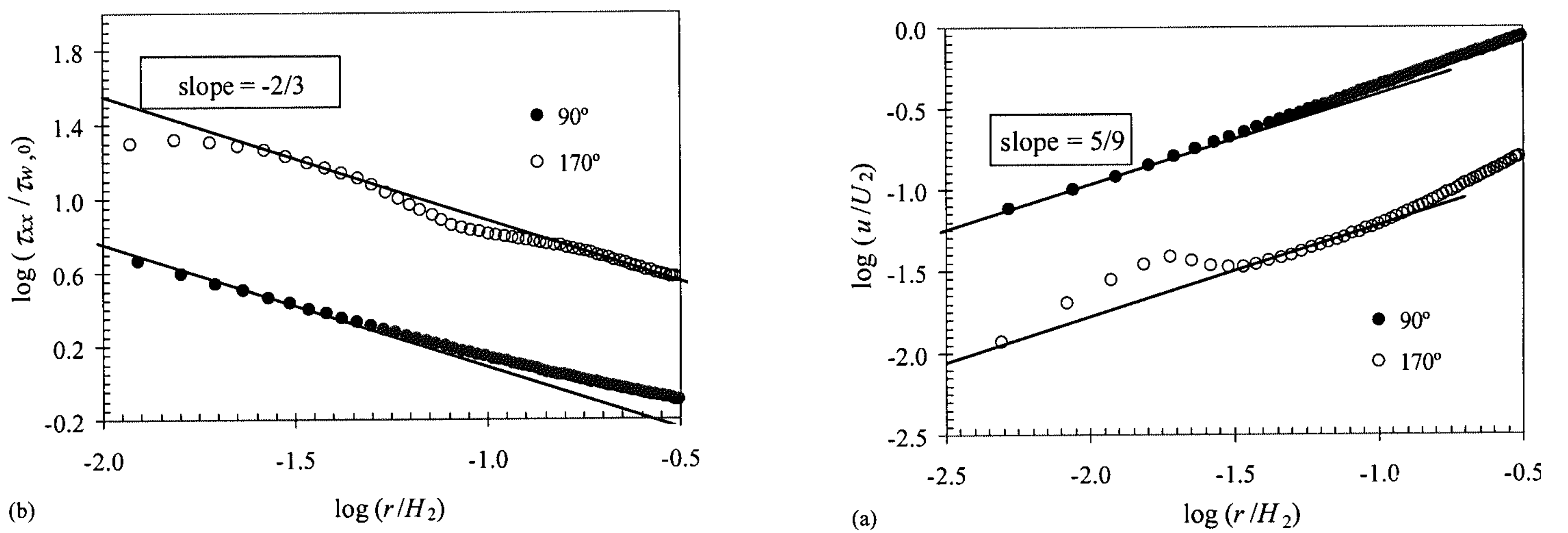
Hence non-Newtonian effects opposite sign to inertial

…very fast

Potential flows $g^{1/2}\mathbf{u} = \nabla\phi$

Flow around sharp 270° corner: Hinch 1995 JNNFM

$$\phi = r^{2/3}\cos\tfrac{2}{3}\theta, \qquad \sigma \propto r^{-2/3} \qquad \psi = r^{14/9}\sin^{7/3}\tfrac{2}{3}\theta$$



Alves, Oliviera & Pinho 2003 JNNFM

D. Capillary squeezing a liquid filament

also 5. Strongly Elastic

Mass $\qquad \dot{a} = -\frac{1}{2} E a$

Momentum $\qquad \frac{\chi}{a} = 3\mu_0 E + G(A_{zz} - A_{rr})$

Microstructure $\qquad \dot{A}_{zz} = 2EA_{zz} - \frac{1}{\tau}(A_{zz} - 1)$

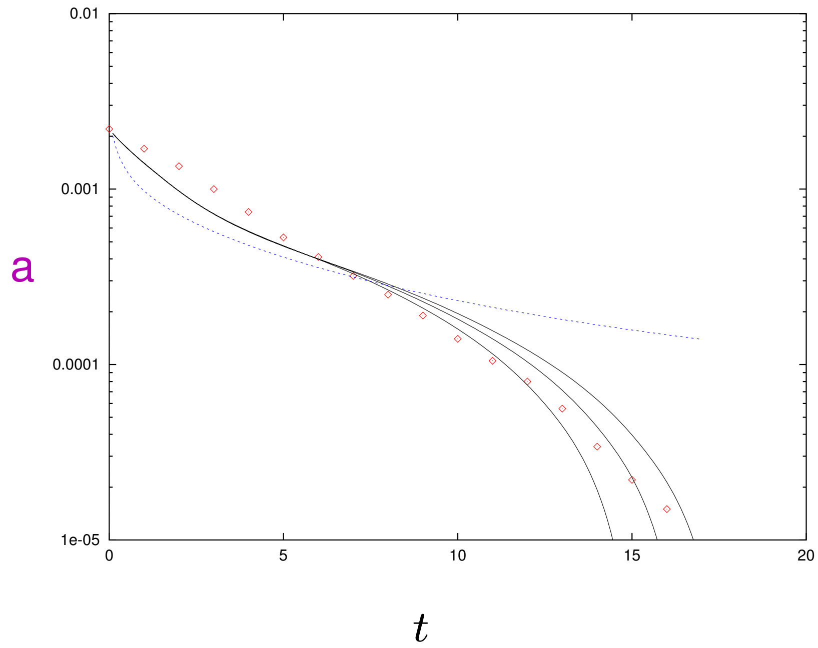
Solution $\qquad a(t) = a(0)e^{-t/3\tau}$

Need slow $E = 1/3\tau$ to stop A_{zz} relaxing from χ/Ga

. . . capillary squeezing

Oldroyd-B $a(t) = a(0)e^{-t/3\tau}$ does not break

Experiments S1 fluid



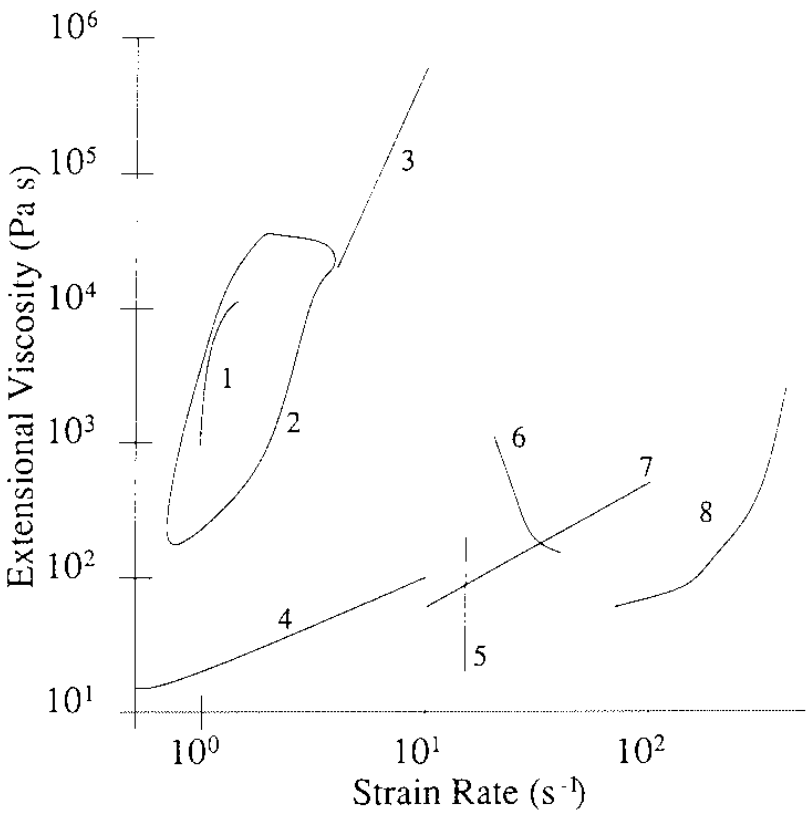
Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

but filament breaks in experiments

C. M1 project

no simple extensional viscosity



1. Open syphon
2. Spin line
3. Contraction
4. Opposing Jet
5. Falling drop
6. Falling bob
7. Contraction
8. Contraction

Keiller 1992 JNNFM

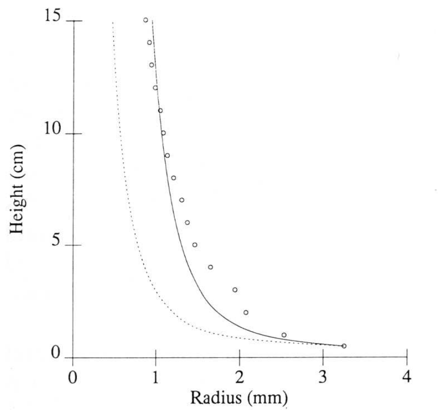
really elastic responses

…M1 project

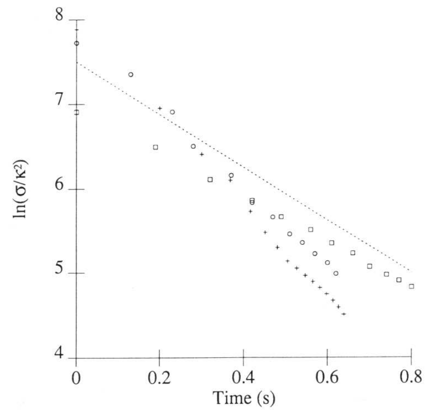
Fit data with Oldroyd-B: $\mu_0 = 5$, $G = 3.5$, $\tau = 0.3$ from shear

Keiller 1992 JNNFM

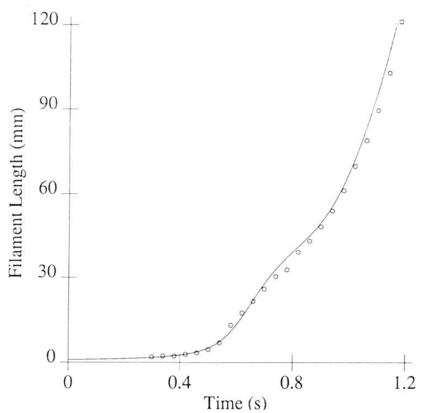
1. Open syphon   Binding 1990



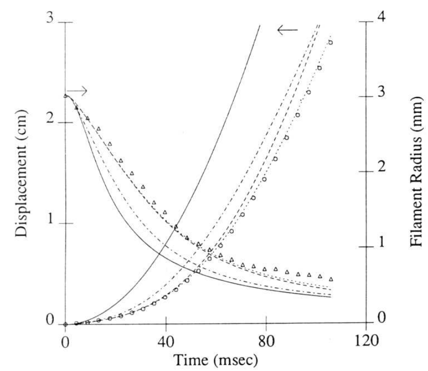
2. Spin line   Oliver 1992



5. Falling drop   Jones 1990



6. Falling bob   Matta 1990



Oldroyd B: Successes & Failures

Simplest viscosity μ_0 + elasticity G + relaxation τ

C. M1 Project

3. Tension in streamlines

A. Contraction: Δp small decrease, no big increase, no large vorticies

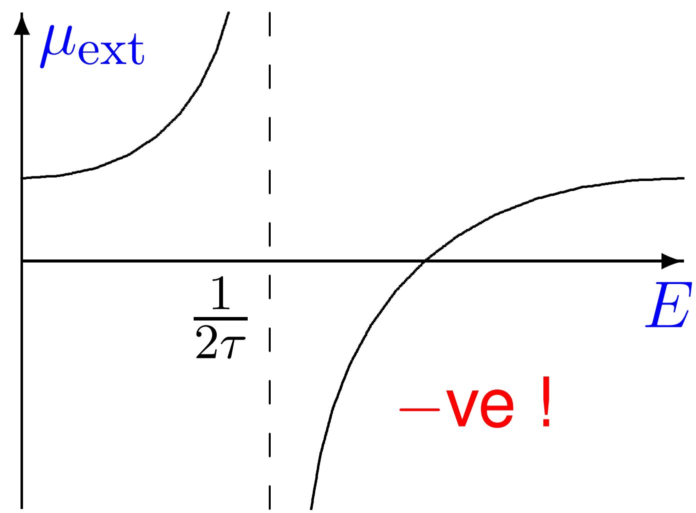
B. Sphere: Drag small decrease, no increase, no long wake
D. Capillary squeezing: long time-scale, no break

Also difficult numerically at $\frac{U\tau}{L} > 1$

Need more physics in constitutive equation

Disaster in Oldroyd-B

Steady extensional flow



$$\dot{A} = 2EA - \frac{1}{\tau}(A-1), \qquad \text{solution:} \qquad A = e^{(2E-\frac{1}{\tau})t}$$

Microstructure deforms without limit if $E > \frac{1}{2\tau}$

Need to limit deformation of microstructure

FENE modification

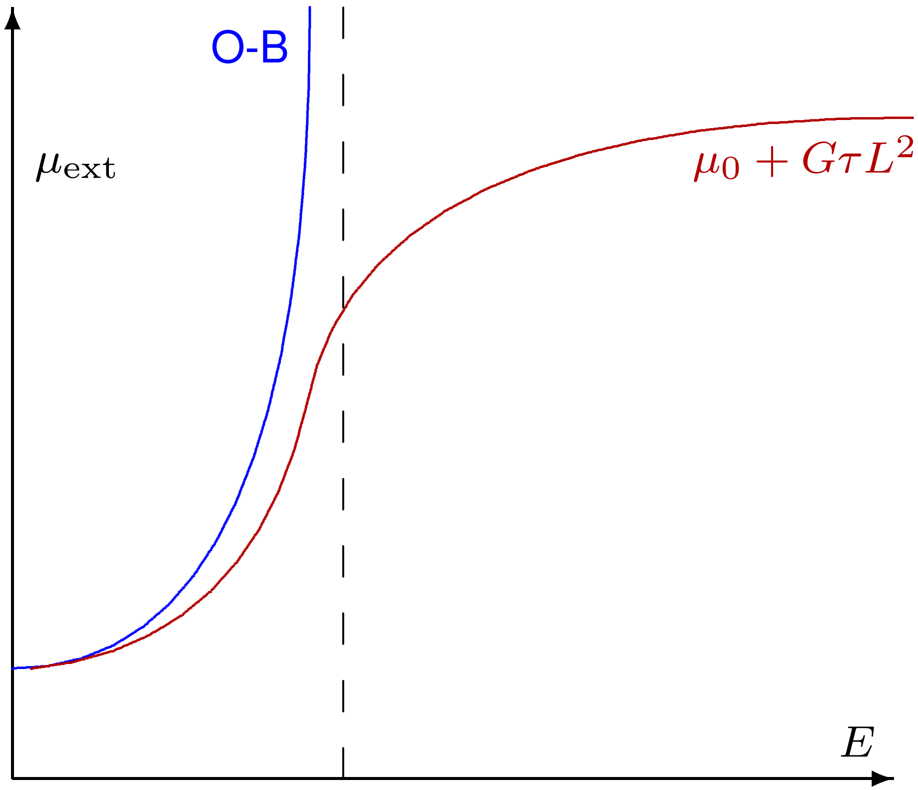
Finite Extension Nonlinear Elasticity

$$\frac{DA}{Dt} = A \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot A - \frac{f}{\tau}(A - \mathbf{I})$$

$$\sigma = -p\mathbf{I} + 2\mu_0 E + G f A$$

$$f = \frac{L^2}{L^2 - \mathrm{trace}\, A}$$ keeps $A < L^2$

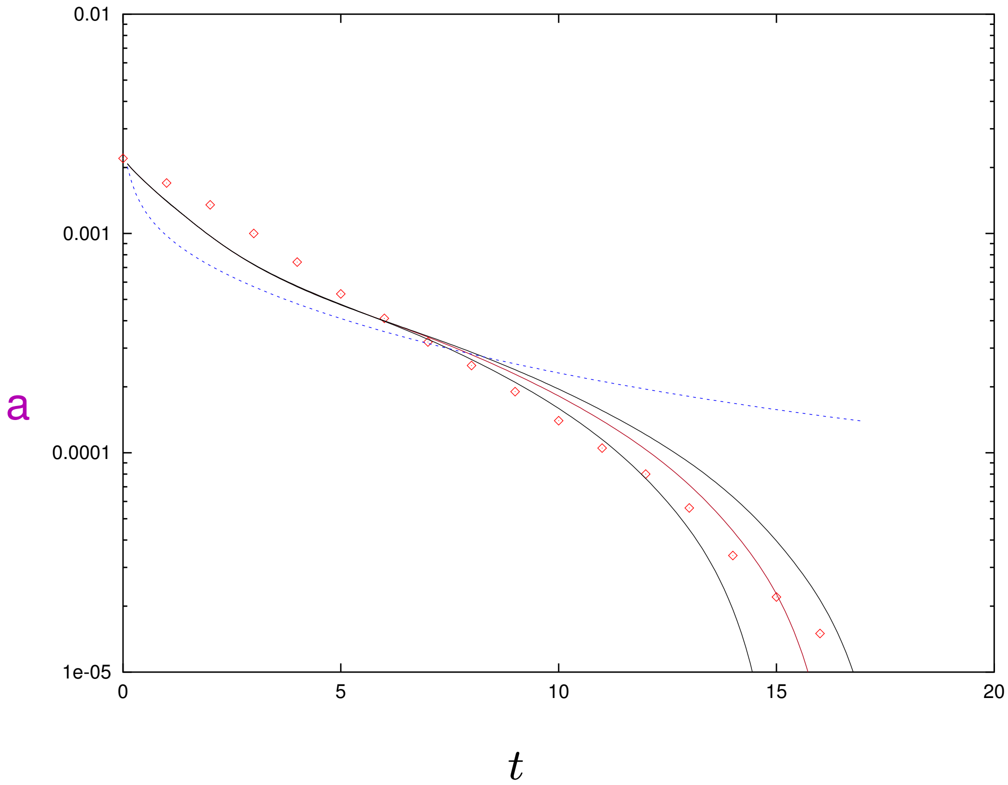
…FENE



Large extensional viscosity $\mu_0 + G\tau L^2$, but small shear viscosity μ_0

D. FENE capillary squeezing

Filament breaks in with FENE $L = 20$



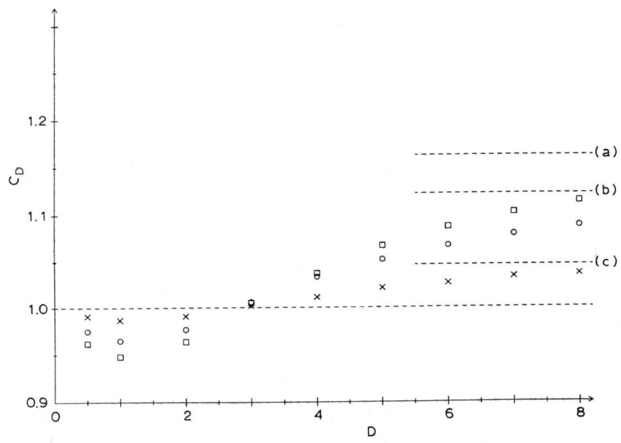
Exp: Liang & Mackley 1994 JNNFM

Thy: Entov & Hinch 1997 JNNFM

B. FENE flow past a sphere

FENE

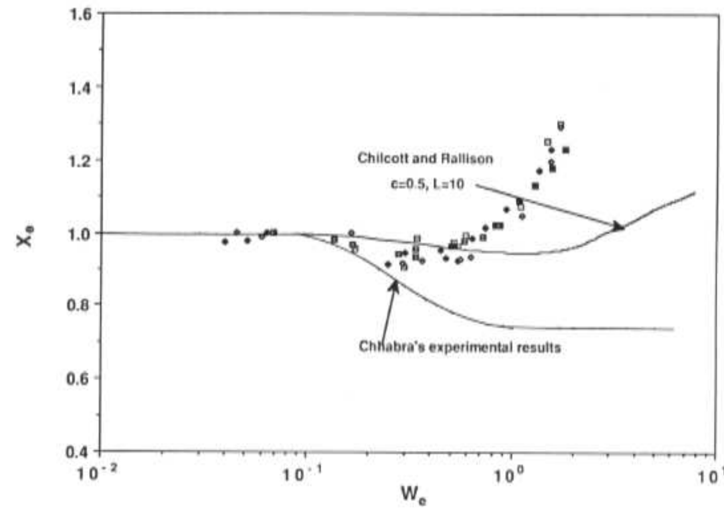
F



De

Chilcott & Rallison 1988 JNNFM

Experiments M1



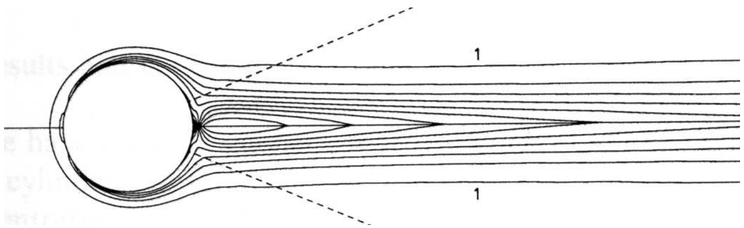
De

Tirtaatmadja, Uhlherr & Sridhar 1990 JNNFM

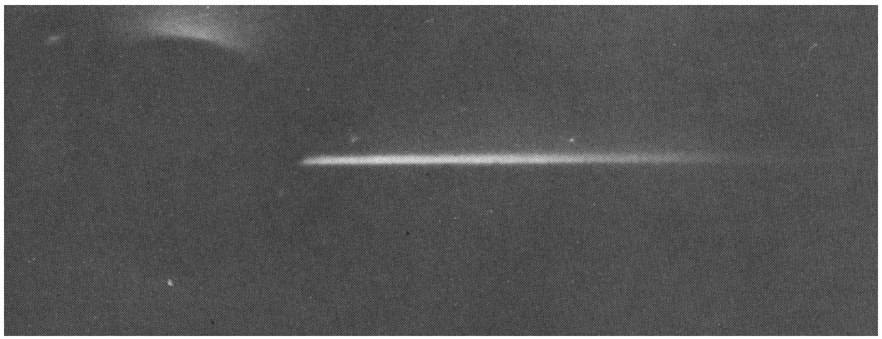
FENE gives drag increase

. . . FENE flow past sphere

FENE drag increase from long wake of high stress



Chilcott & Rallison 1988 JNNFM

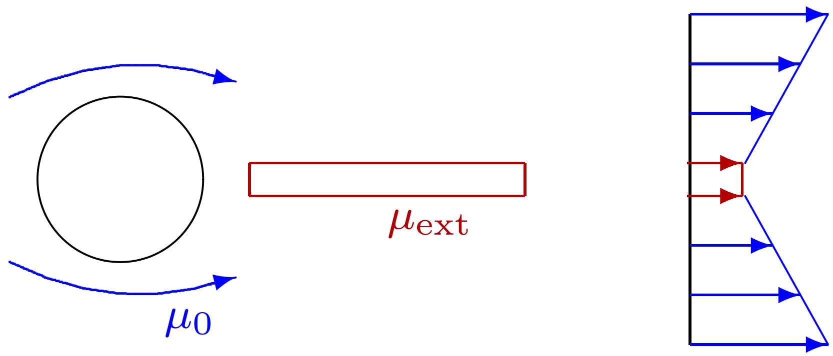


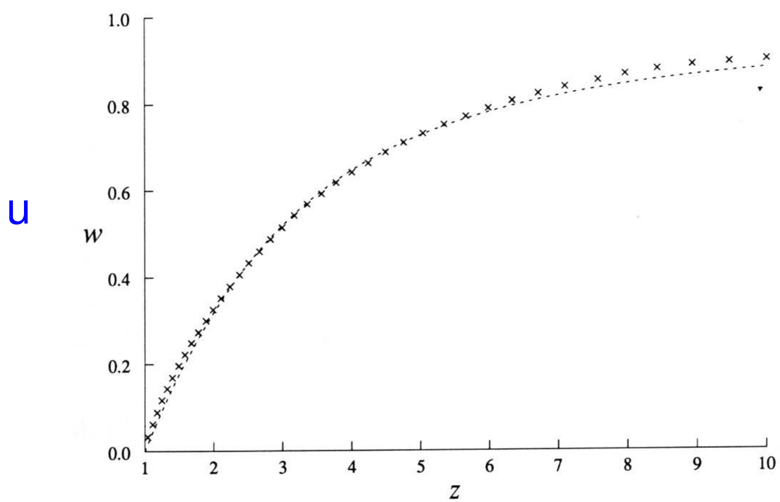
Cressely & Hocquart 1980 Opt Act

“Birefringent strand”

. . . birefringent strands

Boundary layers of high stress: μ_{ext} in wake, μ_0 elsewhere.



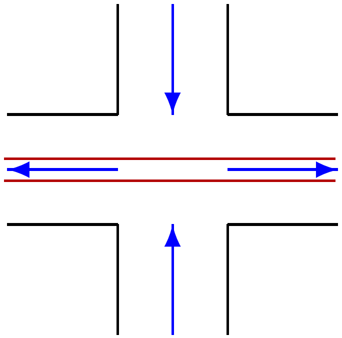


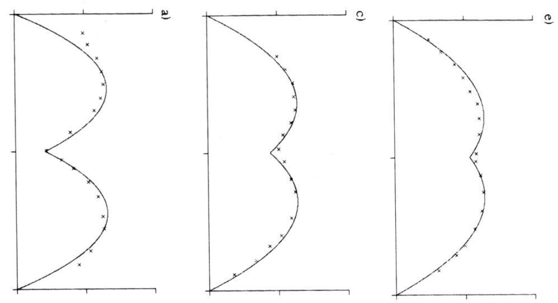
Harlen, Rallison & Chilcott

1990 JNNFM

...birefringent strands

Can apply to all flows with stagnation points, e.g.





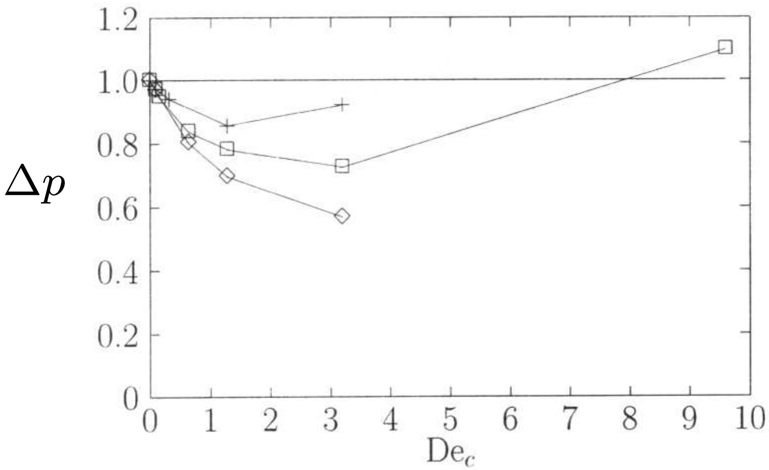
Thy. Harlen, Rallison & Chilcott 1990 JNNFM

Exp. Cressely & Hocquart 198n Opt Act

Also cusps at rear stagnation point of bubbles.

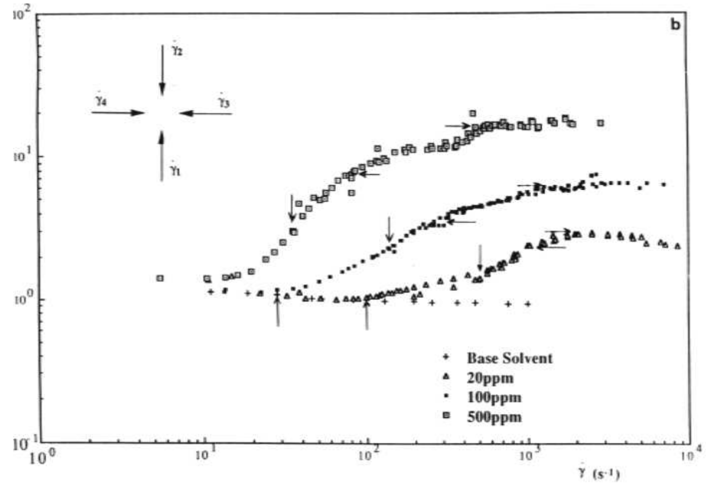
A. FENE contraction flow

FENE $L = 5$



Szabo, Rallison & Hinch 1997 JNNFM

Experiments



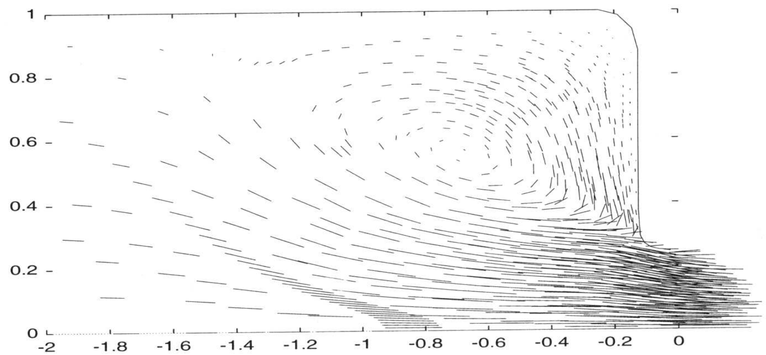
Cartalos & Piau 1992 JNNFM

FENE gives increase in pressure drop

. . . FENE contraction flow

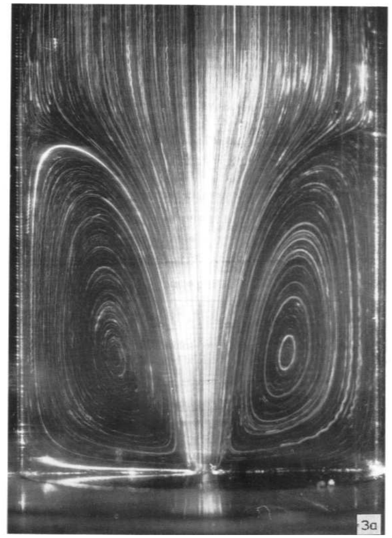
Increase in pressure drop from long upstream vortex

FENE $L = 5$



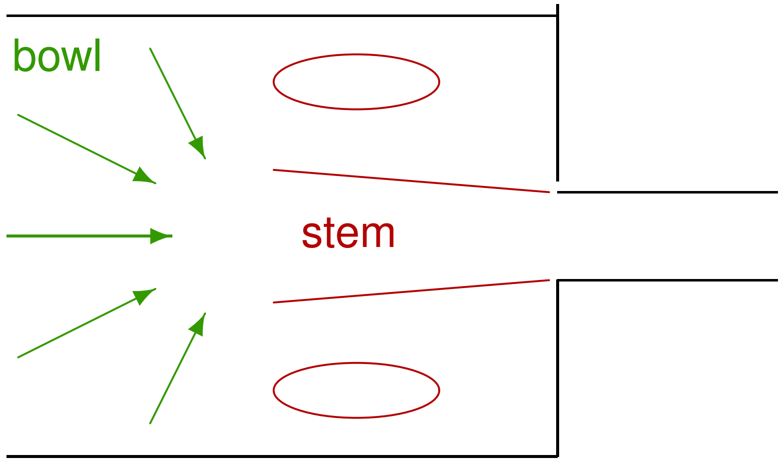
Szabo, Rallison & Hinch 1997 JNNFM

Experiments



Cartalos & Piau 1992 JNNFM

…a champagne-glass model



Bowl: point sink flow, full stretch if $De > L^{3/2}$.

Stem: balance $\mu_{\text{ext}} \dfrac{\partial^2 u}{\partial r^2} = \mu_{\text{shear}} \dfrac{1}{r^2} \dfrac{\partial^2 u}{\partial \theta^2}$

if small cone angle $\Delta\theta = \sqrt{\dfrac{\mu_{\text{ext}}}{\mu_{\text{shear}}}}$

Flow anisotropy from material anisotropy: $\mu_{\text{ext}} \gg \mu_{\text{shear}}$

Conclusions for FENE modification

- A. Contraction: Δp increases, large upstream vortex
- B. Sphere: drag increase, long wake
- D. Capillary squeezing: filament breaks
- Numerically safe

But sometimes need small L to fit experiments.

Understanding flow of elastic liquids?

In Oldroyd B

- Tension in streamlines
- Stress relaxation – transients see $\mu_0 < \mu_{\text{steady}}$
- Flows controlled by relaxation – E to stop relaxation, very slow

In FENE – deformation of microstructure limited

- μ_{ext} large – increase Δp & drag
- $\mu_{\text{ext}} \gg \mu_{\text{shear}}$ – flow anisotropy

independent of details of model?

More than viscous $+$ elastic