This draft is an expanded version of the published Note 58 of *Lucidity and Science, Part I* (M. E. McIntyre 1997, *Interdisc. Science Reviews* **22**, 199). After revision it may become part of a book in preparation, with the same or a similar title, perhaps *Lucidity, Science, and Music.* **Revision is essential because the draft below contains two factual mistakes.** First, it repeats the myth that J. S. Bach's well-tempered clavier was tuned in equal temperament. In fact Bach probably used a temperament in which fifths on the flat side of the circle of fifths were kept pure, and most of the fifths on the sharp side were compressed slightly more than in equal temperament, making the C major and nearby triads purer in their tuning, and the triads of remote keys harsher. Second, more importantly, and very embarrassingly, there are some **wrong statements about the tuning of plain minor triads**, arising from my failing to check the facts experimentally with suitably precise pitch-generating equipment. On finally getting round to doing these checks, in late 2003, I found that my ear, and the ears of other musicians I consulted, prefer the wide minor third (316 cent) in a plain minor triad, contrary to what is implied below. However, the narrow or 'septimal' minor third (267 cent) is still favoured in a closely-spaced Tristan chord (the ubiquitous 'minor triad with added sixth'), in middle to high tessitura.

The startling implication is that the closely-spaced Tristan chord is psychophysically **simpler** than the plain minor triad! It is simpler in the sense that the brain's auditory model-fitting system seems to prefer to fit a single harmonic series to the closely-spaced Tristan chord, but **two different** harmonic series to the plain minor triad. In this sense the minor triad is the simplest **polychord** of Western music.

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58. **A journey into musical hyperspace. Preamble:** In Parts I and II (or pp. 12–14 and Appendix 2 above), I touched on the peculiar power of music to go somewhere, or take us somewhere. This is especially characteristic of Western polyphonic music. But in what kind of space does it move? Indeed, why does music exist at all? There are intriguing clues from psychoacoustics and neurophysiology, and from the practicalities of how harmony works. They connect with two central threads in my discussion: the relation between play and survival, and the perceptual and cognitive importance of organic change.

Many of the rules or guidelines about powerful harmonic progression, or motion, or function, in Western music — representing discoveries made by composers in a vast range of so-called tonal and atonal, classical, jazz and other styles, from the simplest popular songs to Bach, Debussy, Schoenberg, and beyond — can be summarized, and generalized, by reference to the idea of organic change. This captures much of what is relevant in a fundamentally simpler way than one might think possible from the music-theoretic literature. Indeed, the perspective developed here, relating musical practicalities to biological fundamentals, may well seem strange to a music theorist. But the simplicity must have been noticed by many composers, some of whom may well have regarded it as a trade secret.

Wagner might be a case in point, as suggested by the much-discussed *Ring* and *Tristan* examples below. From the present perspective these examples bear a close family resemblance to examples from Mozart and Gershwin, as I shall demonstrate. Related to all this, though left aside here, is a view of so-called 'consonance' and 'dissonance' that avoids any absolute dividing line between them purporting to be independent of timing, context, and musical motion including contrapuntal motion, the power of voices moving against each other.[22] Also left aside are the larger-scale or 'architectural' aspects of harmonic function. What I shall focus on is elementary. But it is also far-reaching because, in a strong sense to be explained, it is culture-independent.

The last point is worth stressing because of today's common assumption, or article of faith, that 'Western music' is an arbitrarily determined cultural affair, dependent on an arbitrary subdivision of the octave into 12 intervals. Indeed, the idea that art and culture are *arbitrary* constructions — as distinct from the reasonable idea that culture is important — seems so widespread that in order to show exactly how and why it is wrong, for present purposes, I need to go into a little more detail than elsewhere in this book. The point is so important, at a deeply fundamental level — beyond music as such, or particular types of music — that the digression would seem well justified. The discussion is self contained, and should be intelligible to any interested person who responds to music and who can pick out a few notes on a keyboard, or can ask a keyboard player to do so. Audio clips will be provided on the Internet and in the multimedia version [not yet done, sorry!].

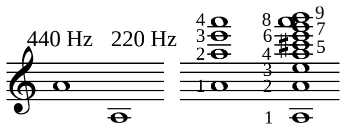
Two kinds of perceptual proximity: By powerful harmonic motion or function I mean sequences of harmony changes that, with suitable timing — and the timing is always crucial — are powerful in reinforcing the melodic and contrapuntal motion so as to help the music to 'go somewhere'. A sense of going somewhere depends not only on a sense of continuity but also, still more fundamentally, on a sense of what is nearby and what is further away. Harmonic motion has the interesting property of depending not on one but on two basic, style-independent, culture-independent, and subjectively very different kinds of perceptual proximity. In this respect musical space has some kinship with the 'hyperspace' of science fiction stories. It is possible to go somewhere that is both nearby and far away.

The first kind of perceptual proximity is the obvious one, that of semitones or adjacent notes on the Western keyboard, or adjacent notes in other scale systems, or of the continuous pitch variations that are natural to the human voice. Psychologists call this ‘height’ proximity, or ‘height’ similarity. The second is what psychologists call ‘chroma’ proximity or similarity, especially in connection with the near-identity of octaves, echoed in the repeating pattern of black and white keys on Western musical keyboards. It has also been called Pythagorean, circle-of-fifths, long-pattern, or, most plainly for present purposes, *harmonic-series* proximity. Here I will stick to the last term, because it is the clearest and most self-explanatory as will be seen in a moment.

Whatever we call it, harmonic-series proximity is just as basic — just as primordial and culture-independent — as height proximity. This is an inescapable consequence, I will argue, of the model-building and model-fitting that is involved in auditory perception. Different musical cultures differ only in the ways in which they exploit the two kinds of perceptual proximity.

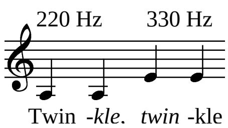
Two notes are near each other in the harmonic-series sense if, to good approximation, they have vibration periods or cycle times in a simple ratio. This means that their vibrations have a high degree of synchrony. Equivalently, two such notes have overlapping harmonic series, in the sense that some of the members of one series coincide to good approximation with members of the other. The members of a harmonic series, it will be recalled, are the fundamental frequency and its integer multiples. Thus the 1st, 2nd, 3rd, 4th,... members have frequencies equal to the fundamental frequency multiplied by 1, 2, 3, 4,..., and cycle times equal to the fundamental cycle time multiplied by $1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4}$,...

Unisons aside, pairs of notes an octave apart are the closest possible in this harmonic-series sense. The fundamental cycle times are in the ratio 1 : 2. Every member of the harmonic series of the upper note coincides with a member of the lower. Every second vibration of the upper note synchronizes with a vibration of the lower. Notes an octave apart are perceptually so similar that musicians give them the same name, as with 440 Hz A or *la*, the note to which orchestras tune, and 220 Hz A, an octave lower, shown here together with the first four and the first nine members of their respective harmonic series, enough to show part of the pattern of coincidences:



Harmonic-series overlap implies that the two notes produce strongly overlapping excitation patterns on the basilar membrane of the inner ear. Still more important, they produce synchrony or near-synchrony of many features in the trains of nerve impulses sent to the brain, and in the reverberating neural circuits within the brain that must be involved in the model-fitting. Neurophysiological research has provided strong evidence that such synchrony should be relevant.[153] This idea leads to what is called the ‘long pattern hypothesis’,[153a] which says that to produce the octave and other musical percepts of ‘consonance’ the synchrony must be good enough to keep approximately in step over a time interval that is ‘long’ in some suitable sense. The results of psychophysical experiments suggest that ‘long’ means time intervals up to one or two hundred milliseconds,[150b,153a] though circumstance-dependent. Such time intervals are roughly of the right order to be consistent with musicians’ pitch perception accuracies, as far as one can say in the present state of knowledge, i.e., without a detailed understanding of the brain’s pitch perception mechanisms.[155]

The next closest in the harmonic-series sense are pairs of notes a musical perfect fifth apart, as at the start of ‘Baa, *baa, black* sheep’, ‘Twin*kle, twin*kle, little star’, ‘Ah, *vous dir*ai-je, Maman’, etc., e.g. 220 Hz A or *la* and 330 Hz E or *mi*, four white keys to the right:



Musicians call two such notes ‘harmonically close’, as distinct from ‘melodically close’. They have only slightly weaker overlap and synchrony. The fundamental cycle times are in the ratio 2 : 3. Every even-numbered member of the harmonic series of the upper note coincides with a member of the lower, because every even integer gives another integer when multiplied by $\frac{3}{2}$. Every third vibration of the upper note synchronizes with every second vibration of the lower. Similarly, the perfect fourth, the major third, etc., with cycle-time ratios 3 : 4, 4 : 5, etc., represent successively greater distances or dissimilarities in the harmonic-series sense, though successively smaller in the height sense.

But why is harmonic-series proximity just as basic, primordial, and culture-independent as height proximity? The reason is simple: it is biological necessity. It is simply that the harmonic series, or equivalent long-pattern time-domain information, has to be represented among the brain's model building blocks. Such information is needed to build internal models that can fit, hence make sense of, the very complex spatiotemporal excitation patterns produced by 'a jungleful of animal sounds'.* More precisely, the harmonic series or equivalent time-domain information is needed as a model building block whenever the jungleful of sounds, or any other sounds, have amongst them the almost-periodic signals produced by vibrating syrinxes, larynxes, or other acoustic oscillators — whistles, hoots, yowls, and so on, all the way to human speech and song. This is a matter not of culture but of the most basic physics — the patterns associated with almost-periodic sound signals — and the most basic biology. Nature–nurture polemics are beside the point. We need not debate whether the ability to recognize yowls and whistles is fully formed at or before birth, or whether unconscious learning from auditory input has a role, which probably it does. One way or another, survival compels the ear–brain system to perform feats of pattern perception in which, inescapably, the harmonic series or equivalent information, internally represented in one way or another, has to be one of the most crucial patterns the system must deal with.

Once again we see the relation between play and survival. It is for survival's sake that the brain takes pleasure in playing with such patterns. This is a biological imperative that is plainly part of why music exists at all. It is not the whole explanation, of course. Music — song and dance — must have been intimately part of our ancestors' tribal cohesion. It is no accident that the *inner musician* in us has emotional power, and is both a dancer and a singer. In a prehistoric world full of large predators, group bonding was another survival imperative.†

Much of what composers have discovered about harmonic motion can now be summarized in just one sentence. Powerful, continuous harmonic motion involves organic change — some things changing while others stay invariant — with the amount of change, usually small, referring to either or both kinds of perceptual proximity. This is part of why 'good voice-leading' in a musical score can include surprisingly large height jumps in some voices. Jumping by a fifth, for instance, is a small change in the harmonic-series sense even though large in the height sense.

Let me be more specific. The two most basic techniques for powerful, continuous harmonic motion — out of which more elaborate structures can be built and against which other harmonic effects can be contrasted — are **(a)** to change some pitches by small amounts (small in either sense), keeping other pitches invariant, and **(b)** to change all pitches by the same small amount (small in either sense), keeping the chord shape invariant — i.e., to use parallel motion of an invariant chord shape. This summarizes, and generalizes, much of what can be learnt from a good practical harmony textbook. Examples of techniques **(a)** and **(b)** will be given shortly. Claude Debussy (1862–1918) may have been the first great composer to use them both with something like the present day freedom.[150,150a]‡

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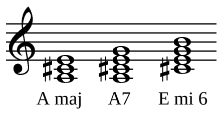
*Because the model-fitting aims at identifying sound sources,[22] it must be phase-sensitive for the purpose of direction-finding but *insensitive* to phase differences between harmonic-series members. Such phase differences vary wildly as the source and receiver move, causing variations in the ensemble of (frequency-dependent) sound transmission paths. This bears on a much-discussed point in the psychoacoustical literature.[150b,153] The 'jungleful' allusion is to the introductory discussion on p. 13 above, or p. 206 of the published Part I. See also the fuller discussions on pp. 7ff. and in Appendix 2 herein, or in Part II and in Note 142 of Part III (`lucidity3.ps`). Parts I–III appeared in *Interdisciplinary Science Reviews* **22**, 199–210, **22**, 285–303, and **23**, 29–70, 1997–8.

†This obvious point seems to be missed surprisingly often, even by professional scientists, perhaps through the unconscious persistence of naive ideas about Darwinian natural selection. There is a mind-set that 'nature red in tooth and claw', 'every man for himself', and so on, is the whole story — that 'selfish genes' are selfish in a simplistic sense. This might be more reasonable if every species had a solitary lifestyle. It forgets that our own ancestors, in particular, had to survive predation in open country, as forests shrank thousands of millennia ago. Group cohesion must have been essential. To think that social skills, bonding through gesture and vocalization, love, altruism, and so on, are difficult to explain in Darwinian terms is a bit like thinking it difficult to explain why birds have wings. Darwin's hypothesis is that anything a species has come to depend on for survival *must have developed with the aid of selective pressures.* And group bonding was essential to our own ancestors' survival. I return to this point in Part III. It is further discussed by Cross, I., 1999: Is music the most important thing we ever did? Music, development and evolution. In: *Music, Mind and Science* ed. Suk Won Yi, Seoul: Seoul, National University Press. Also http://www.mus.cam.ac.uk/~cross/MMS/ and snubook@plaza.snu.ac.kr.

‡Changes that are small in the height sense give rise to what musicians call 'chromatic' effects, and 'leading note' effects. Changes that are small in the harmonic-series sense give rise to what musicians call diatonic or 'non-chromatic' effects, especially 'dominant', 'subdominant', and associated 'neighbouring key' effects. The 'dominant' and 'subdominant' of a given note are its nearest non-octave neighbours in the harmonic-series sense, a fifth above and a fifth below; these neighbours are used with or without added notes that reinforce, decorate, elaborate, or contrast with their harmonic series. It should be cautioned that,...

Debussy's frontier: Debussy also seems to have been the first great composer to exploit systematically the other most basic corollary of the foregoing, of the biological imperative just explained. Multi-note chordal sonorities that correspond to subsets of a harmonic series inherit the long-pattern synchrony of that series. Integers multiply to give integers. This makes such sonorities perceptually special — along with melodic fragments made of harmonic-series subsets.

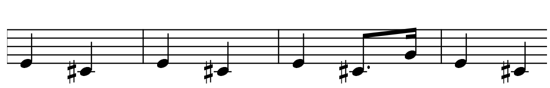
We can now see why, for instance, chords like the following have long been recognized by musicians as perceptually special:



They occur very frequently throughout classical and popular music. In this example, each chord corresponds to a subset of the harmonic series of 55 Hz A. The first corresponds to members 4, 5, and 6 (220 Hz A, 275 Hz C♯, and 330 Hz E), the second to members 4, 5, 6, and 7 (add 385 Hz G), and the third to members 5, 6, 7, and 9 (remove 220 Hz A and add 495 Hz B). Melodic fragments made from members 6, 7, and 8 are commonplace in children's chants:

This is no accident: members 6, 7, and 8 form the harmonic-series subset that is easiest to sing recognizably. The musical effect is close to that of the next easiest, also heard, though less often:

or



Here the two versions are made from members 5, 6, and 7 of two harmonic series. The second of the two series is again the harmonic series of 55 Hz A, of which members 5, 6, and 7 are the bottom three notes of the third chord shown above. Not surprisingly, harmonic-series fragments like these are heard in the children's chants and folk songs of many different cultures. For instance the composer Robert Walker has lived for many years in Indonesia and South-east Asia. "Wherever I go," he tells me, "children use a falling whole tone and minor third."

Musicians will recognize the three chords above as, first, an A major triad or 'common chord', second, a so-called dominant seventh (A7), and third, a so-called 'Tristan chord' or minor triad with added sixth (E minor 6), also called a half diminished seventh or supertonic seventh. (For the sake of simplicity and,

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It should be cautioned that, while proximity or similarity, in the harmonic-series sense, has a clear meaning for present purposes — whether described as strong harmonic-series overlap or near-synchrony in the time domain — 'large distance' or 'dissimilarity' is something far more complex, ill understood, and probably ill defined.[153b] This is hardly surprising in view of the greater complexity produced by asynchrony. It is doubtful whether the notion of 'distance' or 'metric' really applies in its strict mathematical sense, with the usual additivity properties (triangle inequality etc.), as often hypothesized in experimental psychological studies. The problem is most acute for the case of large distance, better described as large perceptual dissimilarity. It has been argued, by R. N. Shepard and others, that if there is a suitable perceptual space having a metric then it would have to be many-dimensional.[152,154] As many as 7 dimensions have been suggested.[154] If anything like this were correct, it would imply that the perceptual space is far too big to be open to exploration by experimental-psychological methods, and probably far too unstable as well — or ill-fitting, as a model of reality — because of the strong effects of musical context and motion, the musical arrow of time, which may destroy even the presumed equivalence between 'X similar to Y' and 'Y similar to X'.[153b,154a]

The reader venturing into the literature of experimental psychology needs to be warned, too, that there is a clash between the musical term 'chromatic' and the psychological term 'chroma'. What musicians call 'chromatic' effects are associated with large, not small, perceptual dissimilarities in the chroma or harmonic-series sense. Further scope for confusion lies in the use of the term 'chroma' in connection with the two ways in which music theorists tend to arrange the 12 notes of the keyboard in a circle, by semitones on the one hand (i.e. by height proximity) and by fifths on the other (i.e. by harmonic-series, alias chroma, proximity). To close each circle, notes an octave apart are regarded as identical. The term 'chroma circle' is customarily used to mean the first (height-proximity) circle and not the second (chroma-proximity) circle — perhaps because the latter has traditionally been called the circle of fifths. Thus chroma proximity — in all cases *except* the octaves now regarded as identical — corresponds to large, not small, distances around what is called the chroma circle. This is why I am avoiding the term 'chroma', even though it is well established in the psychological literature. I am grateful to Ian Cross and Brian Moore for information and references.

more importantly, for the sake of the widest possible generality, I am going to take what a music theorist might call outrageous liberties with chord names. I will use them in a naive way that refers to the static sonorities alone, in the practical manner of jazz and popular musicians.[§]) Debussy exploited the fact that there are many other subsets of a harmonic series, and therefore many other such special harmonic-series sonorities.

Notice now that parallel motion of any such chordal sonority is related to my primordial whistles and yowls — or at least low pitched versions thereof — revealing them as special cases of technique **(b)** using continuous change. In a deeply unconscious sense, a jungleful of animal sounds is also a jungleful of technique **(b)** in action.

The tuning of chordal sonorities in general, and of special harmonic-series sonorities in particular, involves nontrivial practical questions. This is because of the perceptual relevance of long-pattern synchrony. It takes consummate skill and mental concentration on the part of, say, orchestral musicians, to tune such sonorities accurately enough for best effect. The pitch of a given note has to be slightly flattened or sharpened, depending on how it is contributing to the sonority of the moment. Accuracy in such corrections contributes to the wonderful luminosity of sound that can arise in great performances. (The idea that the best musicians play to a fixed 'scale', persistent though it may be in the literature, has been shown by careful experiments to be wrong.[150c,155b]) I will postpone discussion of imperfect tuning except to warn that the theoretical 'just tuning' enshrined in musical-acoustics textbooks is, despite its name, inaccurate for the second and third chords above — this may be part of why they were traditionally regarded as dissonant — and to make the further points that perception is robust and long patterns are finite. Were this to cease to be the case, orchestras would immediately go out of existence, keyboard instruments would become museum pieces, and music would be playable only with the aid of computers. We may also note that certain kinds of tuning imperfections and fluctuations are themselves perceptually significant. They include the deliberate fluctuations called vibrato, on 'intermediate' timescales between vibration-cycle and long-pattern timescales.[155] Fluctuations over some limited range can help the ear–brain system to disentangle junglefuls and roomfuls of multi-note sonorities.[22] Builders of electronic musical instruments take trouble to create such fluctuations artificially, with varying degrees of success.

The ways in which Debussy developed the unrestricted use of organic-change principles and of special harmonic-series sonorities and melodic fragments, thereby opening up vast new musical vistas — he called it 'crossing a frontier' — have been beautifully illustrated in an essay by the late, and much-loved, composer and musicologist Sir Peter Platt.[150a] As the biological and spiritual significance of music comes to be more widely understood, historians may yet see the crossing of this frontier as the most important turning point of all in the transition from nineteenth to twentieth century music. For it was not the invention of a great intellectual scheme or manifesto. It was not the creation of a sphere of influence, of a new band of disciples or zealots. It was a simpler, rarer, and more significant thing altogether. It was as if someone working in the visual arts had suddenly found out how to use vast numbers of new shapes and colours, all belonging to an organically related family whose existence and significance had remained undiscovered until then — despite being under everyone's nose.

The harmonic series was certainly under the nose of, among others, the great physicist and polymath Hermann von Helmholtz, author of the celebrated treatise *On the Sensations of Tone*, with his sure grasp of mathematical and physical principles and his intense interest in musical sounds. Indeed, Helmholtz is famous for having devised an elegant experimental technique, today called the Helmholtz resonator, for making the separate members of a harmonic series audible in the manner of jew's-harp playing. Yet even in the 4th edition of *Sensations*, published in 1877, one still finds the unconscious, or unstated, assumption that for practical musical purposes the harmonic series stops at member 6.

Perhaps this was bound up with the oft-repeated myth or mind-set that the 7th member is always 'too flat', as if that were a universal principle. Here in a very real sense was Debussy's frontier, even though

[§]The naming of chords involves disparate traditions as well as complex issues of musical motion, and is an area of perpetual disagreement, especially between composers and music theorists (e.g. Piston and de Voto[150], p. 359 & Ex. 23-41, p. 364). The composer Robin Holloway (personal communication) tells me that he steers clear of all this and doesn't concern himself with chord names — the important thing being to find 'the right notes'. The effect of a chord is, of course, context-dependent: it may 'function' in different ways, and theorists favour names that say something about context. Names that give advance warning of the next move are especially favoured. Thus in the view of many theorists the third sonority shown above should be called a 'Tristan chord' only when followed by a certain kind of 'hyperspace move', as I shall call it, in which the key changes to F major via C7. There may also be restrictions on what precedes the chord. But because I am taking a more general view, applicable to many styles of music and to contexts that might or might not be describable as being in a definite key — or as moving toward, or 'revealing themselves as being in' a definite key — I need some way of talking about the static sonorities themselves, independent of particular contexts. *And the number of possible contexts is combinatorially large.*

the common occurrence of the three chords above — to say nothing of plain minor triads, the top three notes of the third chord — shows that practical musicians aware of fine tuning corrections had long made, so to speak, nighttime excursions across that frontier in particular directions. Some especially striking evidence has come down to us from Purcell: the sonority of the third chord above is used very persistently, not to say insistently, in his famous and achingly beautiful aria *Dido's Lament*.[150c] Purcell directs the keyboard to be silent, one consequence of which would have been to allow the singer and string players full freedom to make the finest possible tuning corrections.

From Helmholtz' assumption, by contrast, there flows the whole mythology of 'just tuning', along with an exaggeration and rigidification of the distinction between consonance and dissonance. To ignore the harmonic series beyond member 6 is to think that, for instance, there is only one sort of perfectly tuned minor third, the wide one needed in the first chord displayed above, ratio 5 : 6, as distinct from the narrow one needed in the second and third chords and in plain minor triads, ratio 6 : 7 (e.g. the top pair of the second chord, the middle pair of the third chord, and the bottom pair of a minor triad). The same mythology is perpetuated in today's musical-acoustics textbooks, where it never seems to be pointed out that the equal-tempered tuning of modern keyboards, by a strange and wonderful accident, or stroke of luck (see below), manages to hit a compromise between the wide and narrow minor thirds and between the correspondingly narrow and wide major thirds, such as the bottom pair of the first chord above (cycle-time ratio 4 : 5) and the top pair of the third chord (cycle-time ratio 7 : 9). You cannot, of course, point this out if you think, perhaps unconsciously, that there is only one sort of minor third and one sort of major third.

Some trade secrets: Let us return to the organic-change principle and to techniques **(a)** and **(b)** — the most basic techniques that make musical harmony work, in which some pitches change, usually by small amounts, while others stay invariant. To illustrate how these techniques work in practice and to discover for yourself a few composers' trade secrets, if you don't know them already, the quickest way is to consider the height-proximity neighbours of a 'diminished seventh' chord such as C♯ dim7:

C♯ dim7

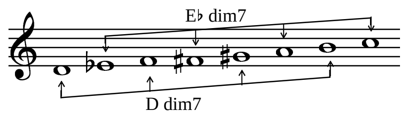
This is not a special harmonic-series sonority. But it is special in another way. It is at the crossroads of a large, powerful, versatile set of harmonic moves using techniques **(a)** and **(b)**. They are an intimate part not only of Debussy's style but also of a vast range of other styles from Purcell and Bach to Stravinsky, Duke Ellington, Lutosławski, and practically everyone else. They provide a rich variety of ways to go somewhere that is nearby in the height sense and almost anywhere you like in the harmonic-series sense, including some of the most thrilling 'hyperspace leaps'. For this reason, some English composers have called the diminished seventh chord the 'Clapham Junction' of harmony, referring to a famous branching of railway lines in London with its many possibilities for going to, and coming from. The power and versatility depend on the symmetry of the diminished seventh. Its adjacent notes are 3 semitones apart, dividing the octave into 4 equal portions: 12 is divisible by 4 to give 3.

If we start by restricting attention to the closest possible height-proximity neighbours, holding three out of the four notes invariant and changing the one remaining note by a semitone — let us call these 'one-note neighbours' — we see that a single diminished seventh already has eight different such neighbours. In other words, the eight ways of moving one or other of the four notes up or down by a semitone all produce different results, illustrating technique **(a)** in eight different ways. It is worth listening to them all, one after another, if you have not tried it before. Notice for instance that E minor 6, the Tristan chord shown above, is one of these neighbours to C♯ dim7. Only the top note differs. So also is A7, the dominant seventh chord shown above, apart from an octave shift. For the moment, let us count notes an octave apart as being the same.

In fact, every one of the eight closest possible one-note neighbours is either a Tristan chord or a dominant seventh. This is a consequence of the abovementioned symmetry. The symmetrical subdivision of the octave implies that a diminished seventh chord is translationally invariant, in the sense that the same four notes are obtained when the whole chord is translated, i.e. transposed or shifted, three steps up or down the keyboard, or any multiple of three steps. It follows that the eight closest neighbours consist of four different Tristan chords, each obtainable from the others by 3-step translations or transpositions, and four different dominant sevenths obtained similarly. Conversely, we might characterize the diminished seventh sonority by saying that it is poised delicately, and unstably, between four closely adjacent pairs of special harmonic-series sonorities.

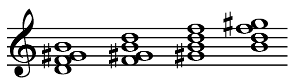
Translational invariance also implies that there are three different diminished sevenths. Between them they give access, as one-note neighbours, via technique **(a)**, to all twelve Tristan chords and all twelve dominant sevenths. (This hints at what I meant by power and versatility, but there is more to come.) Any two diminished seventh chords are, of course, neighbours to each other in the sense of technique **(b)**. The use of such neighbours is conspicuous in, for instance, the music of Mozart and of the jazz violinist Stéphane Grappelli, and in many other styles of music.

By putting two such neighbours together, such as D dim7 and E♭ dim7, say, we discover another interesting structure, the eight notes of what is now called an octatonic scale:



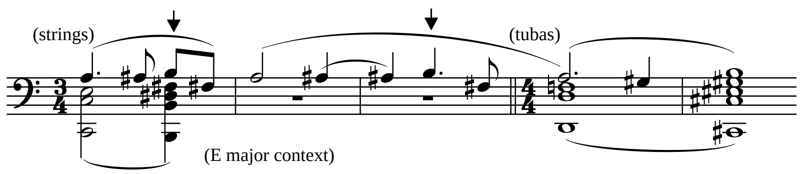
Another great composer, Igor Stravinsky, made brilliant use of this scale from early in his career. The Stravinsky scholar Richard Taruskin tells us that the usefulness of the scale, picked up from Rimsky-Korsakov who had noticed its use in Liszt's music, was for a long time one of Stravinsky's most closely guarded trade secrets. According to Taruskin, Stravinsky carefully avoided talking about it.[155a] This might explain why the octatonic scale is still, even today, hard to find in textbooks or in curricula for basic musical training, despite its close relationship with the diminished seventh including, of course, exactly the same translational invariance property.

A first example: The examples to follow will all be centred for convenience around D dim7, which by translational invariance is the same chord as F dim7, G♯ dim7, and B dim7, if we continue to count notes an octave apart as being the same:


D dim7 and its inversions

Musicians call these 'inversions' of the same chord, meaning that some notes are changed by octaves. It is worth listening carefully to all four. The musical ear perceives them as very similar to each other, though not identical, as expected from the perceptual similarity of octaves. In a nutshell, translational invariance says that a diminished seventh is the same chord as all its inversions.

Here is a famous example that can be understood in terms of the foregoing:



Opera-lovers will recognize the slow, quiet transition into Act 2 scene 4 of Wagner's *Die Walküre*, the second opera of the *Ring* cycle, when, just after Sieglinde has collapsed from fear and exhaustion, the supernatural takes a hand: the Valkyrie Brünnhilde appears to tell Siegmund his fate.

The three-chord motif after the double bar signals Brünnhilde's presence. It recurs throughout the rest of the *Ring* cycle and is usually called the 'fate' or 'destiny' motif. Here it sounds slowly and mysteriously on the tubas, followed by a quiet drumbeat. Its powerful effect depends on the transition from the second chord, D dim7 with B omitted, to the last chord, C♯7, a dominant-seventh closest neighbour.

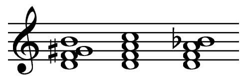
As always, the full power of the effect depends on context. Everything that precedes the move to C♯7 has the feel of coming to rest in or somewhere near the key of E major or A major, with 'near' understood in the sense of harmonic-series proximity. Siegmund is tenderly kissing the sleeping Sieglinde, a moment of peace in a turbulent drama. Instead of the C♯7 chord we could easily have had another closest neighbour that is also close in the harmonic-series sense, namely E7/D (a E dominant seventh with D in the bass). (And this could have been followed by, say, A/C♯ (an A major triad with C♯ in the bass), settling thereafter into a stable A major.) So the move to C♯7 is felt as a hyperspace move — a magical twist to match the supernatural twist in the story. The supernatural is felt as both nearby and far away.[¶]

A few other points are worth making. Notice that the first chord after the double bar, a plain D minor triad, is another closest neighbour to the chord that follows it, the D dim7 with its B omitted. The first chord would have been a Tristan chord if the B had not been omitted from both chords. The minor triad, if accurately tuned, is still a special harmonic-series sonority, as already mentioned, and a very smooth one; and the omission of the B from the second chord does not, in this context, change its D dim7 character very much. This may be partly because the missing B is already very much 'in the air' through its emphasis in the preceding bars (arrows above), and through its immediate reappearance in the last chord.

Two-note neighbours, and a second example: Let us explore further, and consider the next closest types of neighbouring chord. This leads us straight away to some interesting sonorities not yet encountered. For instance, starting from D dim7 you can raise two of its notes by a semitone, taking say F↑F♯ and B↑C, keeping D and G♯ invariant:

If you listen to this, you may well be reminded of Scriabin's later compositions, such as the *Poem of Ecstasy*, or of certain kinds of operatic or film music suggestive of magic, wonderment, or visionary experience. This particular two-note neighbour is an incomplete wholetone cluster, or so-called 'French augmented sixth', the latter term arising historically from its earliest typical uses and contexts. (I continue here with my 'outrageous liberties'.) A moment's thought shows that a single diminished seventh chord has four different such French augmented sixth neighbours. A single French augmented sixth chord is also a one-note neighbour to two dominant seventh chords, D7 and G♯7 in the case shown above, and to two Tristan chords, F minor 6 and B minor 6.

The total number of two-note neighbours, with two notes invariant and the other two raised or lowered by a semitone, in all possible combinations, works out to be 24, not counting inversion. Each one of them is an interesting and much-used sonority.[‖] Two of these two-note neighbours

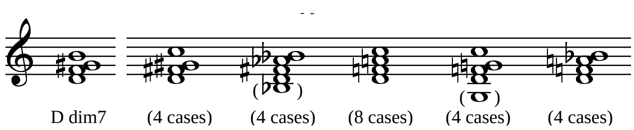


D dim7   1st nbr   2nd nbr

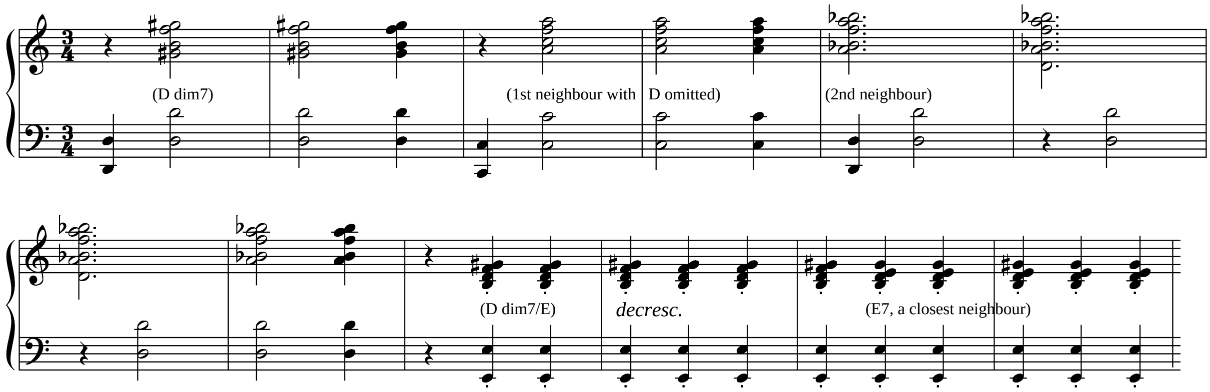
are, for instance, significant for a famous passage in the first movement of Beethoven's *Eroica* Symphony, the magnificent climax at bar 280, shown here transposed and simplified for playability on the piano keyboard:

[¶]In this case the 'hyperspace move' is practically the same thing as what musicians call modulation from one key to another, because there is a clear sense of key at the outset. But 'hyperspace move' is a more general idea than 'modulation'. For instance it applies — see below — to the *Tristan* example, where, famously, there is no clear sense of key at the outset. Moreover, there are some hyperspace moves that are not counted as modulations, by a convention arising from their frequency of occurrence in certain contexts. Notable among these are the standard harmonic gestures using 'Neapolitan sixth' and 'augmented sixth' chords.[150] The sense of 'key' is itself, of course, to do with keeping the emphasis on particular sets of special harmonic-series sonorities, as distinct from more general uses of larger sets of such sonorities.

[‖]Musicians will recognize all these two-note neighbours as one or another of five distinct types, not counting inversion: (1) 'French augmented sixth' (an incomplete wholetone cluster, sometimes named differently depending on context), (2) dominant seventh with raised fifth (= augmented triad plus minor seventh, another incomplete wholetone cluster), (3) major triad with added sixth (= minor triad with added seventh), (4) dominant seventh with raised third (appoggiatura to the third, tonic pedal etc., giving some of the most radiant and widely-used sonorities, whether 'resolved' or not), (5) major triad plus sharpened seventh (as in the Beethoven *Eroica* example to be quoted next, and in countless others; also, this sonority permeates mainstream jazz). A single diminished seventh has 4, 4, 8, 4, and 4 different neighbours of these respective types, 24 in all. Inversion, vertical spacing, and instrumental colour multiply still further the available sonorities, and their precise effects are multplied more vastly yet by context and by counterpoint, the power of contrary motion, the pull of cross-relations, etc., etc. The Mozart, *Tristan* and other examples to follow will again remind us, as did the *Ring* example above, of the importance of context. Here is D dim7 alongside just *one* representative of each of its five types of two-note neighbour:



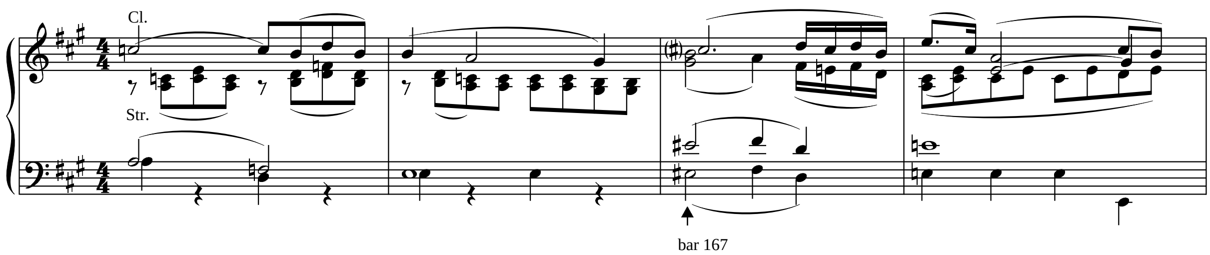
D dim7   (4 cases)   (4 cases)   (8 cases)   (4 cases)   (4 cases)



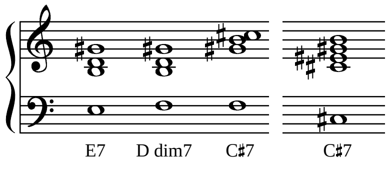
The first neighbour is made more consonant by omitting the note D, turning it into a plain F major triad and increasing the impact of the relatively dissonant second neighbour, whose vertical layout further emphasizes the dissonance. Notice also that, despite the omission of the D, the change from the first to the second neighbour is strongly organic. The two invariant notes A and F offset the impact of the sonority change and the wholetone step that restores the D in the bass. (Invariant notes leading to dissonances in the manner of the A are traditionally called suspensions.) D dim7 continues in a central role after the climax, along with its closest neighbour E7, making the next two moves strongly organic as well, with notes an octave apart counted as being the same for this purpose, as before. The original version for orchestra is scored with a greater vertical span, for maximum effect, and is centred around A dim7 rather than D dim7.

From Mozart to Gershwin to 'the' Tristan chord: Let us look finally at three more examples, from Mozart, Gershwin, and Wagner, the last being the famous *Tristan* opening itself. The three examples may at first seem quite different; but we shall see and hear that harmonically speaking, from the present viewpoint, they are closely related to each other and to the *Ring* example, surprising though this may seem. The Gershwin example illustrates technique **(b)** as well as technique **(a)**.

The Mozart example is from the Clarinet Quintet. For me it remains, over two centuries after it was written, one of the most thrilling examples of a 'hyperspace leap'. Despite long familiarity it still makes my spine tingle — illustrating the independence of the unconscious brain from the consciously intellectual, in this case the deeply unconscious nature of what we simplistically call musical 'expectation'. The example is found at bar 167, or 30 bars before the end of the first movement, at the moment marked by the arrow:

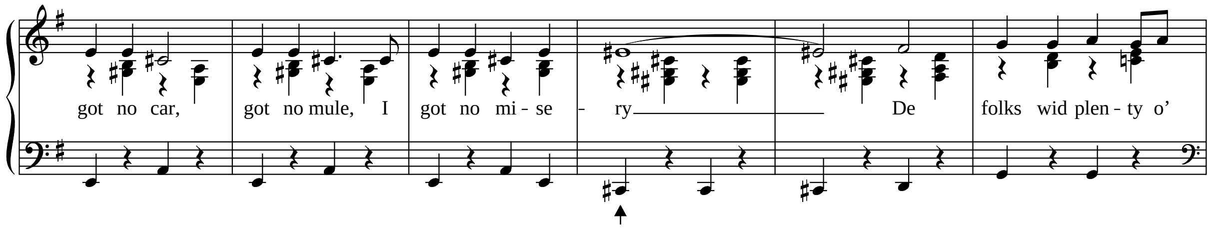


As always, the effect depends for its full power on the skilful preparation of context, on the control of instrumental texture, and on exquisitely judged timing. Once again, the way the harmony works can easily can be understood in terms of the neighbours to D dim7, seen now in an A minor key context. I leave the details as an exercise for the interested reader. But it is quicker to use one's ears. One can listen to the Mozart passage, stopping just before the arrow, and then listen to the first three chords of the following:



It is also interesting to listen to all four chords and then again to the last two chords of the *Ring* example. This makes its resemblance to the Mozart easily audible, despite the telescoping of the latter to omit the D dim7. It could be said, perhaps stretching a point (or should I say warping it?), that in Mozart's version the D dim7 acts as a science-fictional 'hyperspace wormhole'.

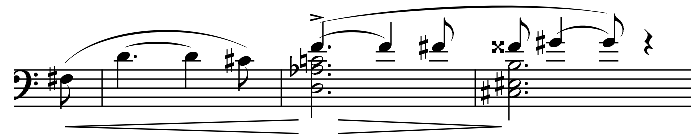
The Gershwin example is from the famous song *I Got Plenty o' Nuttin'*, at the words 'no misery':



Except for the important difference in the bass line, and the omission of the seventh from both chords rather than just the first, this is harmonically very like the *Ring* and Mozart examples, as listening to it will confirm.

The subsequent C♯/C♯ → D/D, leading back to the tonic or key chord G/G, illustrates, in addition, the use of technique **(b)** above, parallel motion of an invariant chord shape including, in this case, two parallel fifths. The first parallel progression uses height proximity. The second uses harmonic-series proximity, somewhat like the famous and very exposed parallel fifth in bars 11–12 of Beethoven's Sixth Symphony, which was 'against the rules' in Beethoven's time even though it, too, works perfectly well musically. (It is when using technique **(a)** that parallel fifths in independent voices often sound clumsy.)

The final example is the famous opening of *Tristan und Isolde* itself. It has special interest here because the notions of organic change, harmonic-series proximity, and 'hyperspace move' are applicable without any trouble, despite the lack of a clear sense of key at the outset — something that has always troubled conventional music theory. The tempo is very slow:



The example has been transposed down a minor third to emphasize certain points of similarity with the *Ring*, Mozart, and Gershwin examples. It then involves, as before, two closest height-proximity neighbours to D dim7. The first neighbour is a Tristan chord — 'the' famous Tristan chord — here an F minor 6 or half diminished seventh on D. (I continue still further with my 'outrageous liberties', referring only to the chordal sonority and not to the subsequent moves.) The second neighbour is the last, C♯7, chord, the same neighbour as in the other examples. The first and last chords are two-note neighbours to each other.

But why does the first chord, as such, have its famous and peculiarly powerful impact? It can only be because of the three bare notes that precede the chord. The chord takes on its peculiar colouration whether or not one listens to what follows, or imagines what follows. Yet the three bare notes establish no unambiguous sense of key.**

Notice, however, that the organic-change principle still applies. In terms of height proximity, the first chord is close to all three of the bare notes that precede it. Indeed the second bare note, D, is invariant in the move to the first chord, apart from dropping an octave. The other two, F♯ and C♯, are height neighbours to notes in the chord.

Notice, furthermore, that none of the three bare notes is close to the chord in terms of harmonic-series proximity. This aspect is rather clear because of the fact, already remarked on, that the chord is a special

**In other words, the effect cannot be convincingly described in terms of modulation from one key to another. To be sure, the key at the beginning *could* be F♯ minor, as the standard analysis assumes.[150] But if, for instance, one were to listen to the first three notes above followed by, say, two-note chord consisting of the opening F♯ with the D just below, then the first three notes would be perfectly comfortable as part of a melody in the key of D major. (This could be continued à la Kurt Weill.) Again, more subtly, one can make it part of a tune in E♭ minor/major, as Debussy did in his well known spoof in the *Golliwog's Cakewalk* from the *Children's Corner* suite. (The original is G♭ minor/major, the melodic line matching Wagner's original opening, a minor third higher.)

harmonic-series sonority. (One wonders whether this was the clue that got Debussy thinking.) In the case shown, the chord corresponds to a subset of the harmonic series of a low B♭ (29.135 Hz). None of the three bare notes have harmonic series that overlap especially strongly with this B♭ harmonic series. The D is closest: there is coincidence at every 10th member of the B♭ series. For the F♯ it is every 32nd member at best, depending on the precise tuning, and for the C♯ it is every 48th member at best.

So the chord is nearby in the height sense, and relatively far away in the harmonic-series sense: the musical motion that produces its impact and peculiar colouration can reasonably be thought of as a hyperspace move.[††] Notice, by the way, that the chord has much the same colouration — it still 'sounds like the Tristan chord' — if the three preceding notes are played in any other order, or even played simultaneously to make a dissonant chord of their own. The Tristan chord also sounds much the same if the three preceding notes are transposed up by a perfect fifth leaving the rest of the example unchanged, i.e. if one replaces the bare F♯, D, and C♯ by a bare C♯, A, and G♯. Apart from changing the melodic line, this leaves us with what the above analysis predicts to be a closely similar situation. That is, the three bare notes are still neighbours to the Tristan chord in the height sense — in fact, one of the three is still invariant — but relatively far away in the harmonic-series sense. In fact they are further away, in that there is even less harmonic-series overlap.

As for the subsequent moves, as already hinted they bear a family resemblance to the earlier examples, though the details are more elaborate. The 'French augmented sixth' sonority is used twice, at the beginning of the last bar and just before it.[‡‡]

12 per octave is not arbitrary: We can now begin to see the full import of J. S. Bach's advocacy, a century earlier, of the equal-tempered tuning of keyboard instruments (**OOPS**[*]) — establishing the symmetrical subdivision of the octave into 12 equal intervals as a central reference system for practical music-making. It was nothing to do with an arbitrary culture, politics, or artistic imperialism. It was partly to do with a reality familiar to the best professional musicians: the ability of first-rate singers and string and wind players to make fine tuning corrections, away from any reference system,[150c] in which the same nominal note has different tunings. Because of the ubiquity of the dominant seventh, such corrections are called for even in simple music in a single key,[155b] implying from the outset that compromise is inescapable for keyboard instruments.

It was also to do with the musical power and manner of functioning of countless examples like those just considered — the power that comes from unrestricted motion in a large musical hyperspace. Bach himself discovered many such examples and used them with unerring mastery, as generations of musicians have acknowledged.

The robustness and the cross-stylistic, cross-cultural penetration of the equal-tempered 12 note system — and its continuing use by prodigiously talented musicians of many kinds, even in the computer age — are remarkable and significant psychophysical phenomena in themselves. They amply vindicate Bach's vision, besides teaching us something profoundly significant about human perception.

The equal-tempered 12 note system has withstood much *avant garde* experimentation, is commercially exploited on a vast scale, and seems destined to occupy an important place in music for the foreseeable

[††] The degree of closeness of the bare D to the B♭ harmonic series could also be thought of as something like the closeness, in the harmonic-series sense, of the two notes of a major third, if we are prepared to discount rather a large number of octaves. So there is some kinship — though I don't find it directly audible — with the standard move from the major tonic to an augmented sixth or to a major triad on the flattened sixth degree, e.g. the jump from the key of D major to the key of B♭ major. This too has the feel of a 'hyperspace move'. With suitable timing, it is the critical move at countless magic moments in classical music, such as the last fortissimo chord before the military-march episode in Beethoven's Ninth Symphony.

[‡‡] It may be noticed that my 'outrageous liberties' with the naming of chordal sonorities have extended to the musical notation as well. Conventionally, for a start, only the first 'French augmented sixth', at the higher F♯, would be named as such. The one immediately following would be described quite differently as an 'appoggiatura' or 'accented passing note' to C♯7. (Notice the continuing illustration of technique **(a)**.) The first chord would not be called a minor triad with added sixth, nor a half diminished seventh, but, rather, an appoggiatura (the top note) to the first French augmented sixth. The top three notes of the first chord would be rewritten (in upward order) as G♯, B♯, and E♯. This gives the theoretically knowledgeable score-reader — though not the listener — advance notice of the subsequent hyperspace move to C♯7.

[*] **I have recently learned that this is historically incorrect!** Some scholars, at least, have believed for quite some time (since 1975, it seems, to my embarrassment) that the 'well-tempered' keyboard tuning used by Bach was probably a compromise different from the equal-tempered tuning that came into use later; see for instance the website of Dr Herbert Anton Kellner, `http://ha.kellner.bei.t-online.de/`, for details. In Bach's system it was certainly possible to play in all keys, but C major and the keys closest to it in the circle of fifths sounded closer to pure tuning than the remotest keys, which sounded harsher. This is probably why, for instance, the preludes and fugues in B major, F♯ major, C♯ major all move briskly, producing a 'grittier' effect that might be enhanced by slight mistuning. Kellner suggests that Bach's system would have compressed each of the 5 fifths C-G, G-D, D-A, A-E, and B-F♯ by 1/5 of the Pythagorean comma, ca. 4.7 cent, leaving the rest pure including E-B. Keeping E-B pure might be related to the special role of E major in Bach's music.

future. It is a practical benchmark, a reference system that captures, by relatively simple means — to a degree of approximation often adequate to the purpose — a large part of the pattern-creating potential that can interest the auditory human brain. Computerized keyboard systems that automatically perform context-dependent fine tuning,[155b] trying to replicate what skilful non-keyboard players do, are beginning to become available. But this interesting development, while potentially important for solo performance on keyboard instruments, seems unlikely to have any profound effect on the procedures of musical composition — particularly composition of the spiritually most important kinds of music, those performable by human musicians, with or without computer assistance but with scope for meaningful aural feedback. Bach would surely have been interested, but not much influenced.

What, then, is not arbitrary about the subdivision of the octave into 12 intervals, rather than some other number? This is the strangest story of all. What Bach and others recognized comes from a remarkable combination of arithmetical and psychophysical happenstances, a strange and wonderful gift from heaven, which does involve approximation but is no more arbitrary than physical principles and our genetic makeup. It has to do with the limits to harmonic and polyphonic complexity set by the finite pitch discrimination of the ear–brain system, together with a set of three numerical accidents, complete flukes of arithmetic, that sharply distinguish the 12 note system from all other equal-tempered systems coarser than 24 notes per octave.

The first accident, a truly breathtaking fluke, is the closeness of $1\!:\!2^{7/12}$ to $2\!:\!3$. As you can easily check on a pocket calculator, $1\!:\!2^{7/12} = 1\!:\!1.4983 = 2\!:\!2.9966$, almost exactly $2\!:\!3$. That is, the perfect fifth is approximated very closely indeed by its equal-tempered counterpart, within 2 cent or 2% of a semitone ($1200 \log(2^{7/12}/1.5)/\log(2) = -1.96$ cent). Because of octave similarity, the perfect fourth is likewise approximated within 2 cent. This is an extremely fine difference, close to being inaudible except by listening for beats, and far smaller than typical pitch fluctuations in even the most stringently controlled human performances. For a working musician it is practically the same as perfection. So the approximations of equal temperament are more than good enough to express the most crucial kinds of harmonic-series proximity beyond the unison and octave.

On top of that we have the second accident. It is is that 12 — the same 12 that generates the number $2^{7/12}$ — is a highly divisible number. This makes possible the translational invariance of diminished seventh chords already emphasized, corresponding to the divisibility of 12 by 4. Because 12 is also divisible by 2, 3, and 6, as well as by 12 itself, there are four more such translational invariances, giving a total of five classes of translationally invariant chords — bare tritones, augmented triads, diminished sevenths, and wholetone as well as semitone clusters. These are all usable, and are much used, in the kind of way just illustrated — an astonishing proliferation of ways of moving in musical hyperspace. They include the wild sounds of hyperspace moves through augmented triads, the headlong *Ride of the Valkyries* and its echoes in Dvořák's *New World Symphony* and in countless other places. They include the gentler hyperspace move that jazz musicians call 'tritone substitution', e.g. G7 followed by C♯7, another case of proximity to D dim7.

It is interesting to ask what other subdivisions of the octave might give remotely comparable results. If we ask what other subdivisions give at least as good an approximation to the perfect fifth and perfect fourth, we obtain only the subdivisions into 29, 41, 53, 58, 65... parts, in addition to the multiples of 12. We have $2^{(17/29)} = 1.5013$, $2^{(24/41)} = 1.5004$, $2^{(31/53)} = 1.4999$, $2^{(34/58)} = 1.5013$, and $2^{(38/65)} = 1.4996$. But 29, 41, and 53 are all prime numbers, hence barren of translational invariances, and 58 and 65 have only two prime factors, $58 = 2 \times 29$, $65 = 5 \times 13$, hardly fertile. The implication is clear: beyond the standard 12 note system, the next four serious candidates for equal-tempered systems that give access to well connected musical hyperspaces are all multiples of 12: the 24 note, 36 note, 48 note and 60 note systems, and those systems alone. It is difficult to imagine human pitch discrimination getting us much beyond 24, the quarter-tone system; and 24 is not nearly enough to make material improvements in fine tuning, because of the nature of the third accident, as we shall see in a moment.

So what is the third accident? It is that all the intervals that occur naturally in the harmonic series up to its tenth member, including the wide and narrow major and minor thirds, can be matched, to an approximation usually acceptable to the ear, by an equal tempered interval. Even though some of the approximations involved are audibly imperfect — which is why they have always been corrected for in the finest performances on non-keyboard instruments — those approximations are not, on the whole, drastically coarser than the ear–brain's typical pitch discrimination; and they are usually quite a bit better than in some of the more routine orchestral performances.

The worst approximations are those to the narrow minor third and the wide major second of intervals 6 and 7 of the harmonic series, with cycle-time ratios $6\!:\!7$ and $7\!:\!8$. The largest mismatch is

33 cent or a $\frac{1}{3}$ semitone, the narrow minor third being 267 rather than the equal-tempered 300 cent ($1200\log(7/6)/\log(2) = 266.87$). Even this $\frac{1}{3}$-semitone difference, though easily audible, is not enough to spoil the musical sense of dominant seventh, minor triads, or Tristan chords when played on an equal-tempered keyboard.

Can we do any better, then, with finer subdivisions? An equal-tempered system of 36 notes per octave would provide accurate narrow minor thirds. But the wide minor third is 316 cent ($1200\log(6/5)/\log(2) = 315.64$). So to make those accurate as well, along with the narrow major third of the simple major triad or common chord, one would need 72 notes per octave! I suspect that this will always be far beyond our aural capabilities, and that 12 notes per octave plus small corrections are here to stay.

Let us reflect once more, then, on how completely off target, how precisely the wrong way round it is to repeat the myth that the seventh member of the harmonic series is always 'too flat' and to think that 'in theory' one should adjust the seventh of a dominant seventh chord toward, not away from, the equal-tempered keyboard pitch — or, even worse, that it should form a traditional 'just', i.e. wide, minor third with the fifth. One only has to close the textbooks, and open one's ears as Purcell did,[150c] as Debussy did,[150a] and many, many others, including members of the world's great orchestras, bands, and chamber groups, to rediscover that the keyboard is the servant, not the master, of the exquisitely subtle human ear–brain system — as Bach well understood.

Note on 'atonality' [this probably wants shortening]: The word 'atonality' seems to be used in so many different ways, and with so many political undercurrents, that it may have outlived its usefulness. Avant-garde composers have always usefully jolted our imaginations. But they sometimes seem to claim, or at least seemed to claim, in the heyday of the mid-twentieth century Darmstadt school,[55] that Future Music Will be Fully Atonal, in the purist or extremist sense that is the most logical sense of the word atonal. This means, or logically should mean, making no use whatever of harmonic-series proximity, or, rather, suppressing all aural reference to the harmonic series and associated patterns apart from the octave. There must be, it was held, no hint of 'tonality': ideally, no glimmer of harmonic colour, and certainly no reliance on any kind of harmonic function. This of course is difficult to take seriously except as a kind of politics or psychological warfare on the one hand, or as a composers' exercise on the other.

It is surely interesting, as an exercise, to try to write atonally in this purist sense, despite the implied restriction — a drastic restriction — on the motion in musical hyperspace. To make the result interesting to the musical ear–brain system is therefore, indeed, a considerable challenge. Because strictly atonal music in this sense has to function with no reliance on pitch relations beyond melodic contour — on motion in the height dimension only — it could be said to be a bit like poetry or prose that avoids the use of verbs, equally an interesting and challenging exercise for a composer of literature or, for the visual artist, drawings in which all lines are to be straight and vertical, or, for the dancer, staying on one spot, with only vertical motion allowed.

One way to make fairly sure of such strict atonality is to write music for instruments none of which produce a clear single pitch, as with some arrays of percussion instruments. Another, it might be thought, would be to use a non-standard scale deliberately chosen to be barren of translational invariances, i.e. in which the octave is divided into a prime number of equal intervals, say 5, 7, or 11. However, the likely result then would not be pure atonality but a restricted tonality, like many kinds of folk music. Whatever the composer's theory or manifesto, the ear-brain system will fit harmonic-series subsets to the incoming acoustic data, to whatever extent it can, as must surely happen with the (wonderful, I think) sound from a gamelan orchestra. A real and interesting musical effect is the tense feeling, and special tone-colour, of intervals that audibly stretch or compress harmonic-series intervals. Ordinary bell sounds involve just such colours. (So too, in a subtler way, do the sounds of the piano — not only because it is tuned in equal temperament with compromise major and minor thirds, but also because it has freely vibrating strings whose overtones are very slightly sharper than the members of a harmonic series — a feature that has been shown, moreover, to have a positive impact on the perceived character of piano sound.[155c])

A third and surer way, for the true purist, is the composer's exercise to try to write something interesting using a single note only, say 440 Hz A, and its octave translations. This makes quite sure of imposing a rigorous and drastic restriction of motion in hyperspace. A fourth way is to sound all notes at once all the time, to write music in maximal tone-clusters, or at least in textures sufficiently dense in spacetime as to guarantee adherence to the Schoenbergian ideal of strict equality of all 12 notes. Filling, or saturating, the harmonic dimensions of hyperspace is another way to make sure that there is no harmonic color or motion; and some composers have achieved striking effects in this way. This is the Schoenbergian ideal taken to its most rigorous logical conclusion (as distinct from Schoenberg's actual music, a different thing altogether, and far more interesting).

As the reader will realize, I am putting things in this outrageously strawmanlike way to expose what I think is the absurdity of the notion of strict atonality, taken literally to its logical extreme — to say nothing of the still greater absurdity, and pretentiousness, of purporting to predict the course of Future Music. Such ideas are taken less and less seriously today. I have even noticed respected composers of 'serious' music daring to use the word 'harmony' in public, with no implicit frown of disapproval. What was, and is, taken seriously is that there was, and is, every reason to look for freedom from 'tonality' in what might be called the Beckmesser sense, the very restricted sense of demanding that the music be in a definite 'key' most of the time and that the composer obey strict 'rules' — here is where culture and politics again come in — rules such as having to end in the same key as you started in, or having to exclude certain cases of organic harmonic change that use technique **(b)**. Such rules are indeed, to a considerable extent, arbitrary, and they, too, restrict the motion in hyperspace, though not nearly so much as the pure atonalist's rule forbidding all harmonic motion. Today is a wonderful time for anyone interested in contemporary music. More and more of today's music moves rather freely in hyperspace, some of it in interesting and exciting ways, just as most of today's best literature uses, I think, verbs in interesting and exciting ways, and drawings and paintings use lines and curves with more than one orientation, and dancers make us airborne.

59. Goehr, A., 1990: Music as communication. In: *Ways of Communication*, ed. D. H. Mellor. Cambridge, University Press, 165 pp., 125–142. Alexander Goehr, one of our leading and most highly respected composers, has reminded me that in music and the other arts one has to include 'becoming coherent': there are many examples of musical 'shapes emerging from a... metaphorical mist'. His essay discusses this and other aspects of musical composition including aspects of the perception of music, and of the uses and associations of music in our own and other cultures.

60. Mozart, W. A., 1787: *A Musical Joke*, K.522. Mozart's light-hearted dig at unskilful amateur composers and performers provides, among other things, examples of musical gratuitous (pseudoelegant) variation, such as the gratuitous modulation or key change at bars 37–8, and the gratuitous change in harmonic colour on the second beat of bar 38. (Refs. 151 discuss, with profound musical insight, the use of modulation and colour in ways that are anything but gratuitous.)

150. Piston, W., DeVoto, M., 1978: *Harmony*, 4th USA and 2nd UK edn. Norton, Gollancz, 594 pp. A beautiful textbook, explanatory rather than prescriptive, and firmly based on cogent illustrations from a broad cross-section of great music from the eighteenth to the twentieth centuries, including some references to jazz as well as to Debussy and Stravinsky (though, surprisingly, not to Nielsen). I am indebted to Andrew V. Jones (personal communication) for confirming that the opening of *Tristan und Isolde* is usually analyzed as it is on pp. 364 and 421 of this book, presuming that the key is A minor in the original, equivalent to F♯ minor in my transposition.

150*a*. Platt, P., 1995: Debussy and the harmonic series. In: *Essays in honour of David Evatt Tunley*, ed. Frank Callaway, pp. 35–59. Perth, Callaway International Resource Centre for Music Education, School of Music, University of Western Australia. ISBN 086422409 5. This is a beautiful discussion, with detailed examples, of some of Debussy's far-reaching innovations. These amounted to recognizing the perceptual importance of the harmonic series and its subsets, with no arbitrary restrictions on the choice of subsets. Debussy thereby opened up a huge range of possibilities not only for new chordal sonorities and note-sequences but also for new forms of powerful, continuous harmonic motion — going far beyond Wagner in both respects — including many examples using what I called 'technique **(b)**', parallel motion of an invariant chord shape. Platt also gives an insightful comparison with aspects of Indian classical music and its styles of organic change. See also Thomson, W., 1991: *Schoenberg's Error*. Philadelphia, University of Pennsylvania Press, 217 pp. Platt quotes this 'controversial but insightful' book for its idea of the harmonic series as an ever-present 'template'.

150*b*. E.g. G. B. Henning, 1970, 1971: A comparison of the effects of signal duration on frequency and amplitude discrimination. In: Frequency Analysis and Periodicity Detection in Hearing, pp. 350–361, ed. R. Plomp and G.F. Smoorenberg, Leiden, Sijthoff, 482 pp. [The years 1970 and 1971 are given on the title and copyright pages respectively.] See also p. 260–263 of Green, D. M., 1976: *An Introduction to Hearing*. New York, Wiley, 353 pp. The relation between duration and frequency discrimination is not simply Heisenbergian, though approximately so in a limited range.

150*c*. What I am calling the sonority of the Tristan chord occurs more than thirteen times in *Dido's Lament*, which was probably written around 1683. Thirteen occurrences can be counted on p. 71 of the score published by the Purcell Society (ed. Margaret Laurie and Thurston Dart; p. 177 of the Norton re-issue). The remarkably persistent use of this particular sonority at a slow tempo, and the scoring for voice and strings only — the continuo keyboard is directed to be silent — suggest that Purcell's fine ear must have anticipated Debussy's[150a] in noticing the subtle beauty of the sonority *when accurately tuned*, as well as its aptness in a harmonic scheme for conveying tragedy and pathos, along with the various related sonorities and other devices: passing notes, appoggiaturas, suspensions, and so on, all used to achingly poignant effect. Accurate tuning, requiring as it does the narrow minor third at nearly every moment of the aria (cycle-time ratio 6 : 7), is in conflict with all the practical keyboard tuning systems that Purcell might conceivably have used — be it just, mean-tone, or equal-tempered, or other compromise accommodating major triads as all keyboards must. I am grateful to Andrew V. Jones for help with this example and for advice as to the authenticity of the published score.

One might count as a fourteenth occurrence, in bar 22 on p. 71 (Norton p. 177), a C minor 6 lightened by omission of the note G. It sounds very similar to, and functions in the same way as, several earlier occurrences of the full sonority, as the 'IV(II)' stage of a 'IV(II)–V–I cadence' in the notation of Piston and DeVoto.[150] (On the notation IV(II), see their discussion on p. 359.) This fourteenth occurrence is reminiscent of the first bar of the Mozart Clarinet Quintet extract quoted above, whose second half uses a similarly lightened D minor 6 sonority (A omitted) functioning in just the same way, as a IV(II) subdominant, and making an effect of supreme beauty when accurately tuned. Not surprisingly, there are countless examples of sonorities like these in the music of J. S. Bach.

151. Simpson, R., 1970: *Beethoven Symphonies* (Music guides, British Broadcasting Corporation), republished by London, Ariel Music, 1986, ISBN 0 563 20484 2, 62 pp., and Simpson, R., 1979: *Carl Nielsen, Symphonist*, revised and expanded edition with an added chapter on 'Sibelius, Nielsen, and the Symphonic Problem', London, Kahn and Averill, 260 pp. Both books are about great symphonic music seen as architecture in musical hyperspace, more aptly spacetime. They well illustrate 'the importance of finding a coherent order of presentation' in which 'each new point is preceded by any necessary preparation'. The technically simplest, and breathtakingly simple, example of the powerful architectural use of 'hyperspace effects' is Beethoven's Seventh Symphony.

152. Cross, I., 1997: Pitch Schemata. In: *Perception and Cognition of Music*, edited by J. A. Sloboda and I. Deliège, Hove, UK, Psychology Press, 357–390. An erudite summary of recent efforts to relate psychophysical to music-analytical studies, noting some of the past and current approaches to defining what I am calling 'musical hyperspace'; see also, for instance, the early paper by Bachem, A., 1950: Tone height and tone chroma as two different pitch qualities. *Acta Psychol.*, **7**, 80–88, which, however, confined itself to noting the perceptual near-identity of octaves; thus 'chroma' was conceived to be the quality of musical pitch that is almost the same for two notes an octave apart.

153. Moore, B. C. J., 1997: An introduction to the psychology of hearing, 4th edn. London and San Diego, Academic, 373 pp, esp. chapters 5 and 7. This is an up to date review of present-day knowledge from psychoacoustics and auditory neurophysiology. On harmonic-series proximity, see also the early experimental and theoretical contribution by:

153*a*. Boomsliter, P. C., Creel, W., 1961: The long pattern hypothesis in harmony and hearing. *J. Mus. Theory* (Yale School of Music), **5**, 2–31. Apart from one slight lapse where the authors forget the context-dependence of tonal 'major–minor' distinctions, this is an outstandingly cogent, lucid, and succinct discussion of the early experimental evidence and biological arguments pointing to temporal correlation and near-synchrony in trains of nerve impulses as, very probably, the primary mediator of harmonic-series proximity and of musical harmonic function in general. They point to the long known ability of the auditory brain to make fine time discriminations for other purposes such as directional hearing, and to the stability of our perception of simple musical intervals whether presented melodically, in successive notes, or simultaneously, and many other telling points, supported by some beautiful psychophysical experiments.

153*b*. Tversky, A., 1977: Features of similarity. *Psychol. Rev.*, **84**, 327–352. This cogently makes the case, supported by many telling examples, that perceptual similarity does not often behave as distance in the strict metric sense.

154. Shepard, R. N., 1982: The structural representation of musical pitch. In: Deutsch, D., *Psychology of Music*, Academic, 344–390. For the suggestion of a 7 dimensional pitch-perceptual space, see p. 365. See also, however, Ref. 153*b*.

154*a*. Krumhansl, C. L., 1990: Cognitive foundations of musical pitch. New York, Oxford, Oxford University Press, 307 pp. See p. 130 for experimentally established examples of 'X similar to Y' not being equivalent to 'Y similar to X'; these are cases in which the order of presentation of two pitches in a given tonal context had an influence on judgements of similarity, just as might be expected from ordinary musical experience, in which certain harmonic gestures point strongly to particular parts of musical hyperspace. So, when using spacelike metaphors, one has to remember these pointing arrows, like the gestures of dancers moving in physical space, reminding us that more than simple 'distance' is involved. See also the very apt general remarks in Ref. 153*b*. I am grateful to Ian Cross for drawing my attention to these references.

155. McIntyre, M. E., and Woodhouse, J., 1978: The acoustics of stringed musical instruments. *Interdisc. Sci. Rev.*, **3**, 157–173. The timescales, of the order of tens of milliseconds, on which 'vibrato' is effective are intermediate between vibration cycle timescales and long-pattern timescales. (Here time*scale* means about a sixth of a vibrato period, because $2\pi \simeq 6$.) The timescale-dependence and the surprisingly large frequency excursions during vibrato — it is always startling to hear a recording of violin sound at half speed or less — has implications for the brain's pitch perception mechanisms. One implication is that pitch perception cannot be equivalent to simple spectral or Fourier transformation. This discourages expectations of a precisely Heisenbergian pitch-uncertainty relation. A more interesting implication is that the brain must be extremely flexible about accommodating temporary asynchrony over intermediate timescales. Some kind of elasticity, time integration, or temporary data cache must be involved.

155*a*. Taruskin, Richard. Interview in a programme about the origins of Stravinsky's *Rite of Spring*, on BBC Radio 3's *Sunday Feature*, 3 October 1999. Also discussed was the origin of the idea for the *Rite*. The idea appears to have been suggested to Stravinsky, courtesy of Diaghilev's network of contacts, by the Russian artist and archaeologist Nikolai Konstantinovich Rerikh (Roerich).

155*b*. Boomsliter, P. C., Creel, W., 1962: Ratio relationships in melody. *J. Acoust. Soc. Amer.,* **34**, 1276–1277. This briefly reports on careful and systematic psychophysical experiments with musically trained subjects, showing that skilled musicians playing variable-pitch instruments do not tune to a fixed scale — even when playing simple monophonic melodies in a single key. Rather, a given nominal note tends to be given slightly different tunings that depend on where it occurs in the melody. Thus the old question 'which scale do musicians prefer' is exposed as a classic case of asking the wrong question.

It is only recently that computer technology has made context-dependent fine tuning possible in practical keyboard instruments, three centuries after Purcell[150c] and a third of a century after Boomsliter and Creel's paper. Such keyboard instruments began to be commercially available in 1997 (e.g., `http://www.justonic.com`), though I have not yet had an opportunity to hear the results.

155*c* Blackham, E. D., 1965: The physics of the piano. *Scientific American*, December issue, p. 88. Reprinted in *The Physics of Music*, San Francisco, W. H. Freeman, 1978.