Resumé of lecture 2

Driven Cavity in ψ–ω formulation.

Finite Differences

Poisson problem. SOR.

Test against theoretical solution: $O(\Delta x^2)$ error?

2.7 Vorticity evolution

$$\frac{\partial \omega}{\partial t} = -\frac{\partial(\omega, \psi)}{\partial(x, y)} + \frac{1}{Re}\nabla^2 \omega$$

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On boundary need $\psi = 0$, and value of ω

Boundary condition on ω – so that $\frac{\partial\psi}{\partial n} = U_{\mathrm{wall}}$

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Starts at $t = 0$ as numerical delta function, then diffuses.

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plot ω for $Re = 10$ at $t = 0.525$ with $\Delta t = 0.035$ and $\Delta x = 0.1$

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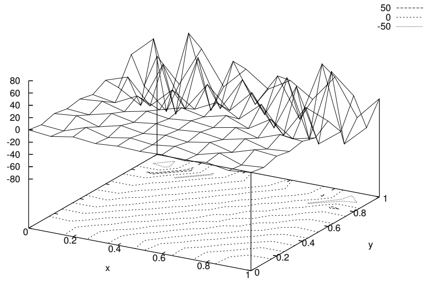

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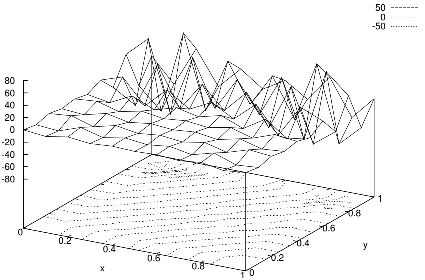

Numerical or physical instability?

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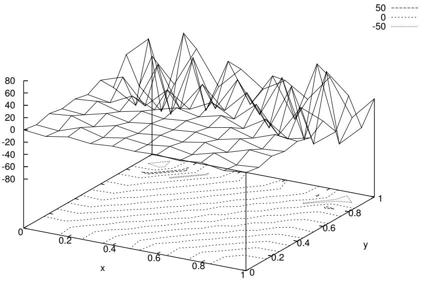
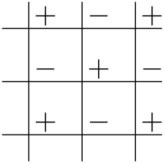


Numerical or physical instability?

Not physically unstable at $Re = 10$ surely?

Time step instability 2

Checker board pattern.



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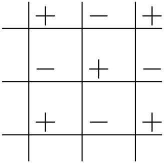

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EJH works at $\frac{1}{5}$.

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(Courant-Friedricks-Lewy)

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'Better' time step algorithms $\rightarrow$ larger Δt, but more accurate?

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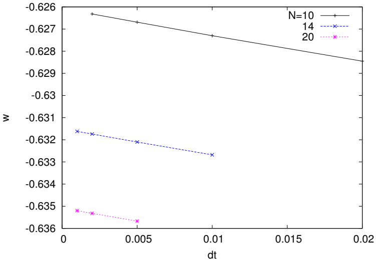
1st order BC for ω_0 with $Re = 10$ and $N = 10$, 14 and 20.



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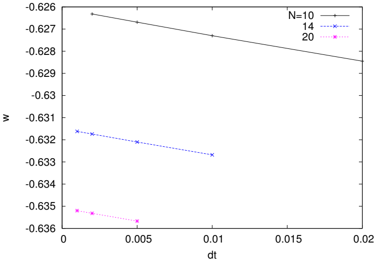

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Large errors in Δx → 2nd order BC for ω_0 better?

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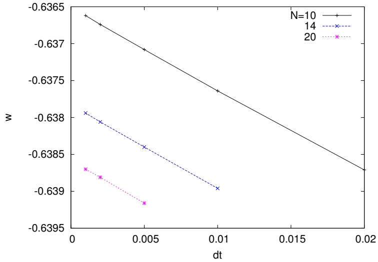

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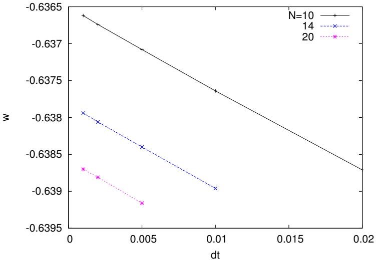

Much smaller errors from Δx.

Well matched design

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Hence no need for second-order time-stepping.

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Set $\Delta t = 0.2 Re \Delta x^2$. Plot $\omega(0.5, 0.5, 1)$ at $Re = 10$ for $N =$ 10, 12, 14, 16, 18, 20, 24 and 28.

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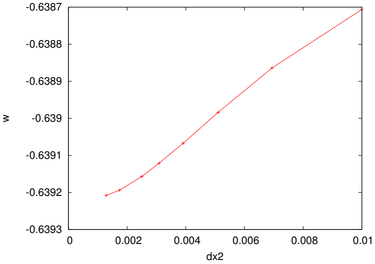

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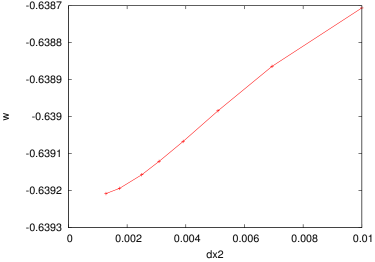

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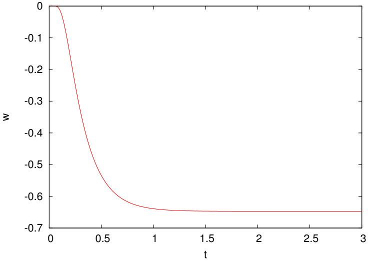
Note linear extrapolation in Δx^2 from $N = 10$ and 14 gives same accuracy as 28 at $\frac{1}{32}$ the CPU.

2.10 Results: time to evolve

Vorticity at centre of box as a function of time, with $N = 20$ and $Re = 10$.



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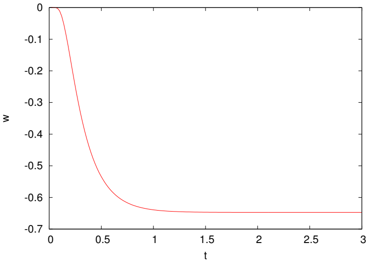

Steady to 10^{-4} by $t = 2$, time to diffuse across box.

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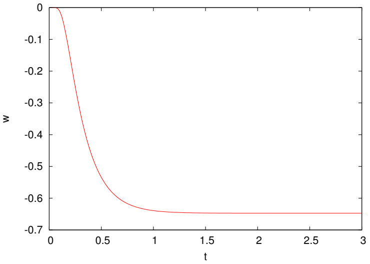
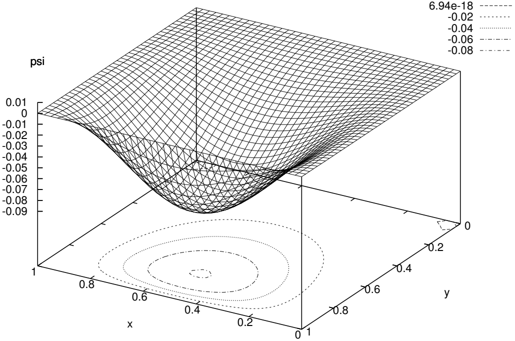


Steady to 10^{-4} by $t = 2$, time to diffuse across box.
For steady state, try reducing to 3 SOR per time step in place of N.

Results: steady streamfunction

At $t = 3$, $Re = 10$ and $N = 40$.



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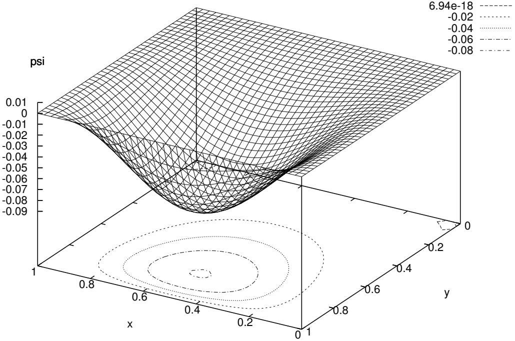

Fast near lid, slow deep into cavity.

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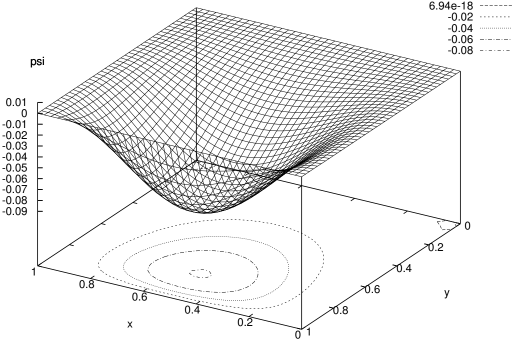

Fast near lid, slow deep into cavity.
Weak reversed circulations in bottom corners

Results: steady vorticity

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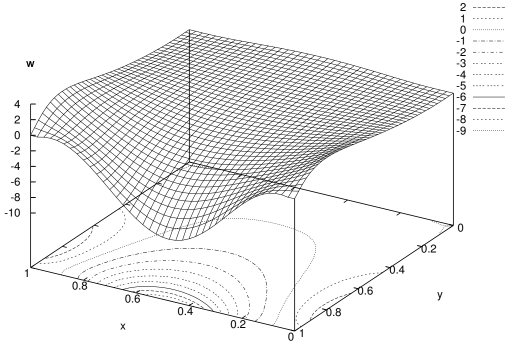

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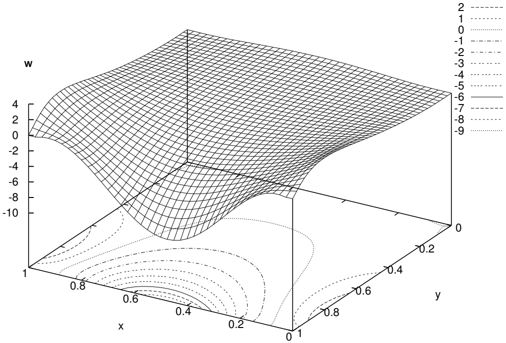

Slight asymmetry downstream

Results: steady mid-section velocity $u(0.5, y)$

$$u_{i\,j+\frac{1}{2}} = \frac{\psi_{i\,j+1} - \psi_{i\,j}}{\Delta x} \quad \text{for} \quad y = (j + \tfrac{1}{2})\Delta x$$

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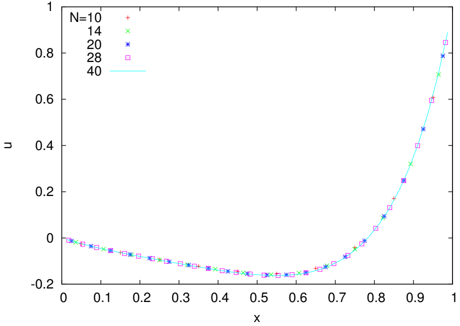

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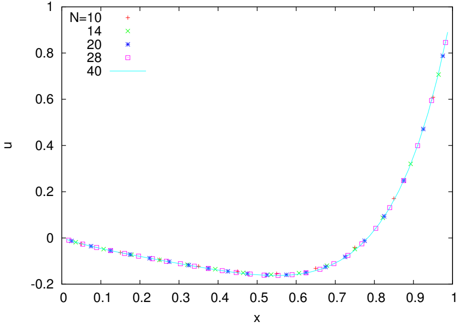

Agree to visual accuracy

Force on lid

$$F = \int_0^1 \left. \frac{\partial u}{\partial y} \right|_{y=1} dx \approx \sum_{i=0}^{N} \left. \frac{\partial^2 \psi}{\partial y^2} \right|_{j=N} \Delta x.$$

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With $O(\Delta x)$ error

$$\left.\frac{\partial^2 \psi}{\partial y^2}\right|_{j=N} \approx \left.\frac{\partial^2 \psi}{\partial y^2}\right|_{j=N-1} = \frac{\psi_{i\,N} - 2\psi_{i\,N-1} + \psi_{i,N-2}}{\Delta x^2} + O(\Delta x).$$

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$$F = \int_0^1 \frac{\partial u}{\partial y}\bigg|_{y=1} dx \approx \sum_{i=0}^{N} \frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N} \Delta x.$$

With $O(\Delta x)$ error

$$\frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N} \approx \frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N-1} = \frac{\psi_{i\,N} - 2\psi_{i\,N-1} + \psi_{i,N-2}}{\Delta x^2} + O(\Delta x).$$

For $O(\Delta x^2)$, linearly extrapolate to boundary

$$\begin{aligned} \frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N} &\approx 2\,\frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N-1} - \frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N-2} \\ &= \frac{2\psi_{i\,N} - 5\psi_{i\,N-1} + 4\psi_{i,N-2} - \psi_{i,N-3}}{\Delta x^2} + O(\Delta x^2). \end{aligned}$$

Force on lid

$$F = \int_0^1 \frac{\partial u}{\partial y}\bigg|_{y=1} dx \approx \sum_{i=0}^{N} \frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N} \Delta x.$$

With $O(\Delta x)$ error

$$\frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N} \approx \frac{\partial^2 \psi}{\partial y^2}\bigg|_{j=N-1} = \frac{\psi_{i\,N} - 2\psi_{i\,N-1} + \psi_{i,N-2}}{\Delta x^2} + O(\Delta x).$$

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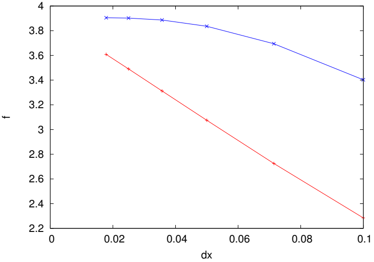
Check: $\psi = 1, y, y^2, y^3 \to 0, 0, 2, 0$

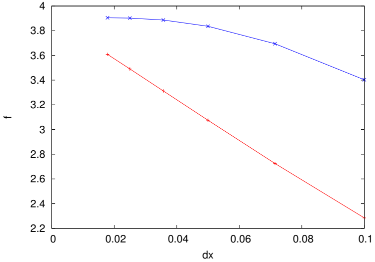
Results: force on lid

At $Re = 10$ for $N = 10$, 14, 20, 28, 40 and 56.



Results: force on lid

At $Re = 10$ for $N = 10, 14, 20, 28, 40$ and 56.



The final answer for the force is

$$F = 3.905 \pm 0.002 \quad \text{at } Re = 10.$$

Results: early times

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Simple $\sqrt{\nu t}$ solution.

Results: early times

Simple $\sqrt{\nu t}$ solution. Plot $F/\sqrt{t/Re}$

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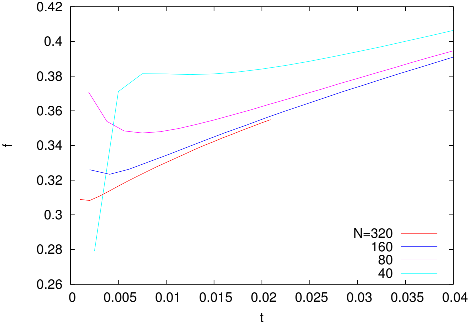
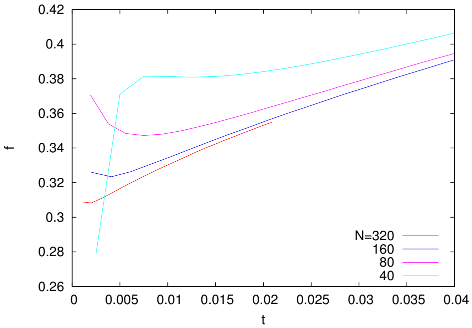


for $N = 40$, 80, 160 and 320.

Failure: Code not designed for $\sqrt{t}$ behaviour.

Results: early times

Simple $\sqrt{\nu t}$ solution. Plot $F/\sqrt{t/Re}$

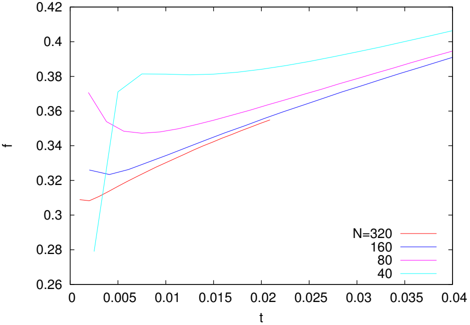


for $N = 40$, 80, 160 and 320.

Failure: Code not designed for $\sqrt{t}$ behaviour.

Results: early times

Simple $\sqrt{\nu t}$ solution. Plot $F/\sqrt{t/Re}$



for $N = 40$, 80, 160 and 320.

Failure: Code not designed for $\sqrt{t}$ behaviour.

Note 0.33, 0.319, 0.307 $\rightarrow \frac{1}{2\sqrt{\pi}} = 0.281$ with $0.4\Delta x^{1/2}$ error.