Lecture 2

Lecture 1: the phenomena.

Now need the intrinsic properties of the material, e.g. viscosity, elasticity.

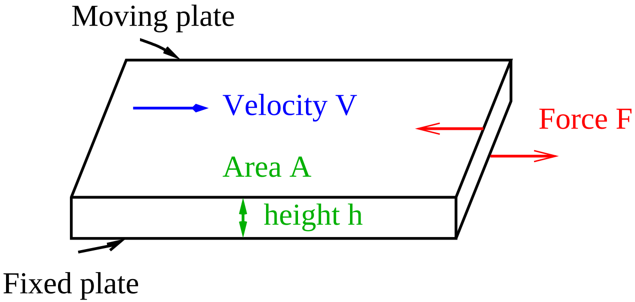
Lecture 2

Rheometry

- Simple shear devices
- Steady shear viscosity
- Normal stresses
- Oscillating shear
- Extensional viscosity
- Scalings
- Nondimensional parameter

Simple shear devices

Conceptual device for simple shear



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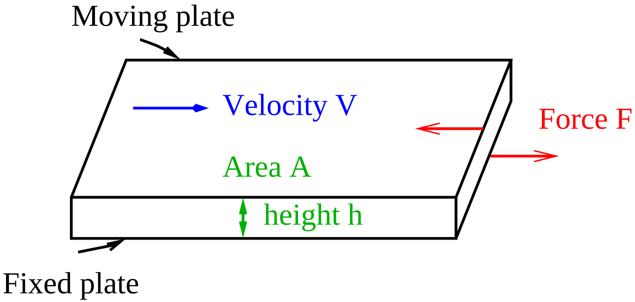

Shear rate $\dot{\gamma} = \frac{V}{h}$

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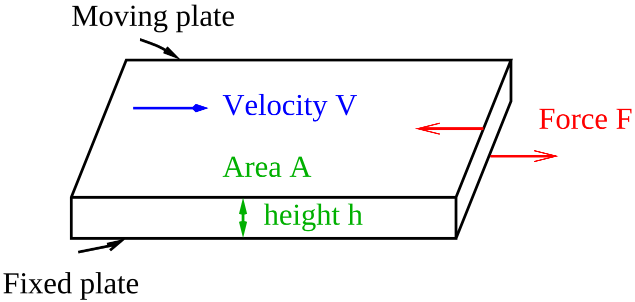

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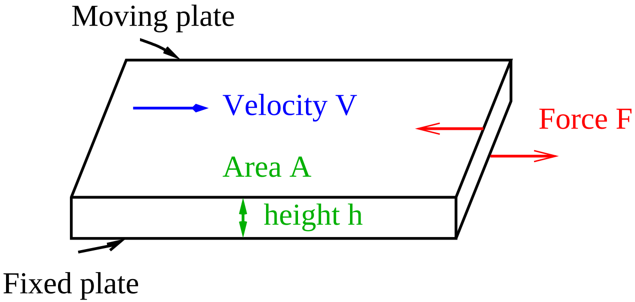

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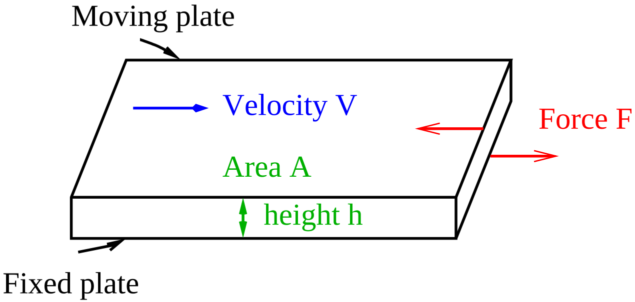

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– works for heavy tars

Simple shear devices 2

Viscometric flows: one layer of fluid slides over another

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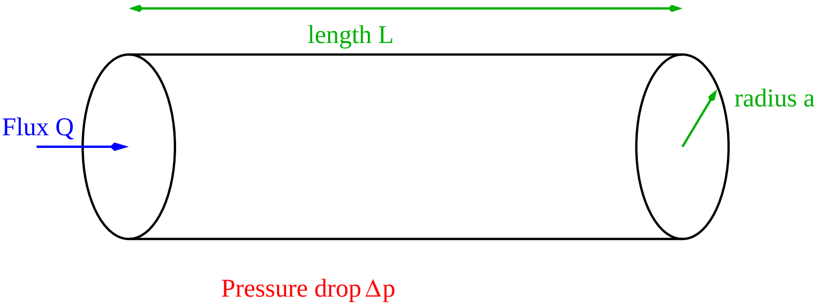

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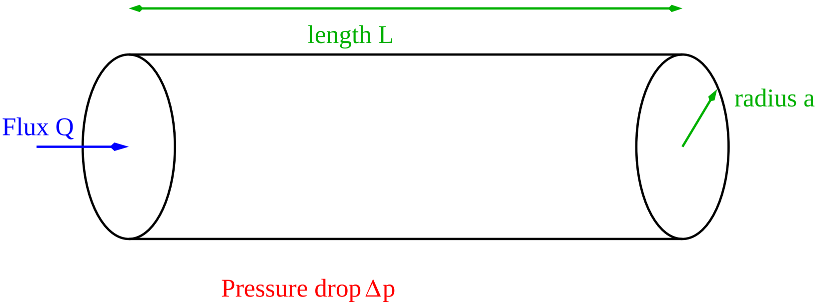

$$\mu = \frac{\pi a^4 \Delta p}{8QL}$$

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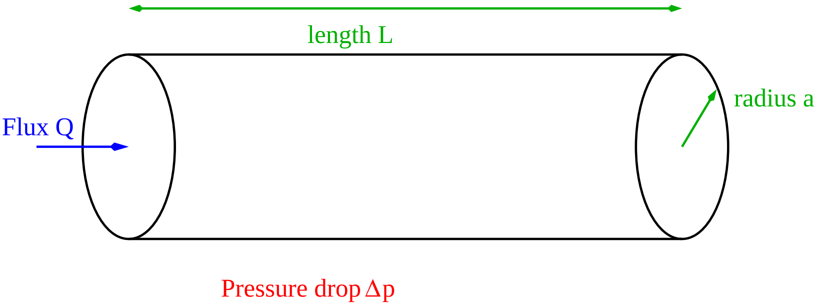

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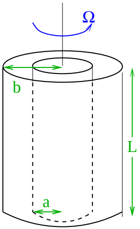
Student Exercise

Simple shear devices 3

Couette experiments in Paris for viscosity of gases, device found in Loire garage.
Unstable if rotate inner too fast.



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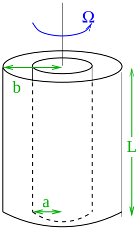

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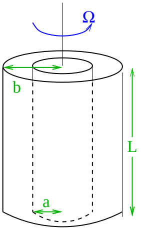

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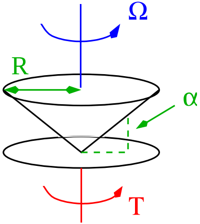

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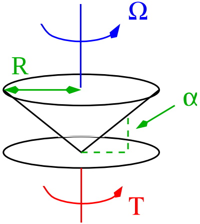

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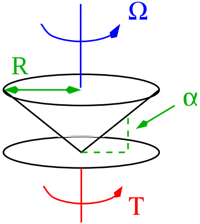

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Angular velocity of top cone Ω. Torque (on lower plate) T.

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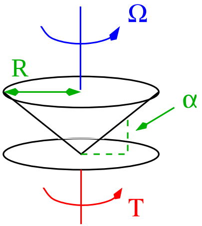

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- μ in $\mathrm{Pa\,s}$
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  - golden syrup 10^{2}
  - molten polymer $10^{3\to 5}$
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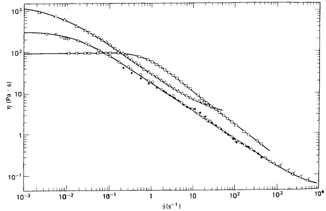
Typically has range of power-law shear-thinning

$$\mu(\dot\gamma) = k\dot\gamma^{n-1}$$

n: 0.6 molten polymer, 0.3 toothpaste, 0.1 grease.

Steady shear viscosity 2

Two polymer solutions and an aluminium soap solution



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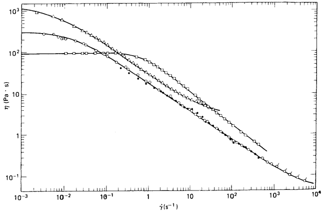

Decades of power-law shear-thinning

Normal stresses

$$\mathbf{u} = (\dot{\gamma} y, 0, 0) \qquad \begin{cases} N_1 = \sigma_{xx} - \sigma_{yy} \\ N_2 = \sigma_{zz} - \sigma_{yy} \end{cases}$$

Stress differences to eliminate incompressibility's isotropic pressure

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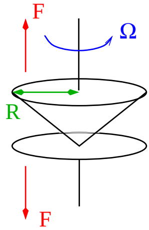
First normal stress difference from axial thrust on plate F.



Axial thrust on plate F.

$$N_1 = \frac{2F}{\pi R^2}$$

Student Exercise

Normal stresses 2

Plot $\Psi_1 = N_1/\dot{\gamma}^2$, as $\propto \dot{\gamma}^2$ at low $\dot{\gamma}$ (indpt sign/direction).

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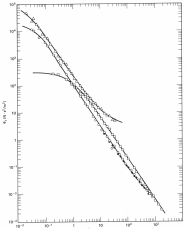

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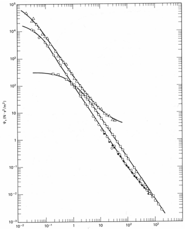

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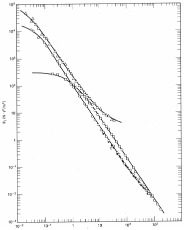

Decades of power-law behaviour.

At low $\dot{\gamma}$, $N \ll \sigma_{xy}$, but at high can be $100\times$.

Normal stresses 3

N_2 normally small and negative.

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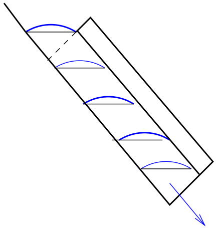
- Bowing of free surface in Tanner's tilted trough



Oscillating shear

Shear:

$$\gamma = \gamma_0 e^{i\omega t} \qquad \text{(real part understood)}$$

Small amplitude: $\gamma_0 < 0.1$.

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Storage modulus G' and loss modulus G''.

$$G^* = G' + iG''$$

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Oscillating because wider range of frequencies, 10^{-3} to $10^{5}\,\mathrm{s}^{-1}$, than steady shear rates.

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Power law behaviour at intermediate ω – probes small scale structure.

Oscillating shear 3

Other unsteady shear flows in modern computer controlled rheometers.

- Switch on stress, measure transient creep
- Switch off stress, measure transient recoil
- Switch on flow, measure build up of stress
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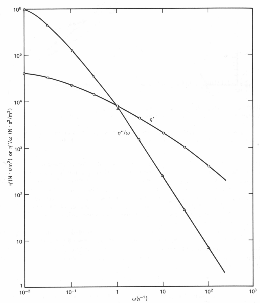
Student Exercise: Connection between these and $G^*(\omega)$?

Oscillating shear 4

Dynamic viscosity $\mu^* = \mu' - i\mu''$



Polyethylene melt (IUPAC Sample A)

Oscillating shear 4

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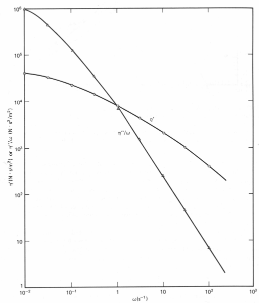

Polyethylene melt (IUPAC Sample A)

At low ω, μ' tends to a constant, and μ'' is smaller by a factor of ω

Extensional viscosity

Uni-axial (axisymmetric) pure straining motion

$$\mathbf{u} = \dot{\epsilon}(x, -\tfrac{1}{2}y, -\tfrac{1}{2}z)$$

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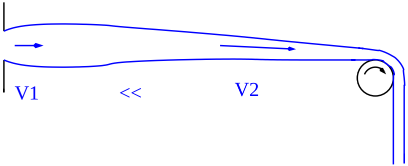
Cannot be steady in time and constant in space, so devices are not perfect.

Extensional viscosity 2

Spinline



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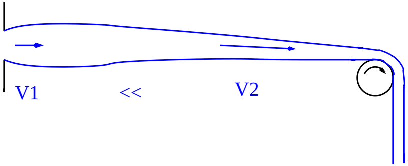
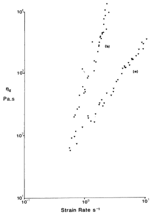
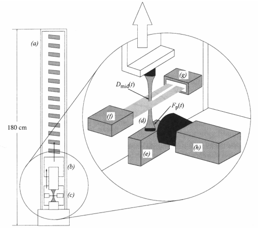


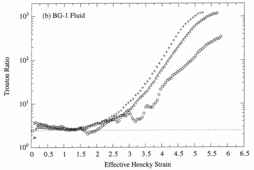
Measure tension T & area $A(x)$ gives stress $\sigma_{xx}(x) = T/A$.
Velocity change & length gives strain-rate $\dot{\epsilon} = (v_2 - v_1)/L$.



Extensional viscosity 3

Filament stretching – Cogswell, Meissner, Sridhar

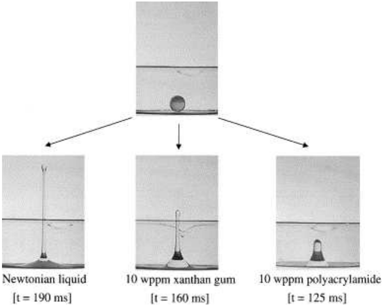




BG-1 Boger fluid: $\dot{\epsilon} = 1.0$, 3.0 and 5.0.

Extensional viscosity 4

Solid sphere hits free surface producing a Worthington jet



Needs theory to interpret splash height.

Extensional viscosity 5

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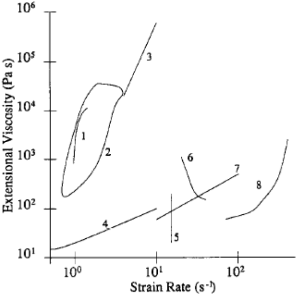
- Squeeze film
- Sag of heap of cement

No agreement between differrent extensional devices!

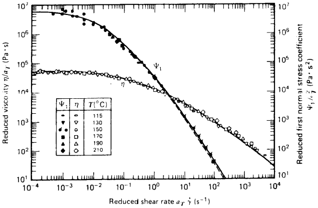
M1 liquid



Temperature scaling

Plot reduced viscosity μ_r as function of reduced shear-rate $\dot{\gamma}_r$

$$\mu_r = \mu(\dot{\gamma}, T)\frac{\mu(0, T_*)}{\mu(0, T)}, \qquad \dot{\gamma}_r = \dot{\gamma}\frac{\mu(0, T)}{\mu(0, T_*)}\frac{T_*\rho_*}{T\rho}$$

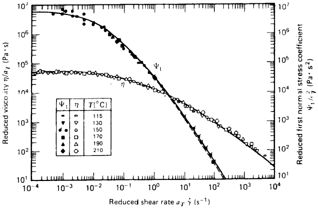


Low density polyethylene melt, reference temp 423K

Temperature scaling

Plot reduced viscosity μ_r as function of reduced shear-rate $\dot{\gamma}_r$

$$\mu_r = \mu(\dot{\gamma}, T)\frac{\mu(0, T_*)}{\mu(0, T)}, \qquad \dot{\gamma}_r = \dot{\gamma}\frac{\mu(0, T)}{\mu(0, T_*)}\frac{T_*\rho_*}{T\rho}$$

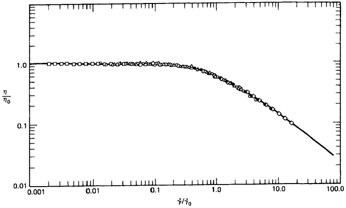


Low density polyethylene melt, reference temp 423K

$\mu(0, T)$ has activation energy around $4000^\circ\mathrm{K}$

Concentration scaling

Plot *intrinsic viscosity* $= \mu(c, \gamma/\gamma_0)/\mu(0,0)$



Cox-Merz ‘rule’

‘Ad hoc’ approximation linking steady and oscillating response, based on oscillation seen if rotate with vorticity in a steady shear.

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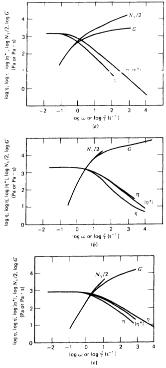
$$\mu_{\text{steady}}(\dot{\gamma}) \approx |\mu_{\text{osc}}(\omega = \dot{\gamma})|, \qquad N_1(\dot{\gamma}) \approx 2G'(\omega = \dot{\gamma})$$

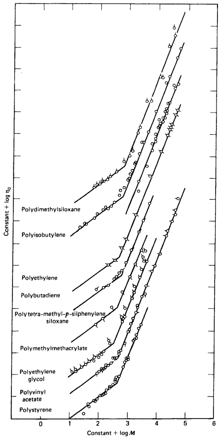


Solutions of polystyrene in 1-chloronaphalene

Molecular weight scaling

At low molecular weight M,
$\mu \propto M^1$

At high molecular weight M,
$\mu \propto M^{3.4}$



Nondimensional parameter

Materials have a time constant τ

- $\mu_{\mathrm{steady}}(\dot{\gamma})$ plateau ends at $\dot{\gamma} = 1/\tau$,
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$De \ll 1$ – fully relaxed, liquid-like behaviour, viscosity
$De \gg 1$ – little relaxed, solid-like behaviour, elasticity