Black-hole binary systems as GW source

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Astro-GR meeting

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CALTECH

Overview

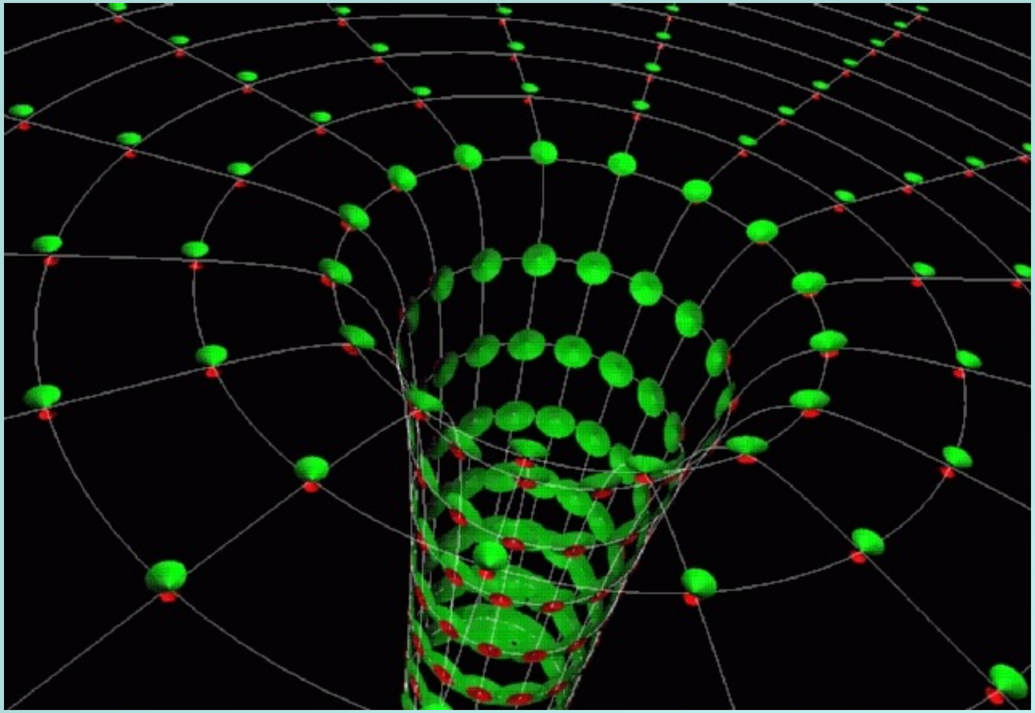
- Motivation
- Introduction
- Ingredients of numerical relativity
- Results
  - A brief history of BH simulations
  - Results following the recent breakthrough
- Summary

1. Motivation

Black Holes predicted by GR

- Black holes predicted by Einstein's theory of relativity
- Term "Black hole" by John A. Wheeler 1960s
- Vacuum solutions with a singularity
- For a long time: mathematical curiosity
  - valuable insight into theory
  - but real objects in the universe?
- That picture has changed dramatically!

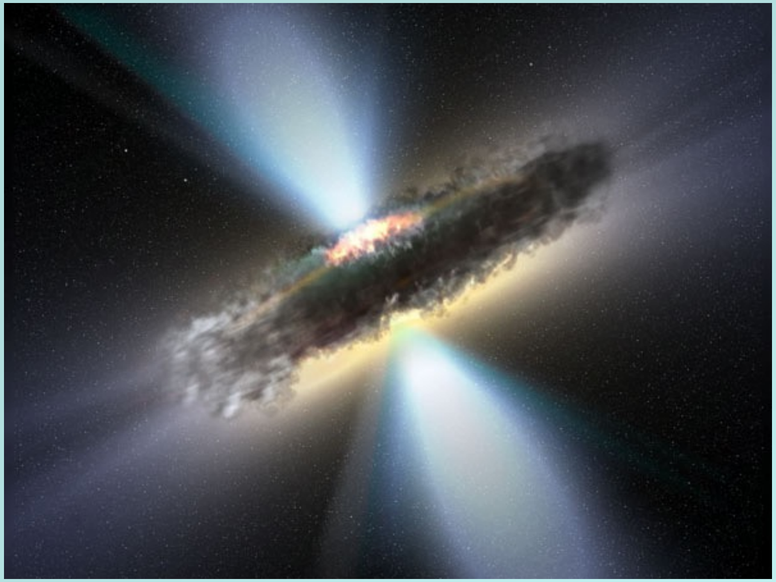
How to characterize a black hole?

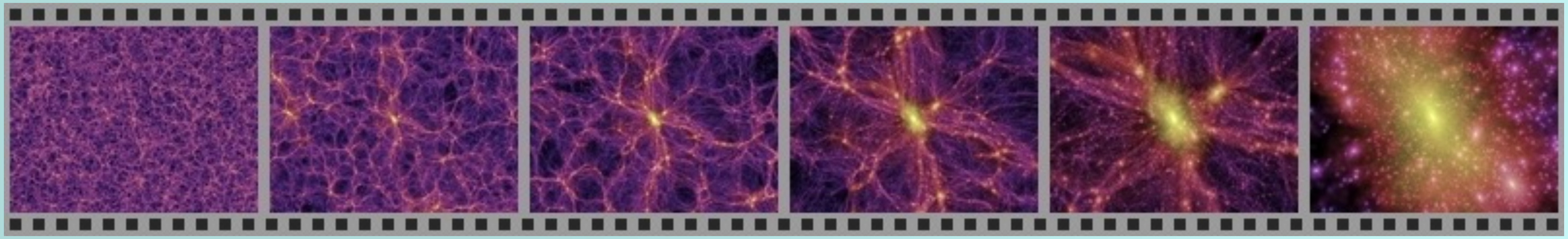


- Consider light cones
- Outgoing, ingoing light
- Calculate surface area of outgoing light
- Expansion:=Rate of change of that area
- Apparent horizon:= Outermost surface with zero expansion
- "Light cones tip over" due to curvature

Black Holes in astrophysics

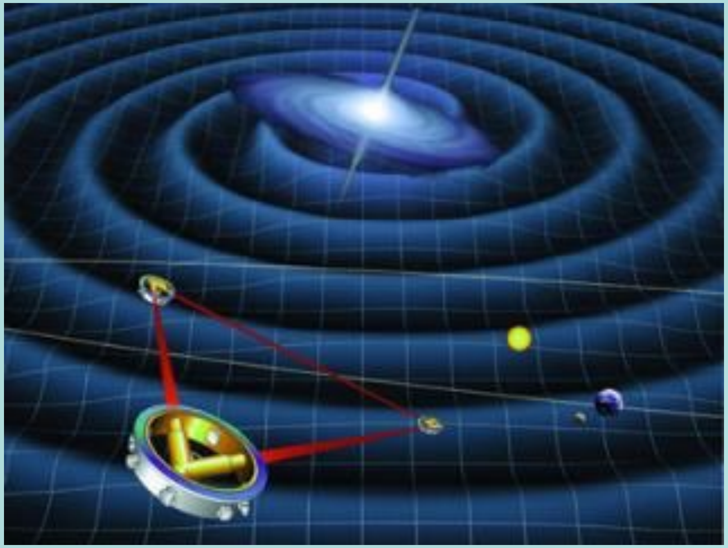
- Black holes are important in astrophysics
  - Black holes found at centers of galaxies
  - Structure of galaxies
  - Important sources of electromagnetic radiation
  - Structure formation in the early universe

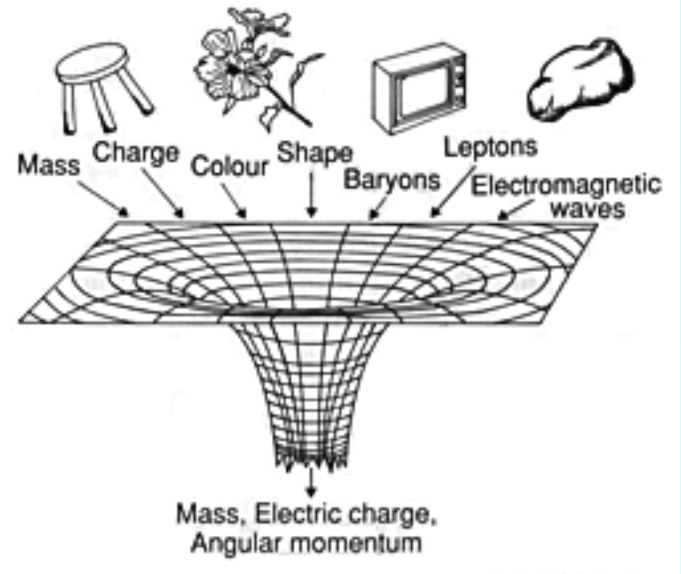




Fundamental physics of black holes

- Allow for unprecedented tests of fundamental physics
  - Strongest sources of Gravitational Waves (GWs)
  - Test alternative theories of gravity
  - No-hair theorem of GR
  - Production in Accelerators

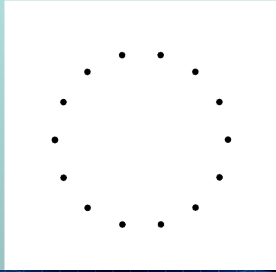


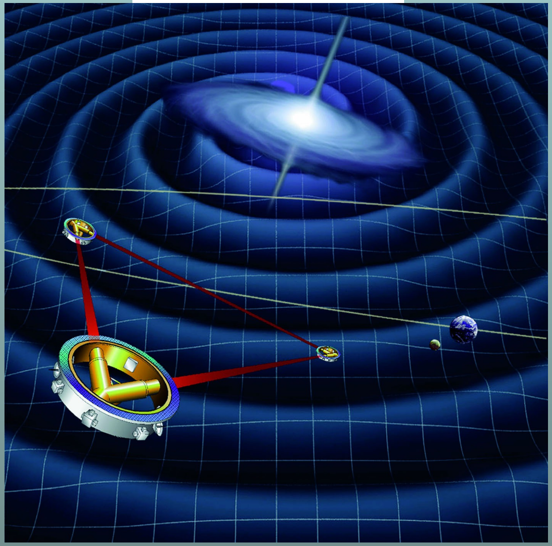


Gravitational wave (GW) physics

- Einstein ⇒ GWs; Analog of electromagnetic waves
- Strongest source: coalescing black holes
- GWs ⇒ change in separation

  < Atomic nucleus over 1 km



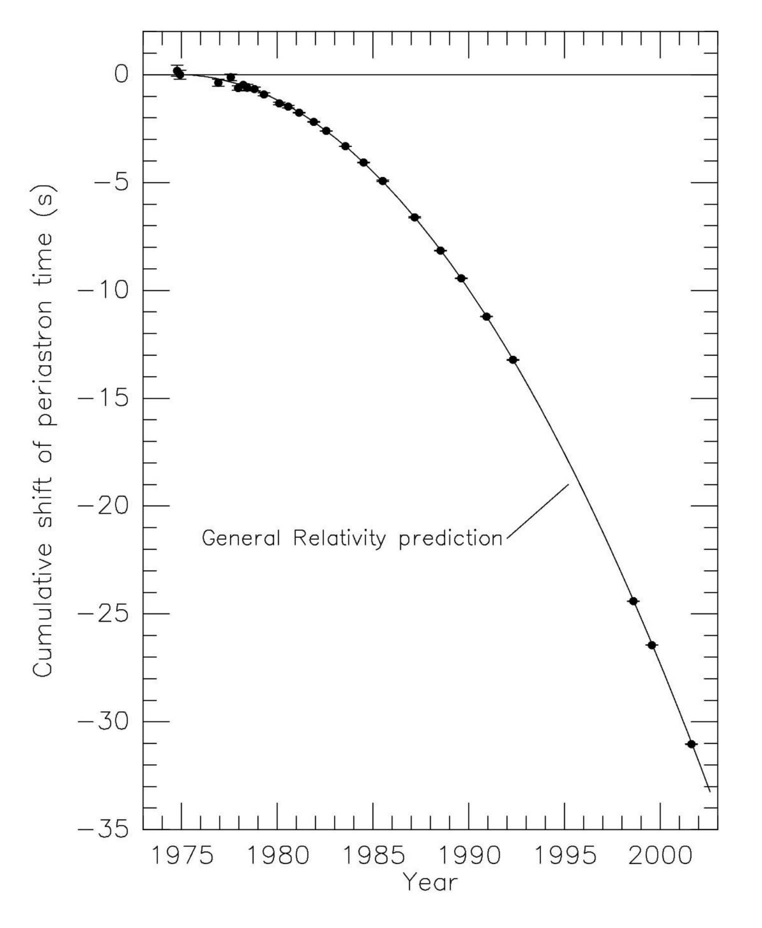


- Latest laser technology: GEO600, LIGO, TAMA, VIRGO
- Space mission: LISA

Targets of GW physics

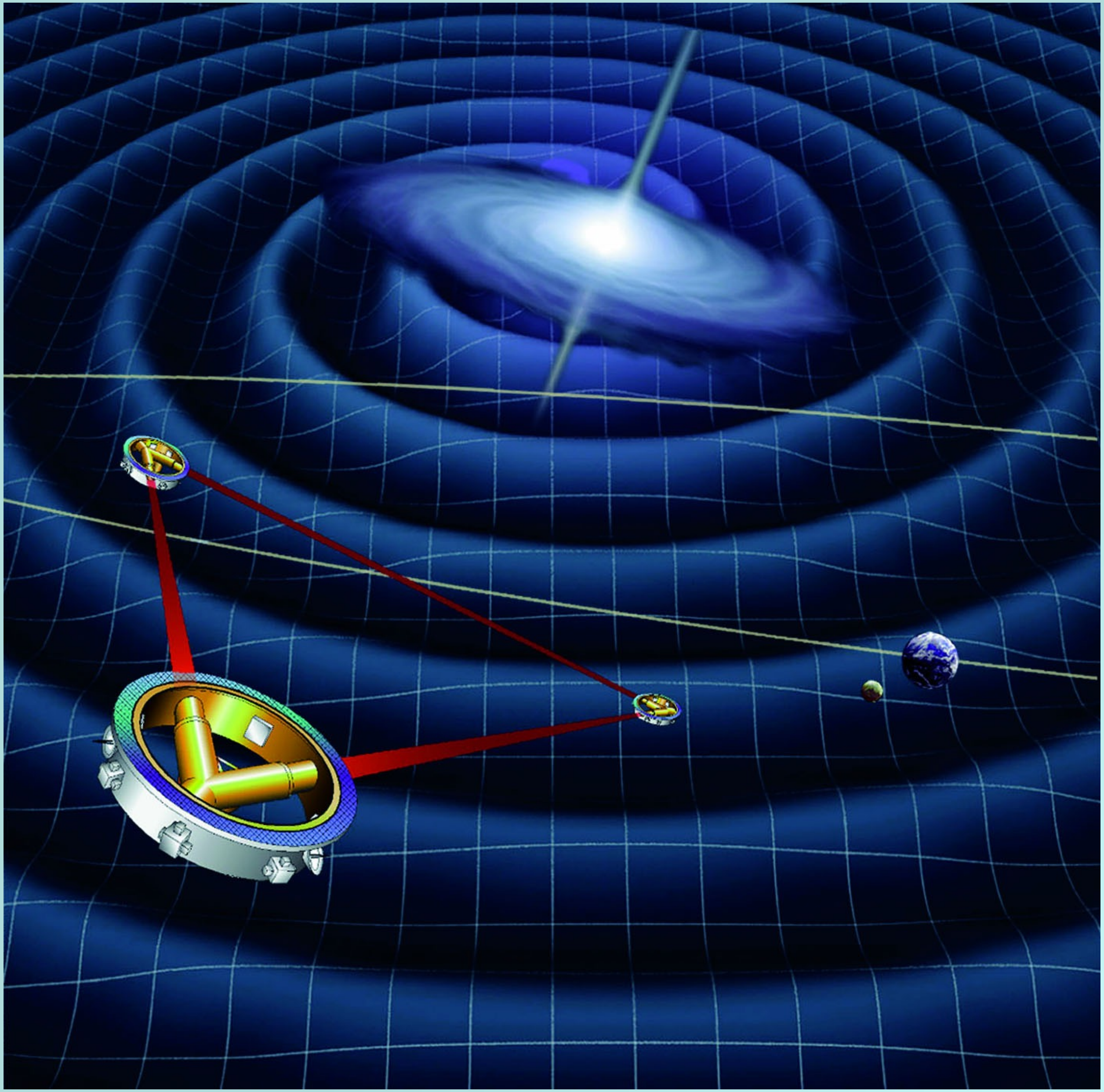
- Confirmation of GR

  Hulse & Taylor 1993 Nobel Prize

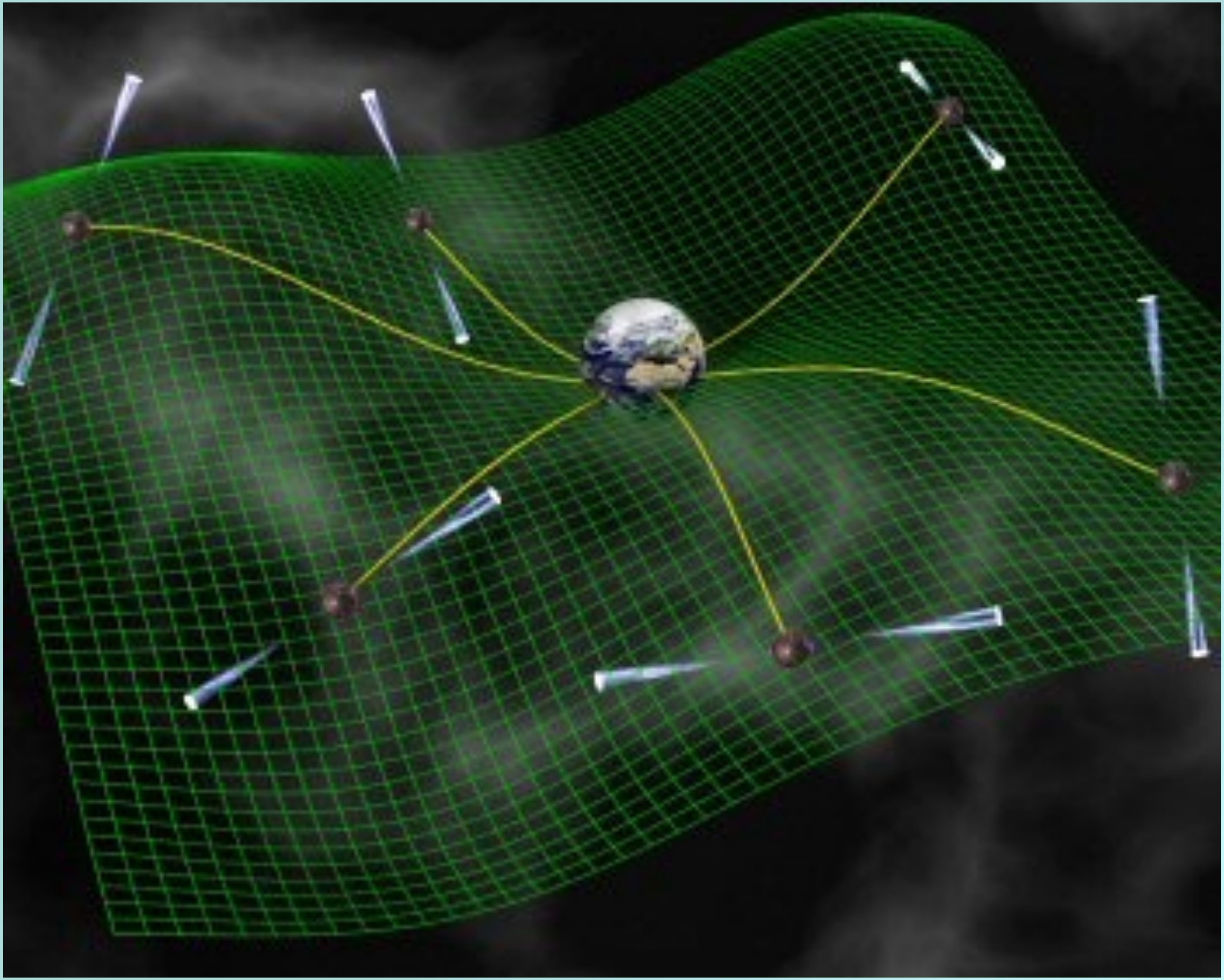


- Parameter estimation of black holes $M, \vec{S}$
- Optical counterparts
  - Standard sirens (candles)
  - Graviton mass
- Test the Kerr nature
- Cosmological sources
- Neutron stars: Equation of state
- Waveforms crucial for detection and parameter estimation

Space interferometer LISA

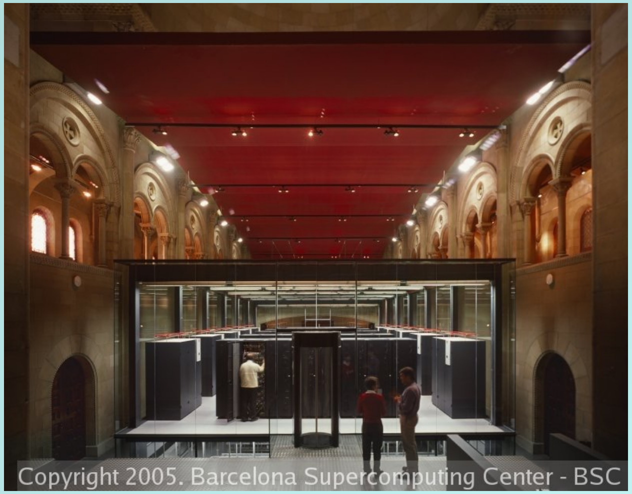


Pulsar timing arrays



Modelling of black-hole binaries

- Analytic solutions for dynamic systems: Hopeless!!!
- Modelling:
  - Approximation theories (PN, Pertrubation theory,…)
  - Numerical Relativity (this talk!)



- Strenght and weaknesses

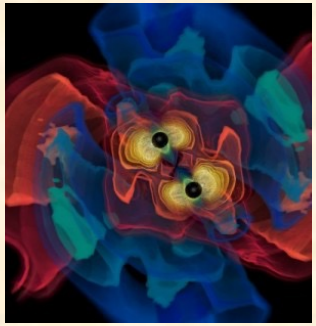
Appr.: efficient, approximative, works?, available

NR: slow, “exact theory”, works, availability?

- Good GW modeling uses both!

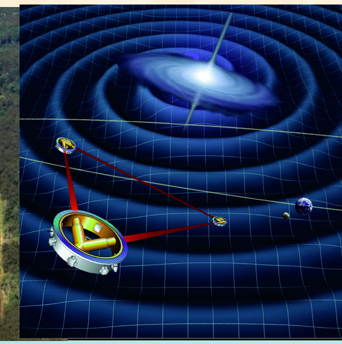
The big picture

Physical system



observe

Detectors



describes

test

Help detection

Provide info

Model

- GR (NR)
- PN
- Perturbation theory
- Alternative Theories?

External Physics

- Astrophysics
- Fundamental Physics
- Cosmology

2. The basics of numerical relativity

A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations:
  - Cast as evolution system
  - Choose specific formulation
  - Discretize for Computer
- Choose coordinate conditions: Gauge
- Fix technical aspects:
  - Mesh-refinement / spectral domains
  - Excision
  - Parallelization
  - Find large computer
- Construct realistic initial data
- Start evolution and wait…
- Extract physics from the data

Gourgoulhon gr-qc/0703035

2.1. The Einstein equations

Theoretical framework of GW course modeling

- Description of spacetime

Metric $g_{\alpha\beta}$

- Field equations: $G_{\alpha\beta}\left(g_{\alpha\beta}, \partial g_{\alpha\beta}, \partial^2 g_{\alpha\beta}\right) = 8\pi\, T_{\alpha\beta}$

MTW: "Spacetime tells matter how to move,
matter tells spacetime how to curve"

In vacuum: $G_{\alpha\beta} = R_{\alpha\beta} = 0$

10 PDEs of 2^{nd} order for the metric

- System of equations very complex: Pile of paper!

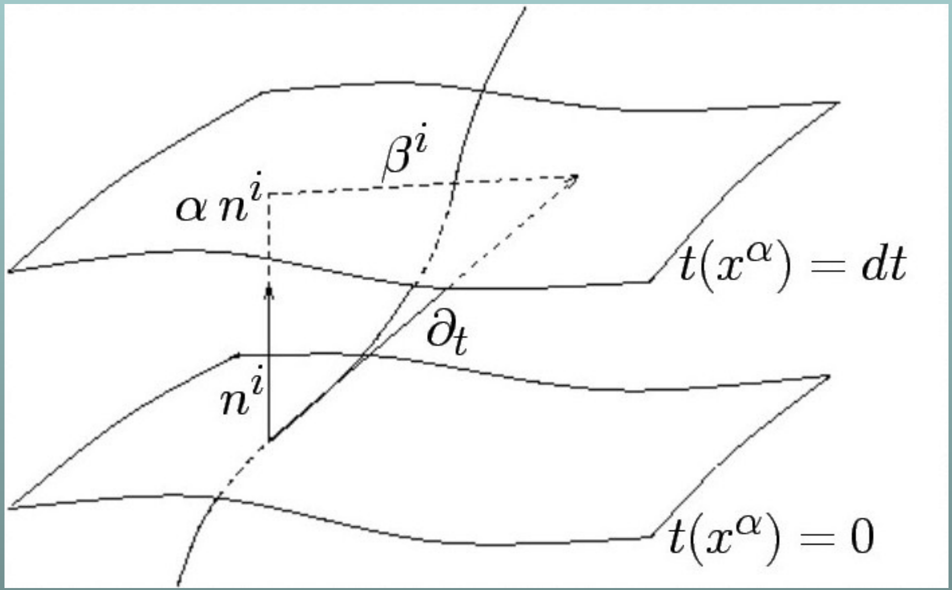
Numerical methods necessary for general scenarios!

3+1 Decomposition

- GR: “Space and time exist as unity: Spacetime”
- NR: ADM 3+1 Split

Arnowitt, Deser, Misner (1962)
York (1979)
Choquet-Bruhat, York (1980)

3-Metric γ_{ij}
Lapse α
Shift β^i



- lapse, shift $\Rightarrow$ Gauge
- Einstein equations
  $\Rightarrow$ 6 Evolution eqs.
  4 Constraints
- Constraints preserved under time evolution!

ADM Equations

- Evolution equations

$$(\partial_t - L_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

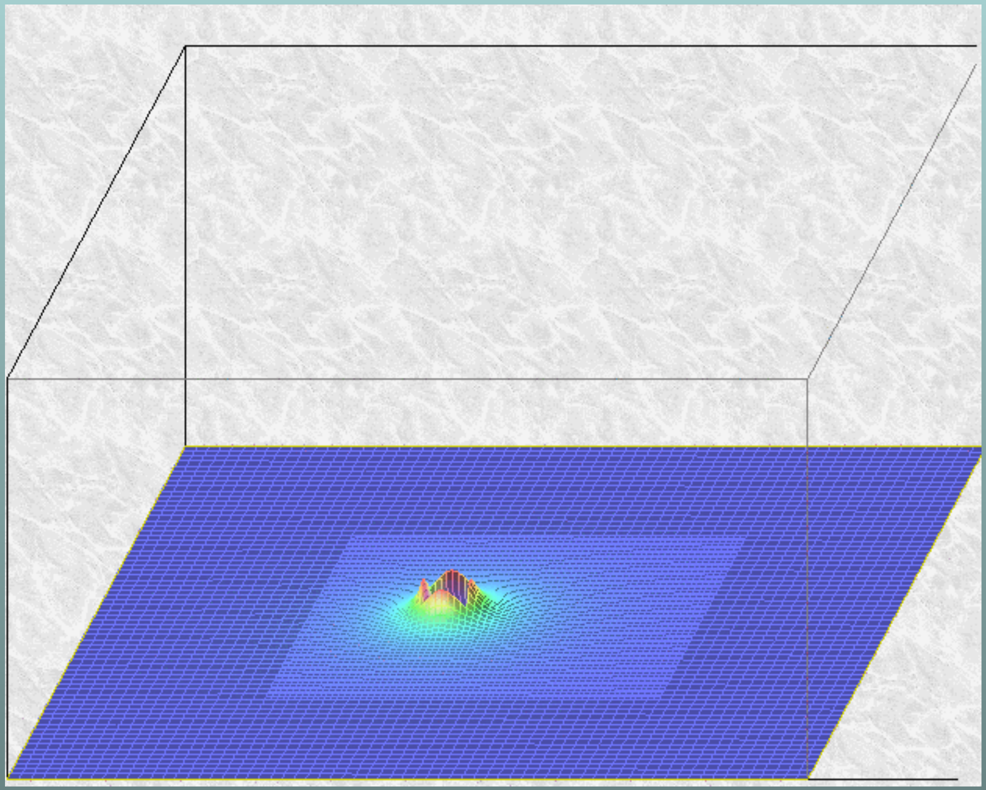
$$(\partial_t - L_\beta)K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K_{im}K^m{}_j + K_{ij}K]$$

- Constraints

$$R + K^2 - K_{ij}K^{ij} = 0$$

$$-D_j K^{ij} + D^i K = 0$$

- Evolution
  - Solve constraints
  - Evolve data
  - Construct spacetime
  - Extract physics



Sperhake et al., PRD 69, 024012

GR specific problems

- Initial data must satisfy constraints

  $\Rightarrow$ Numerical solution of elliptic PDEs

  Here: Puncture data   Brandt & Brügmann '97

- Formulation of the Einstein equations

- Coordinates are constructed $\Rightarrow$ Gauge conditions

- Different length scales $\Rightarrow$ Mesh refinement

- Equations extremely long $\Rightarrow$ Turnover time
  Paralellization, Super computer

- Interpretation of the results? What is "Energy", "Mass"?

Gourgoulhon gr-qc/0703035

Eqs.: Baumgarte, Shapiro, Shibata, Nakamura (BSSN)

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\phi = \tfrac{1}{12}\ln\gamma \qquad \hat{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}$$

$$K = \gamma_{ij}K^{ij} \qquad \hat{A}_{ij} = e^{-4\phi}\left(K_{ij} - \tfrac{1}{3}\gamma_{ij}K\right)$$

$$\hat{\Gamma}^i = \gamma^{ij}\hat{\Gamma}^i_{jk} = -\partial_j\hat{\gamma}^{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\hat{\gamma}_{ij} = -2\alpha\hat{A}_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\tfrac{1}{6}\alpha K$$

$$(\partial_t - \mathcal{L}_\beta)\hat{A}_{ij} = e^{-4\phi}(-D_iD_j\alpha + \alpha R_{ij})^{\mathrm{TF}} + \alpha(K\hat{A}_{ij} - 2\hat{A}_{ik}\hat{A}^k{}_j)$$

$$(\partial_t - \mathcal{L}_\beta)K = -D^iD_i\alpha + \alpha\left(\hat{A}_{ij}\hat{A}^{ij} + \tfrac{1}{3}K^2\right)$$

$$\partial_t\hat{\Gamma}^i = 2\alpha\left(\hat{\Gamma}^i_{jk}\hat{A}^{jk} + 6\hat{A}^{ij}\partial_j\phi - \tfrac{2}{3}\hat{\gamma}^{ij}\partial_jK\right) - 2\hat{A}^{ij}\partial_j\alpha + \hat{\gamma}^{jk}\partial_j\partial_k\beta^i$$
$$+\tfrac{1}{3}\hat{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\hat{\Gamma}^i + \tfrac{2}{3}\hat{\Gamma}^i\partial_j\beta^j \underbrace{- \left(\chi + \tfrac{2}{3}\right)\left(\hat{\Gamma}^i - \hat{\gamma}^{jk}\hat{\Gamma}^i_{jk}\right)\partial_l\beta^l}_{\text{Yo et al. (2002)}}$$

Alternative: GHG Pretorius '05

Generalized harmonic (GHG)

- Harmonic gauge: choose coordinates so that

$$\nabla_\mu \nabla^\mu x^\alpha = 0$$

- 4-dim. Version of Einstein equations

$$R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots$$ (no second derivatives!!)

Principal part of wave equation

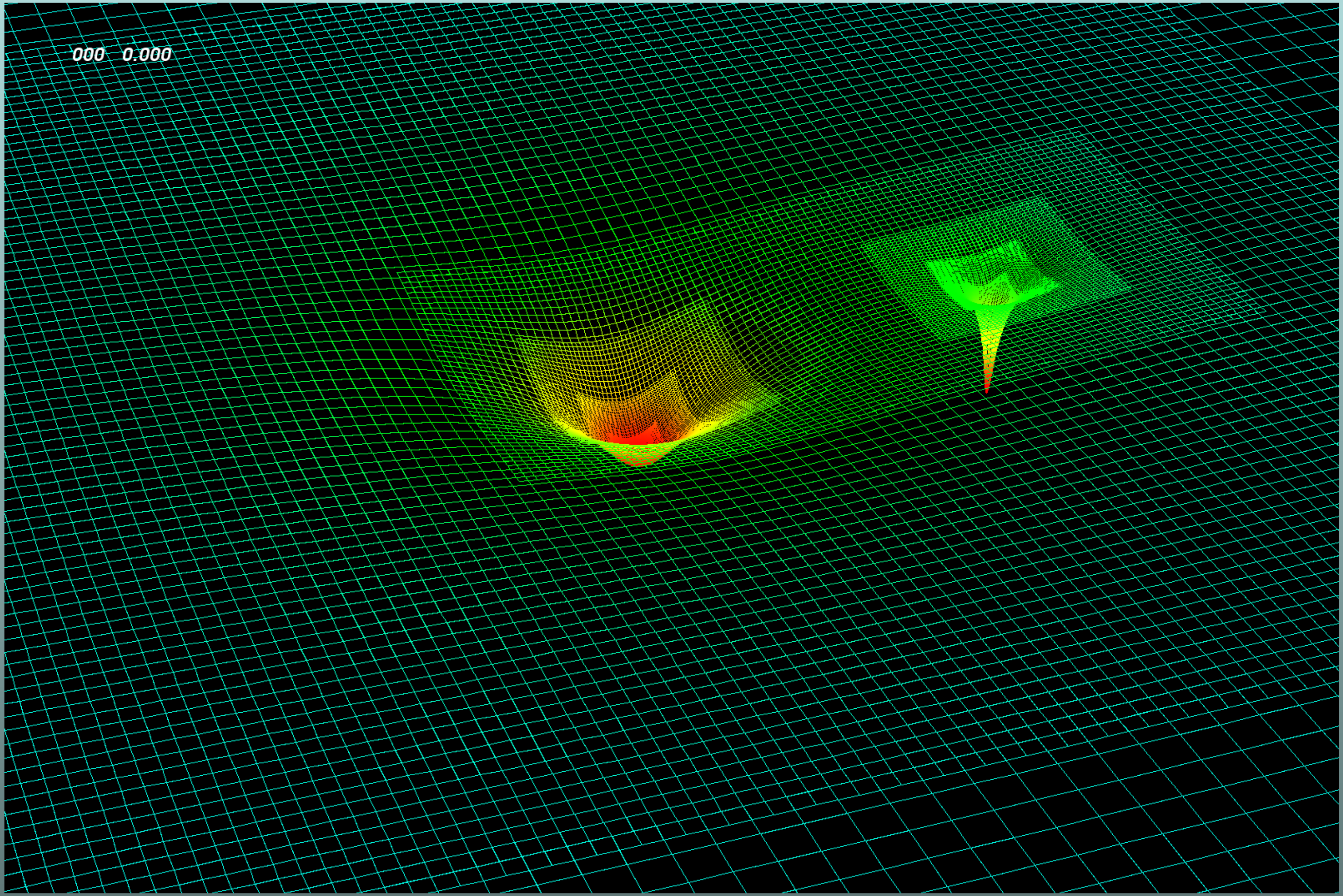
- Generalized harmonic gauge: $H_\alpha := g_{\alpha\nu} \nabla_\mu \nabla^\mu x^\nu$

$$\Rightarrow R_{\alpha\beta} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu g_{\alpha\beta} + \ldots - \frac{1}{2} \left(\partial_\alpha H_\beta + \partial_\beta H_\alpha \right)$$

Still principal part of wave equation!!!

Coordinate and gauge freedom

- Reminder: Einstein Eqs. Say nothing about $\alpha, \ \beta^i$
- Avoid coordinate singularities! González et al. '08



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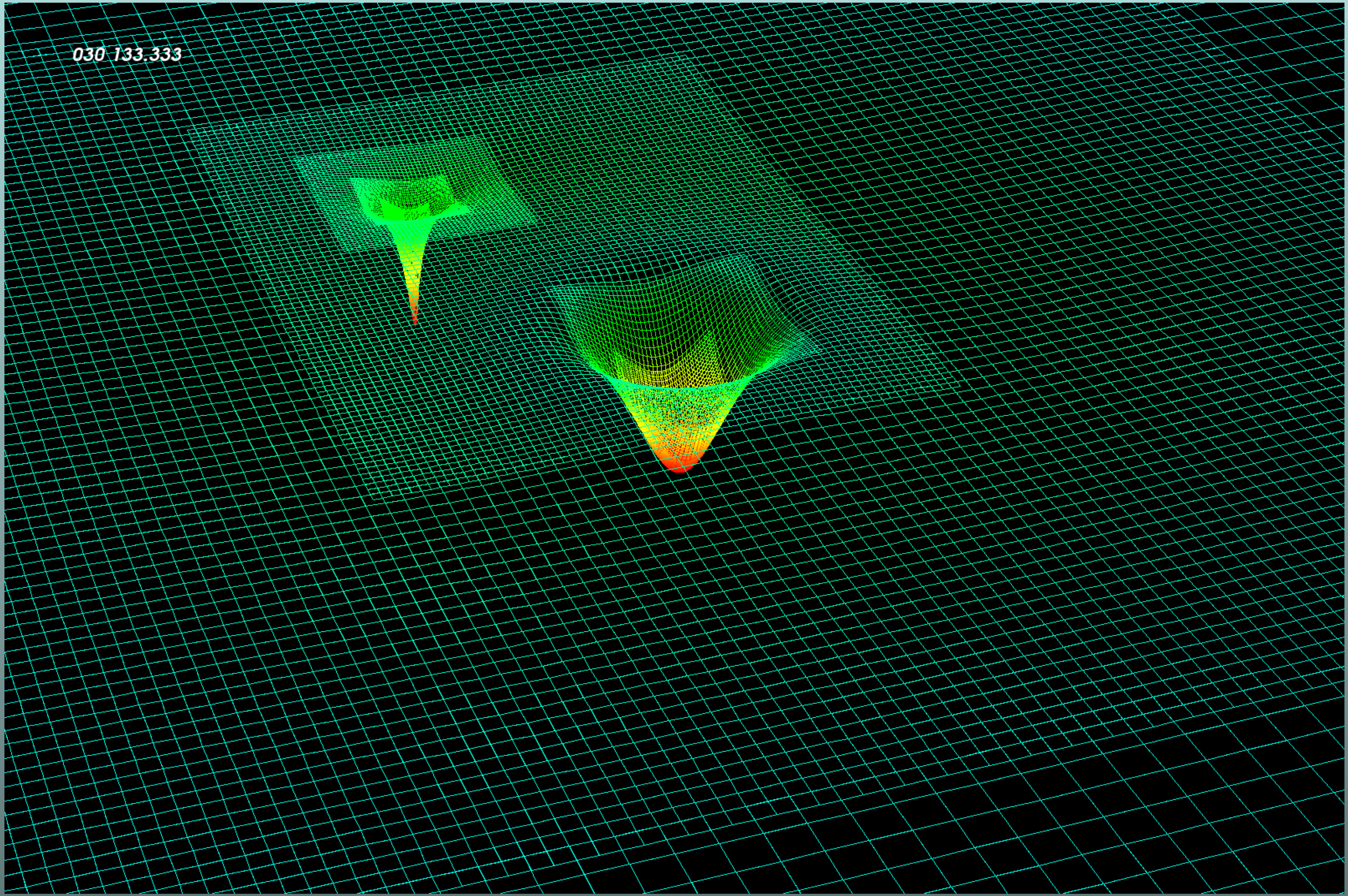

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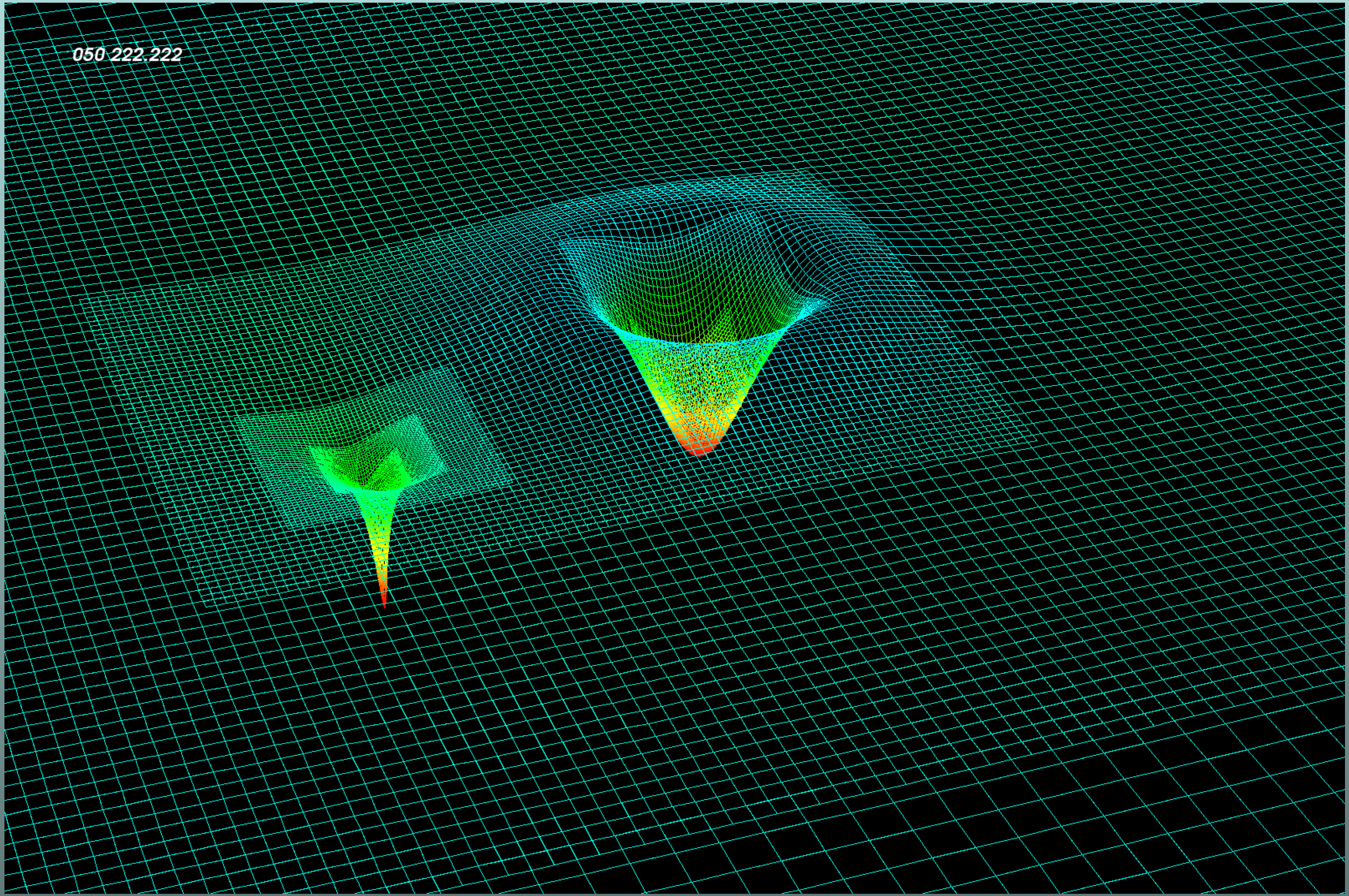

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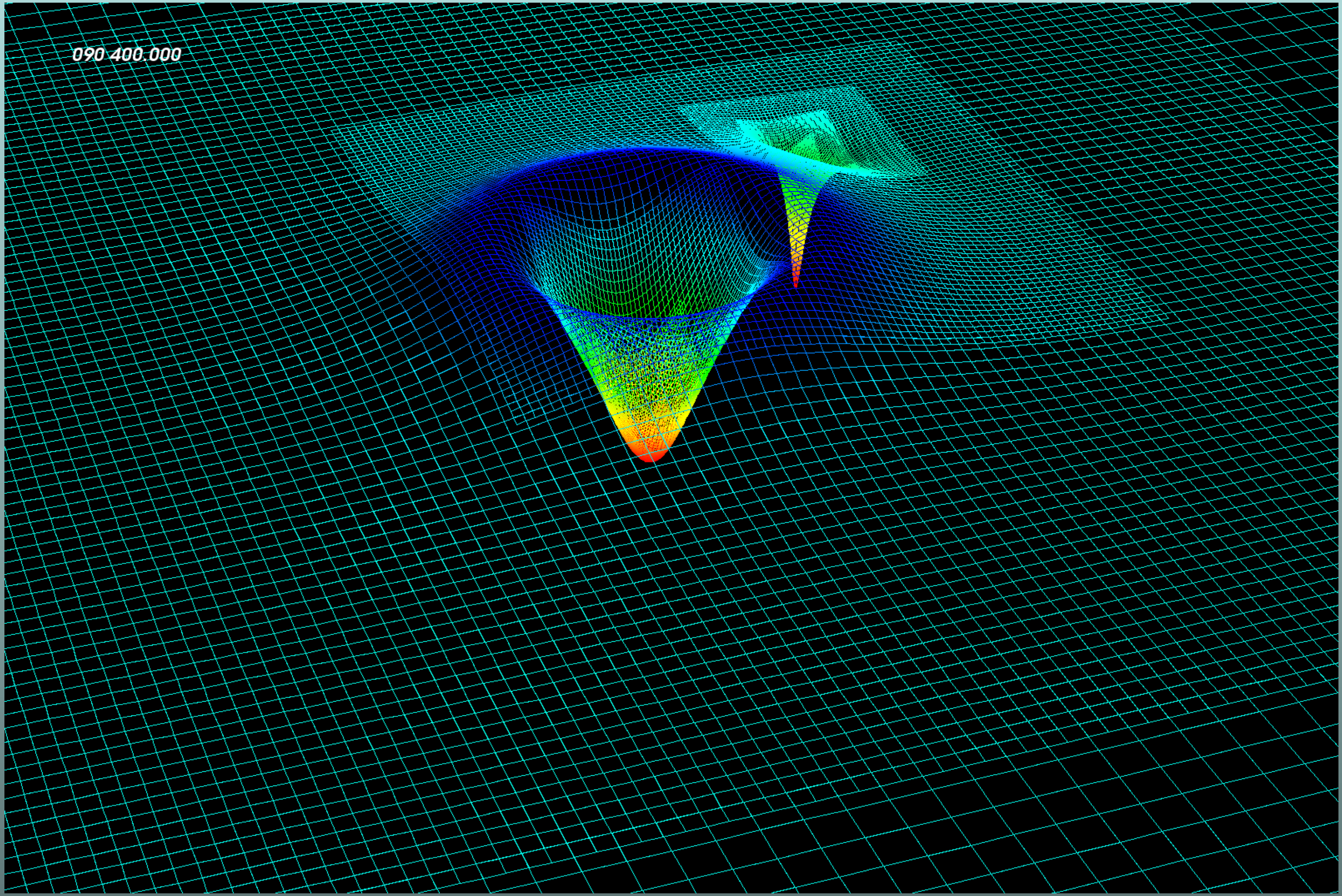


Coordinate- and Gauge freedom

- General scenarios require "live" conditions

$$\partial_t \alpha = F(\alpha, \beta^i, \gamma_{ij}) \qquad \partial_t \beta^i = G(\alpha, \beta^i, \gamma_{ij})$$

  Hyperbolic, parabolic or elliptic PDEs

- Pretorius '05 Generalized Harmonic Gauge

- Goddard, Brownsville '06 moving punctures

  $1 + \log$ slicing, Γ driver

- based on Alcubierre et al. (AEI)

  Bona, Massó 1990s

Diagnostik: Wellenformen

- In and outgoing direction are specified via
  Basis vectors $n^\alpha, \ \ell^\alpha, \ m^\alpha, \ \overline{m}^\alpha$ Kinnersley '69
- Newman-Penrose scalar

$$\Psi_4 = R_{\alpha\beta\gamma\delta} \, n^\alpha \overline{m}^\beta n^\gamma \overline{m}^\delta$$

- At Null-Infinity ! But cf. Nerozzi & Ellbracht '08
- Waves are normally extracted at fixed radius

$$\Rightarrow \Psi_4 = \Psi_4(t, \theta, \phi)$$

- Decompose angular dependence

$$\Psi_4 = \sum_{\ell m} \psi_{\ell m}(t) \, Y^{-2}_{\ell m}(\theta, \phi)$$ "Multipoles"

- Gives directly $E_{\mathrm{rad}}, \ P_{\mathrm{rad}}, \ \mathrm{J}_{\mathrm{rad}}$

A brief history of BH simulations

- Pioneers: Hahn, Lindquist '60s, Eppley, Smarr et.al. '70s
- Grand Challenge: First 3D Code Anninos et.al. '90s
- Further attempts: Bona & Massó, Pitt-PSU-Texas, …

  AEI-Potsdam Alcubierre et al.

  PSU: first orbit Brügmann et al. '04

  Codes unstable

- Breakthrough: Pretorius '05 "GHG"

  UTB, Goddard '05 "Moving Punctures"
- Currently: ≈ 10 codes, a.o.

Pretorius, UTB/RIT, Goddard, PSU/GT, Sperhake, Jena/FAU, AEI/LSU, Caltech-Cornell, UIUC, Tainan/Beijing

3. Animations

Animations

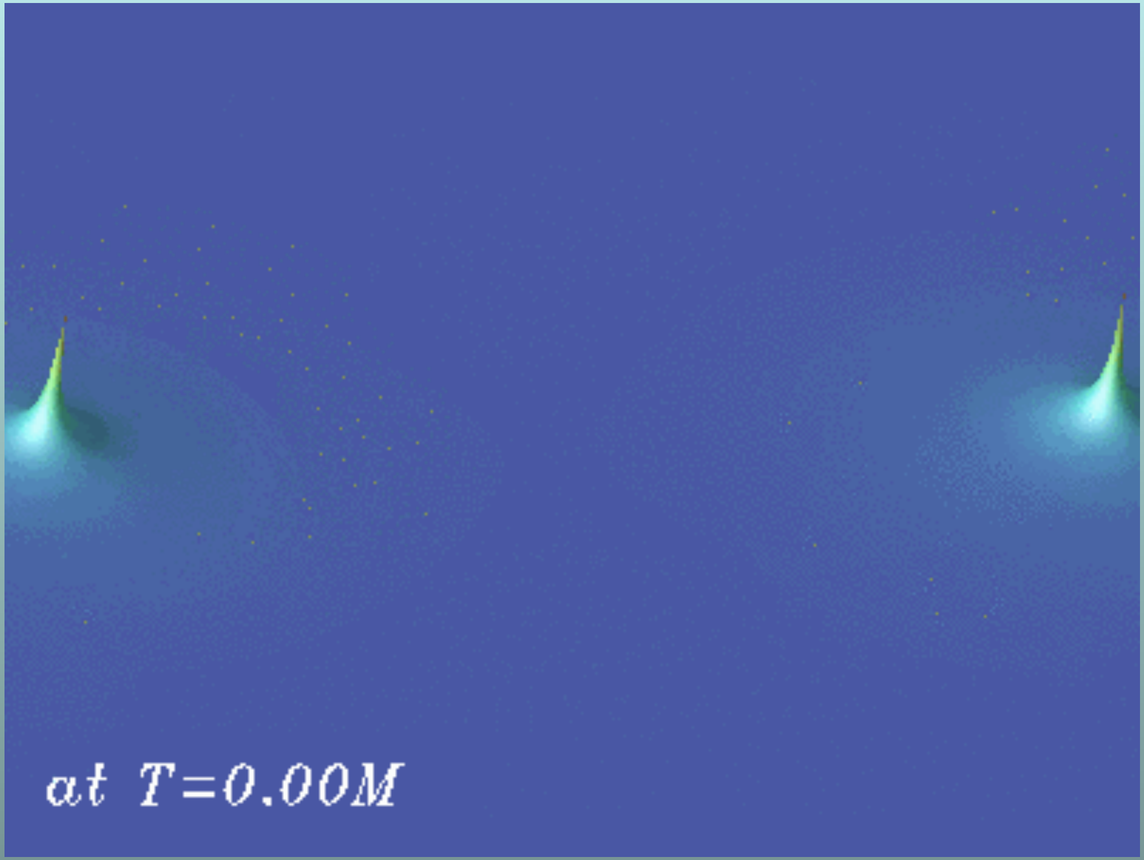
- Lean Code
  Sperhake '07

- Extrinsic curvature

  $\mathrm{tr}K$

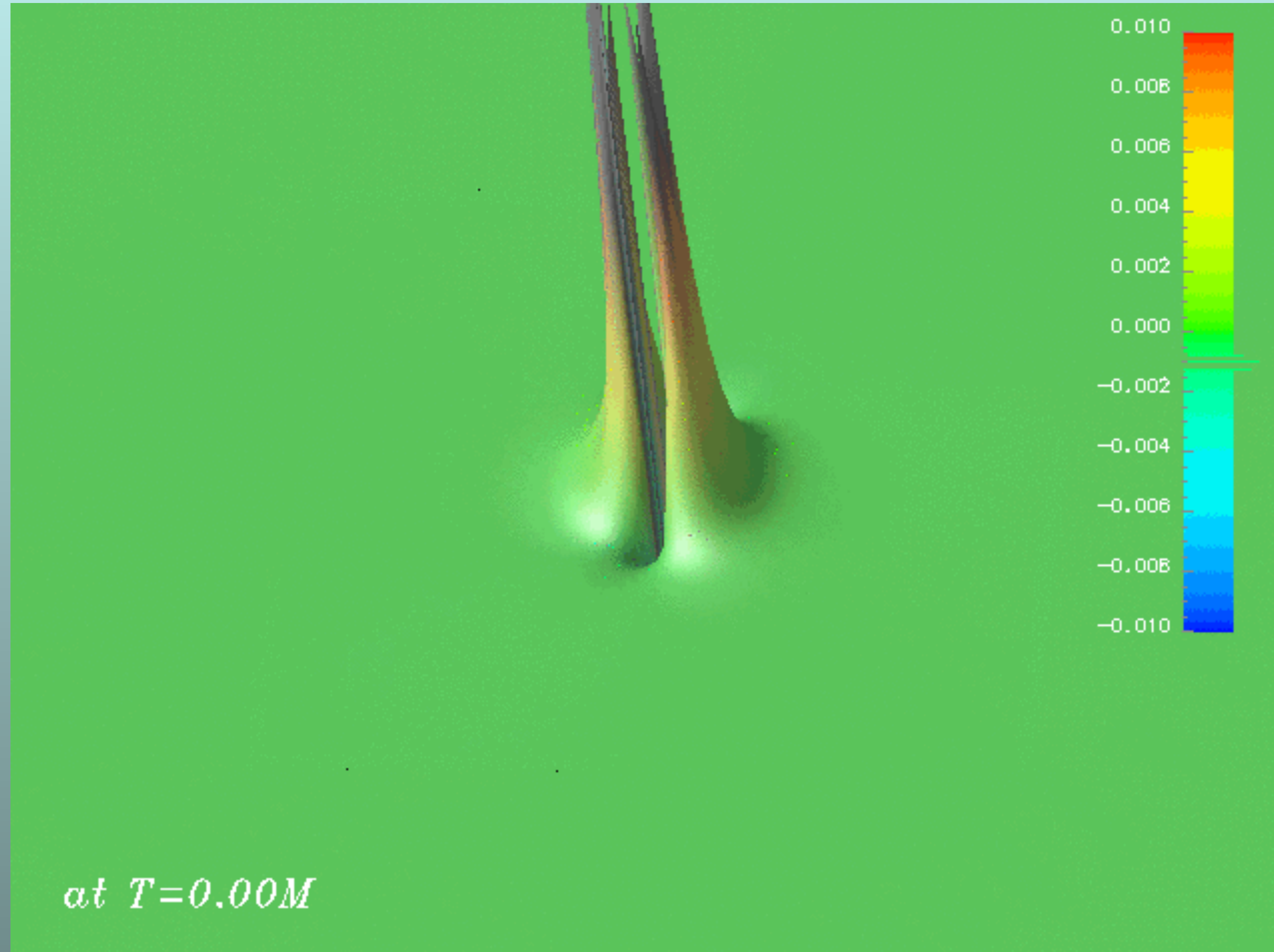
- Apparent horizon

  AHFinderDirect
  Thornburg



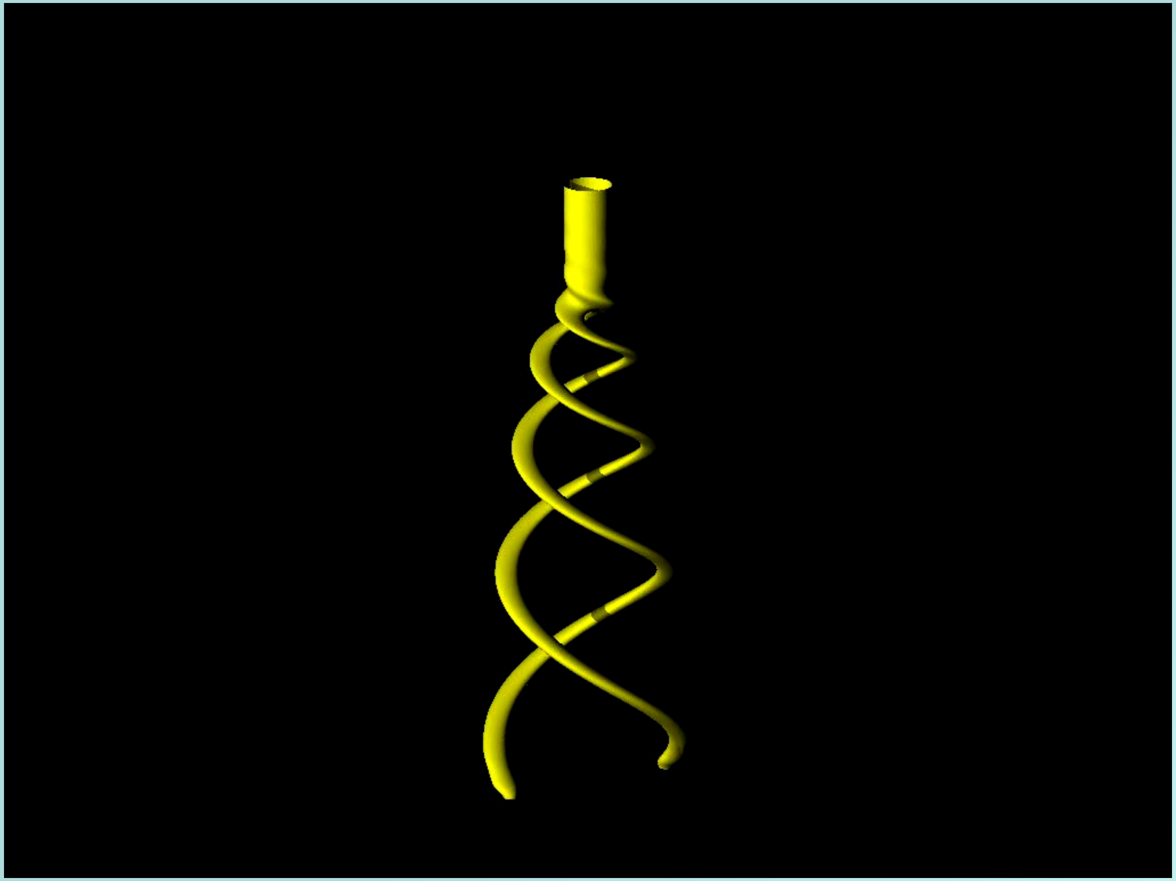
Animations

- $\mathrm{Re}[\Psi_4]$
- $\ell = 2, m = 2$ dominant



Animations

- Event horizon of binary inspiral and merger   BAM



Thanks to Marcus Thierfelder

7. Results on black-hole binaries

Free parameters of BH binaries

- Total mass M_{ADM}
  - Relevant for detection: Frequencies depend on M_{ADM}
  - Not relevant for source modeling: trivial rescaling
- Mass ratio $q = \frac{M_1}{M_2}, \qquad \eta = \frac{M_1 M_2}{(M_1 + M_2)^2}$
- Spin $\vec{S}_1, \vec{S}_2$
- Initial parameters
  - Binding energy E_{b} — Separation
  - Orbital angular momentum L — Eccentricity
  - Alternatively: frequency, eccentricity

7.1. Non-spinning equal-mass holes

The BBH breakthrough

- Simplest configuration
- GWs circularize orbit
  $\Longrightarrow$ quasi-circular initial data

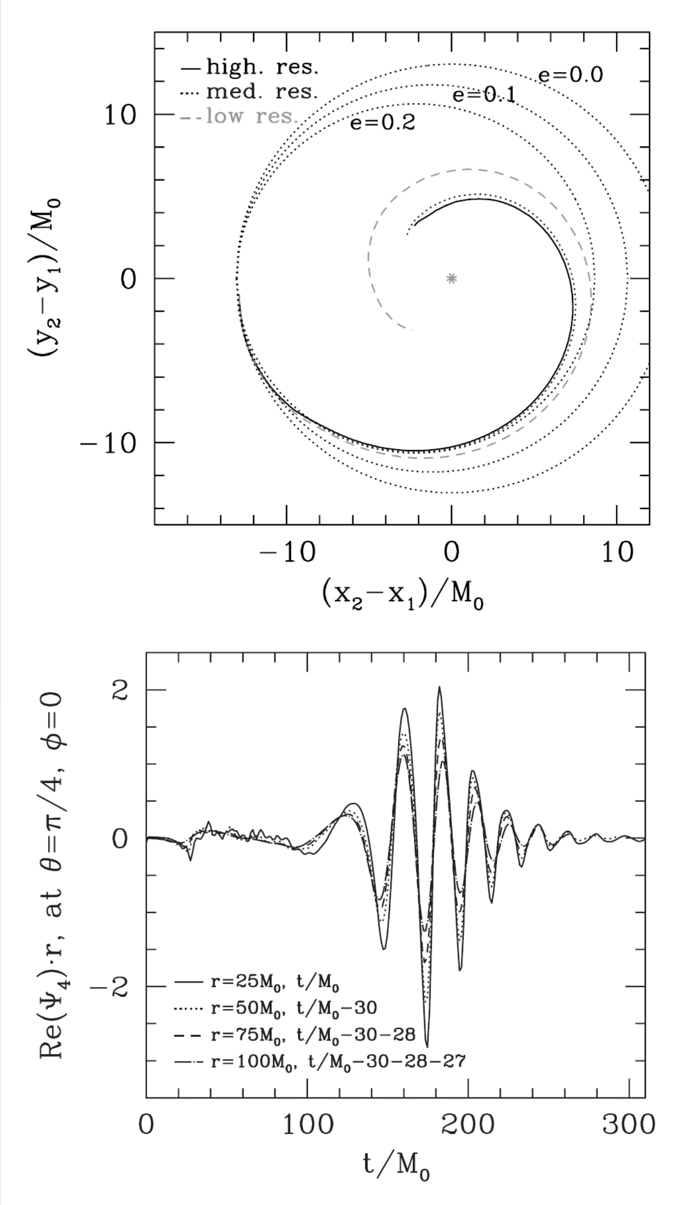
Pretorius PRL ‘05

- BBH breakthrough
- Initial data: scalar field
- Radiated energy

$R_{ex}\,[M] =$	25	50	75	100
$E\,[\%M] =$	4.7	3.2	2.7	2.3

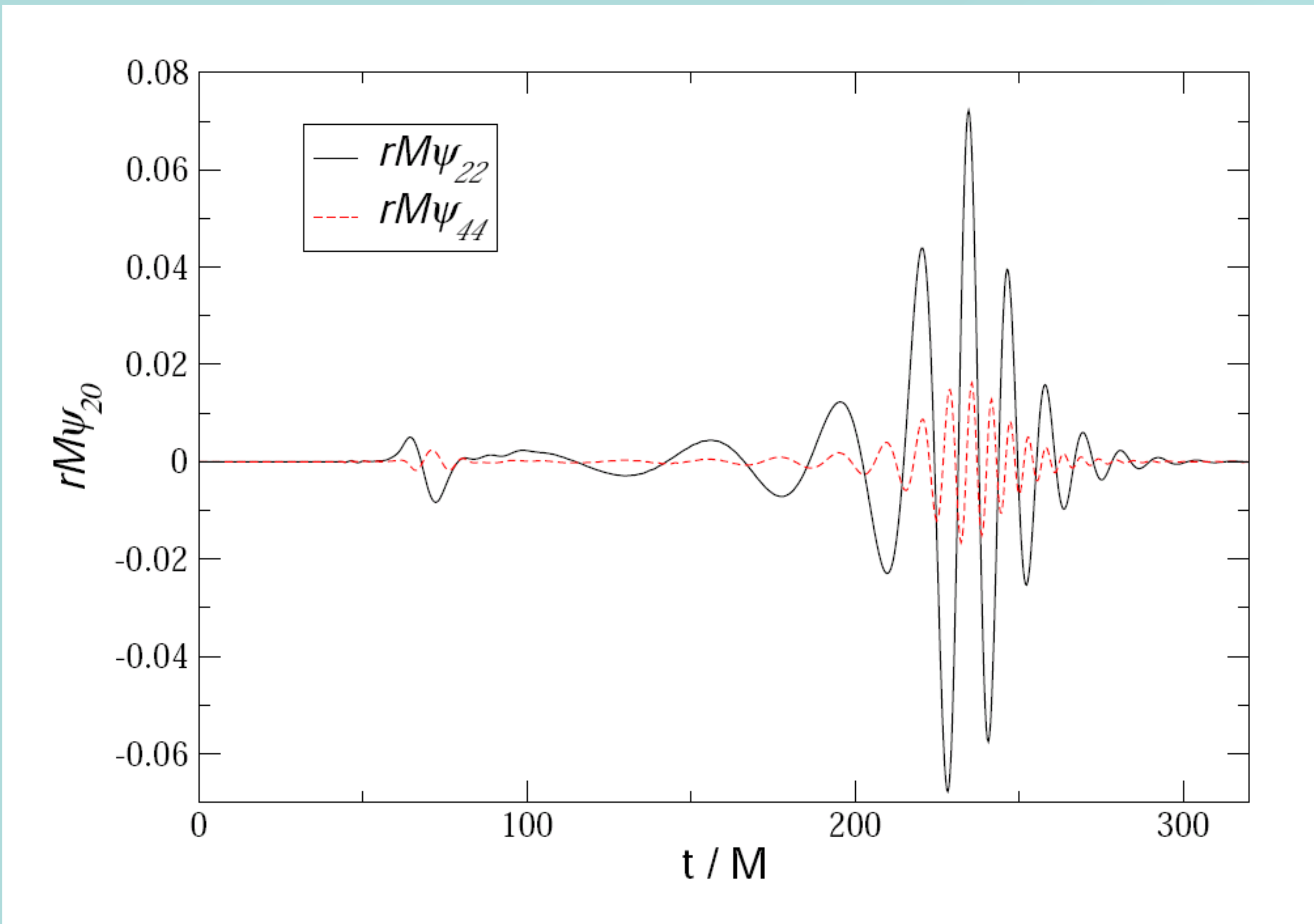
- Eccentricity

$$e = 0 \ldots 0.2$$



Non-spinning equal-mass binaries

- Total radiated energy: $3.6\% \, M_{\mathrm{ADM}}$
- $\ell = 2, m = 2$ mode dominant: $> 98\%$



The merger part of the inspiral

Buonanno, Cook, Pretorius '06 (BCP)

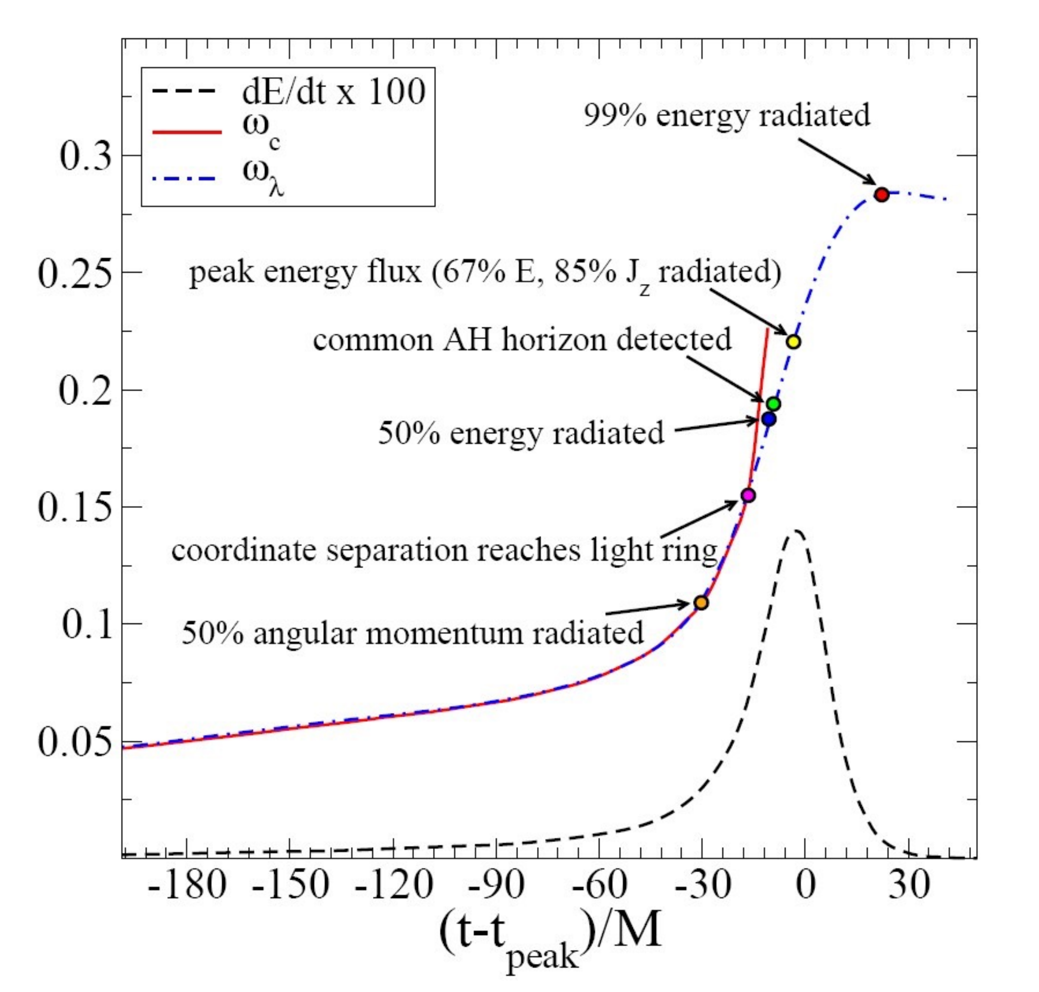
- merger lasts short:
  0.5 – 0.75 cycles

- Eccentricity small

  ≈ 0.01

  non-vanishing

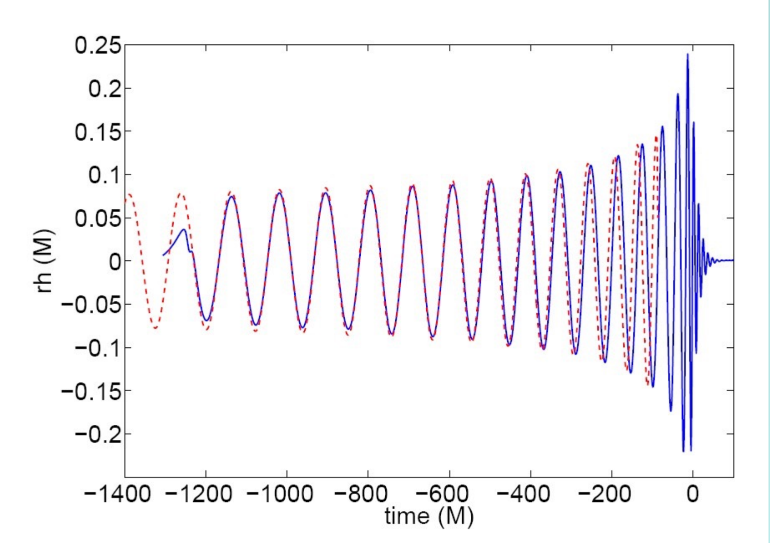
  Initial radial velocity



Comparison with Post-Newtonian

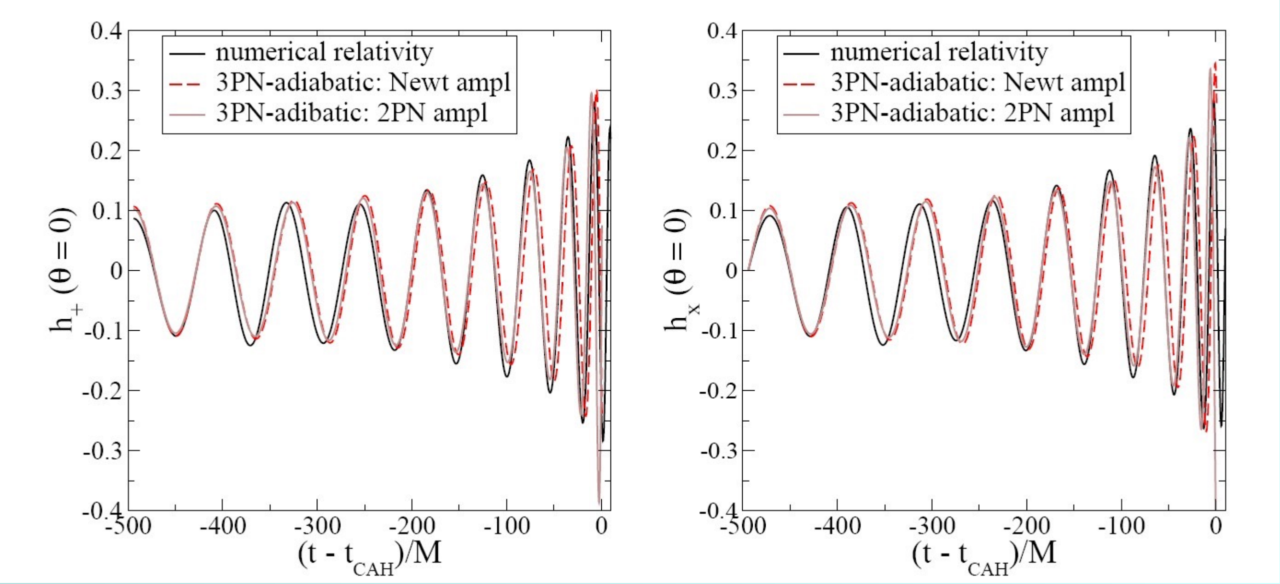
Goddard '07

- 14 cycles, 3.5 PN phasing
- Match waveforms: φ, φ
- Accumulated phase error
  1 rad



Buonanno, Cook, Pretorius '06 (BCP)

- 3.5 PN phasing
  2 PN amplitude

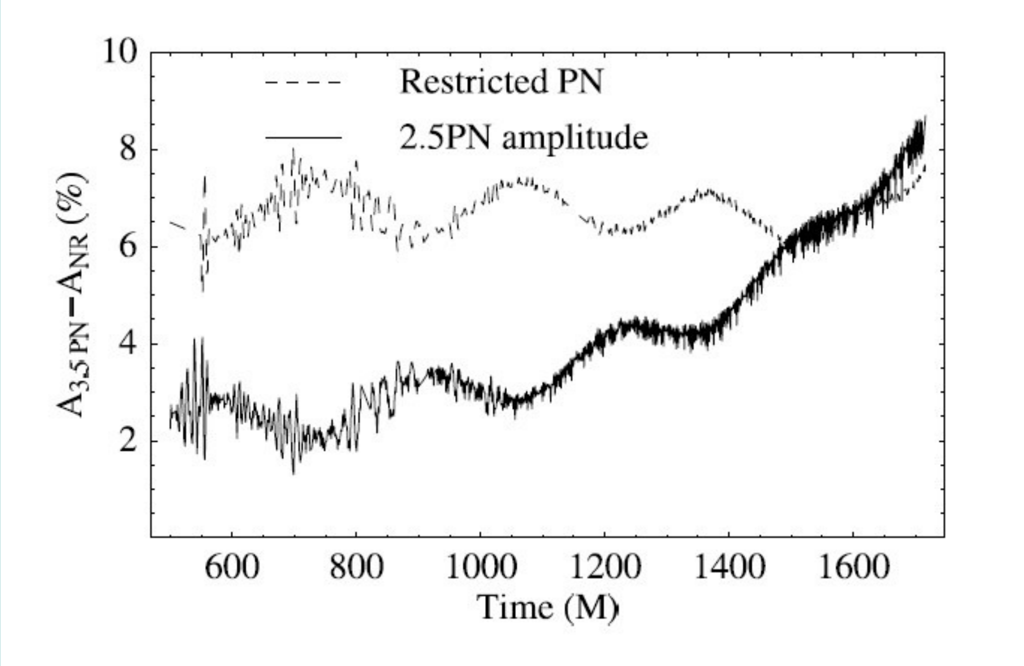


Comparison with Post-Newtonian

Jena '07

- 18 cycles
- phase error $< 1\,\mathrm{rad}$

  6th order differencing !!
- Amplitude: % range



Cornell/Caltech & Buonanno

- 30 cycles
- phase error $\approx 0.02\,\mathrm{rad}$
- Effective one body (EOB)

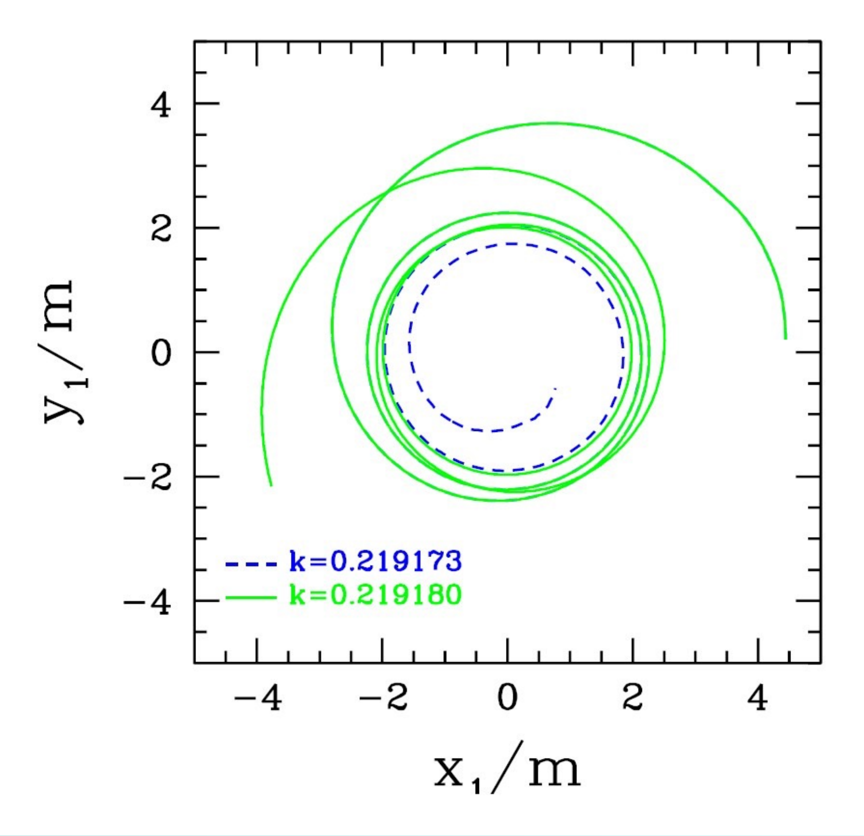
RIT

- First comparison with spin; not conclusive yet

Zoom whirl orbits

Pretorius & Khurana ‘07

- 1-parameter family of initial data: linear momentum
- Fine-tune parameter
  ⇒ ”Threshold of immediate merger”
- Analogue in gedodesics !
- Reminiscent of ”Critical phenomena”
- Similar observations by PSU



Max. spin $j_{\mathrm{fin}} = 0.78$ for $L \approx M^2$

7.2. Unequal masses

Unequal masses

- Still zero spins
- Astrophysically much more likely !!
- Symmetry breaking
  - Anisotropic emission of GWs
  - Certain modes are no longer suppressed
- Mass ratios
  - Stellar sized BH into supermassive BH $\approx 10^6$
  - Intermediate mass BHs $\approx 10^3$
  - Galaxy mergers $\approx 1 \ldots 10^3$
  - Currently possible numerically: $\approx 1 \ldots 10$

Gravitational recoil

- Anisotropic emission of GWs radiates momentum
  $\Rightarrow$ recoil of remaining system

- Leading order: Overlap of Mass-quadrupole
  with octopole/flux-quadrupole

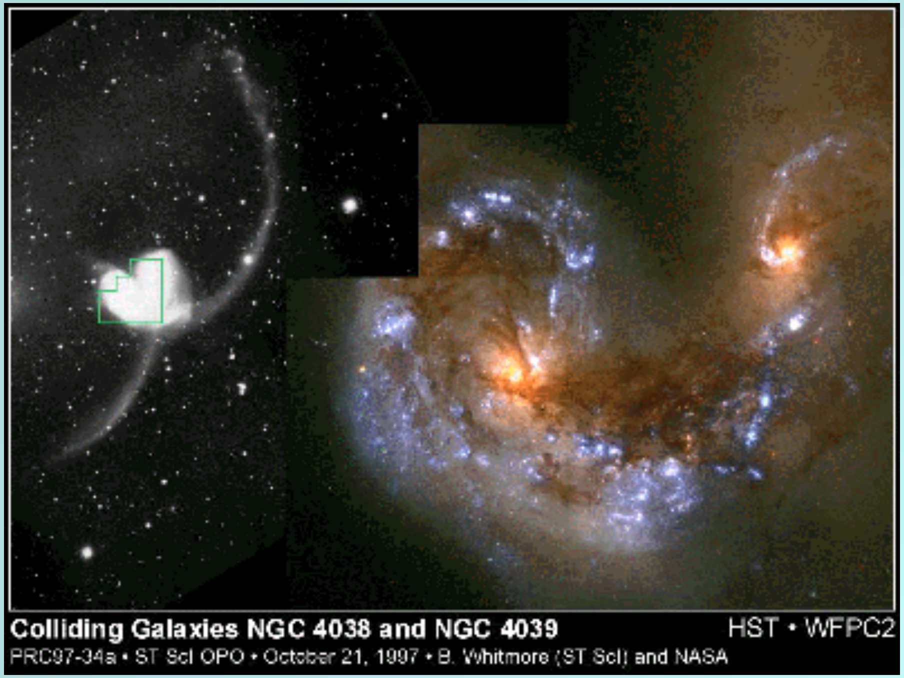
Bonnor & Rotenburg ’61, Peres ’62, Bekenstein ‘73

- Merger of galaxies

$\Rightarrow$ Merger of BHs

$\Rightarrow$ Recoil

$\Rightarrow$ BH kicked out?



Gravitational recoil

- Escape velocities

Globular clusters	$30\ \mathrm{km/s}$
dSph	$20-100\ \mathrm{km/s}$
dE	$100-300\ \mathrm{km/s}$
Giant galaxies	$\approx 1000\ \mathrm{km/s}$

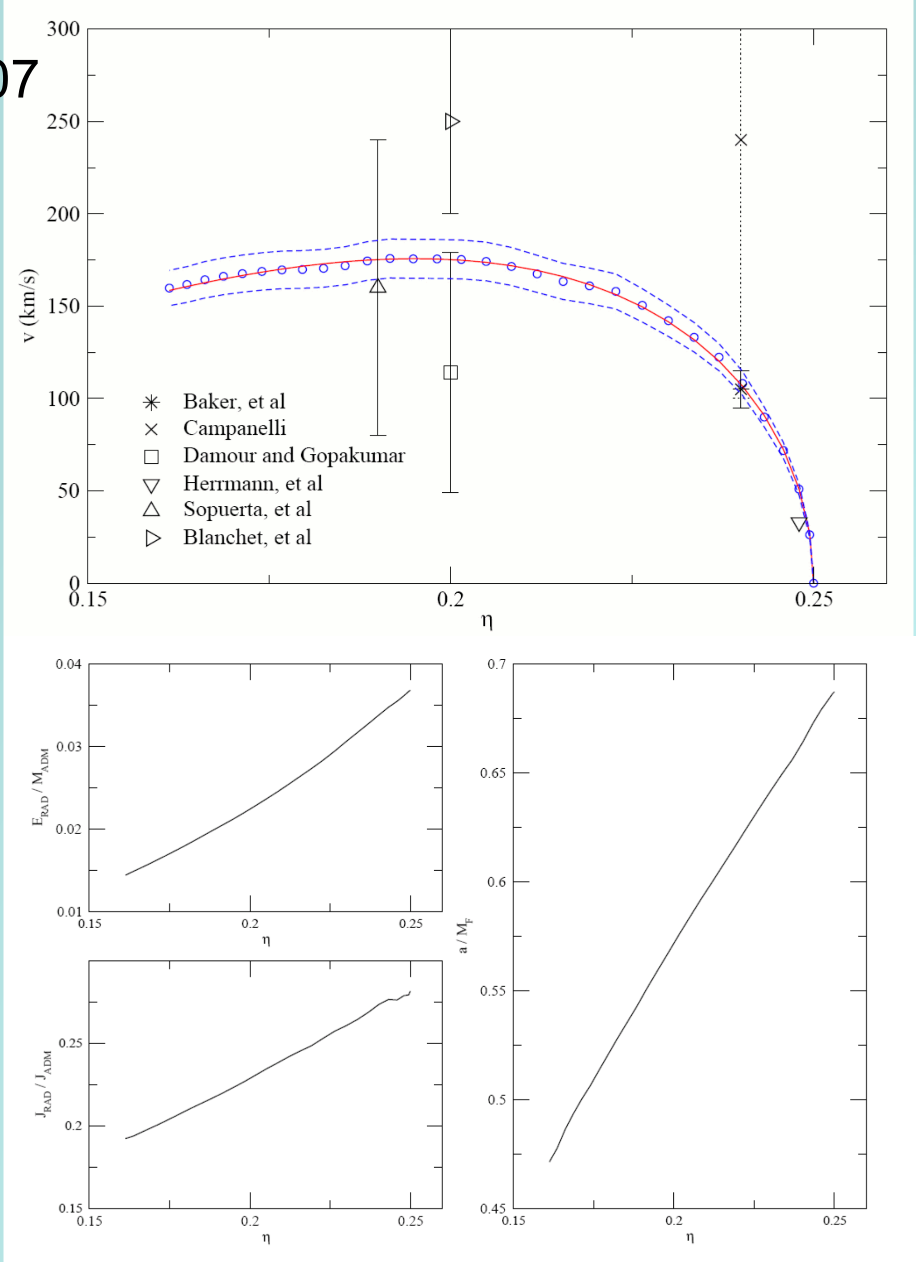
Merrit et al '04

- Ejection or displacement of BHs has repercussions on:
  - Structure formation in the universe
  - BH populations

    IMBHs via ejection?
  - Growth history of Massive Black Holes
  - Structure of galaxies

Kicks of non-spinning black holes

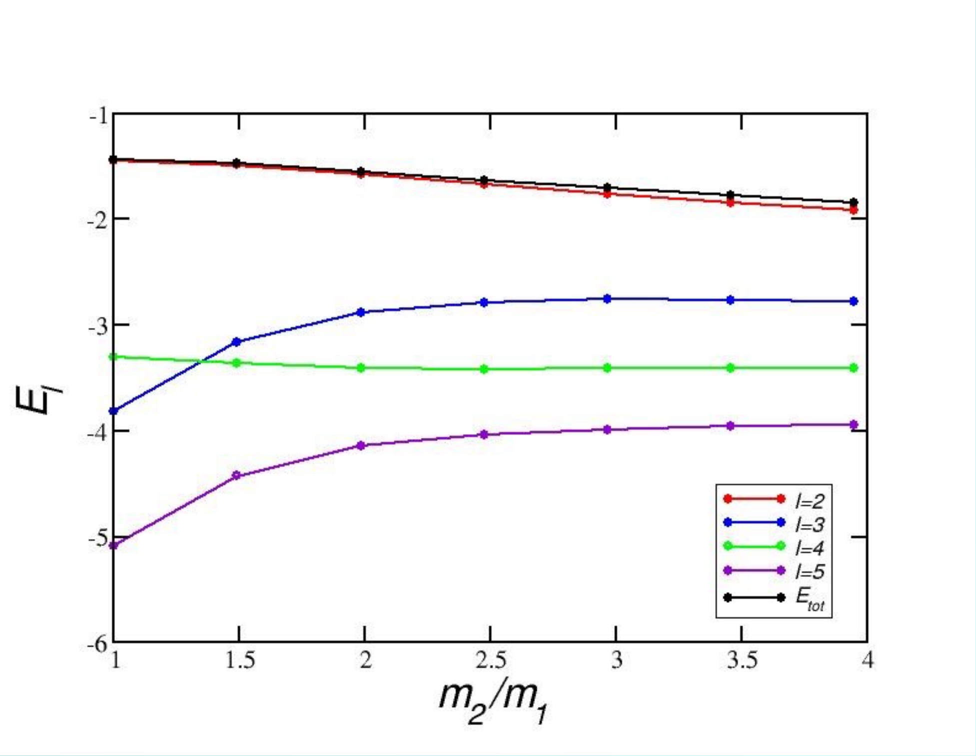
- Simulations PSU '07, Goddard '07
- Parameter study Jena '07
- Target: Maximal Kick
- Mass ratio: $M_1 / M_2 = 1 \ldots 4$
- 150,000 CPU hours
- Maximal kick 178 km/s for $M_1 / M_2 \approx 3$
- Convergence 2nd order
- $E_{\text{rad}} \approx 3\%, \ J_{\text{rad}} \approx 25\%$
- Spin $0.45 \ldots 0.7$



Features of unequal-mass mergers

Berti et al '07

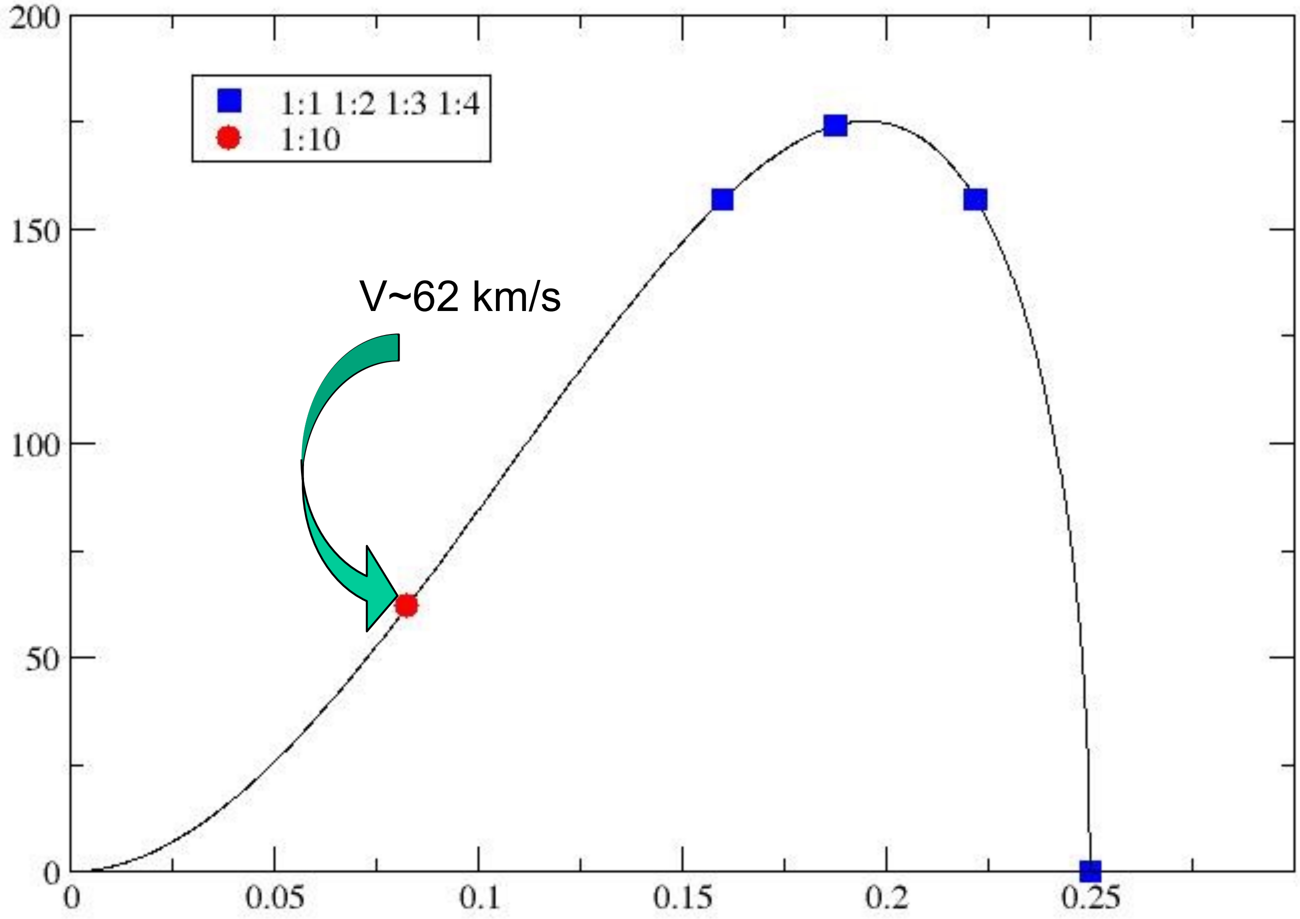
- Distribution of radiated energy
  - More energy in higher modes
  - Odd ℓ modes suppressed for equal masses
- Important for GW-DA



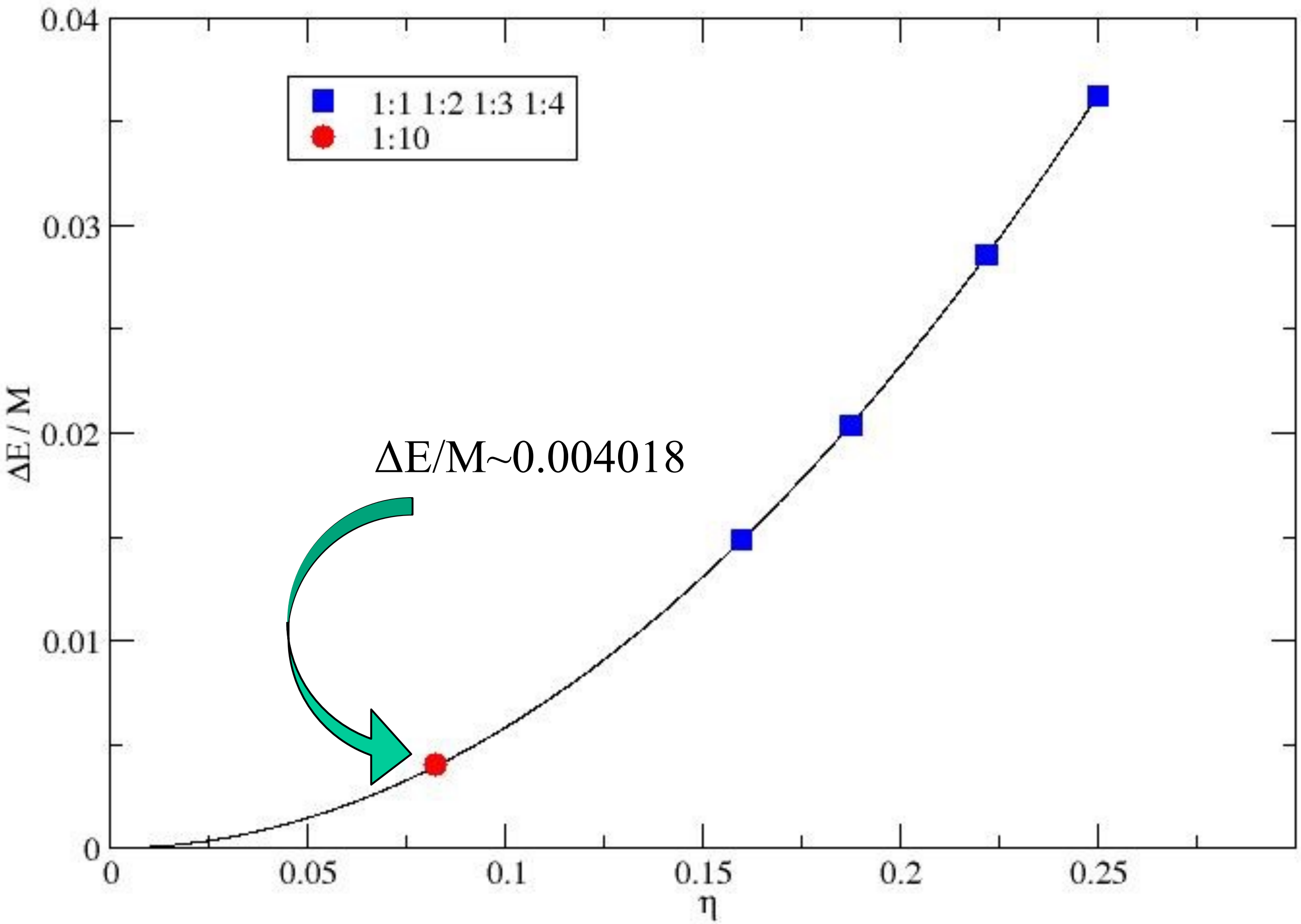
Mass ratio 10:1

- In preparation: González, U.S., Brügmann
- Mass ratio $q = 10$; Bam
- 4th order convergence
- Astrophysically likely configuration: Sesana et al. '07
- Test fitting formulas for spin and kick!

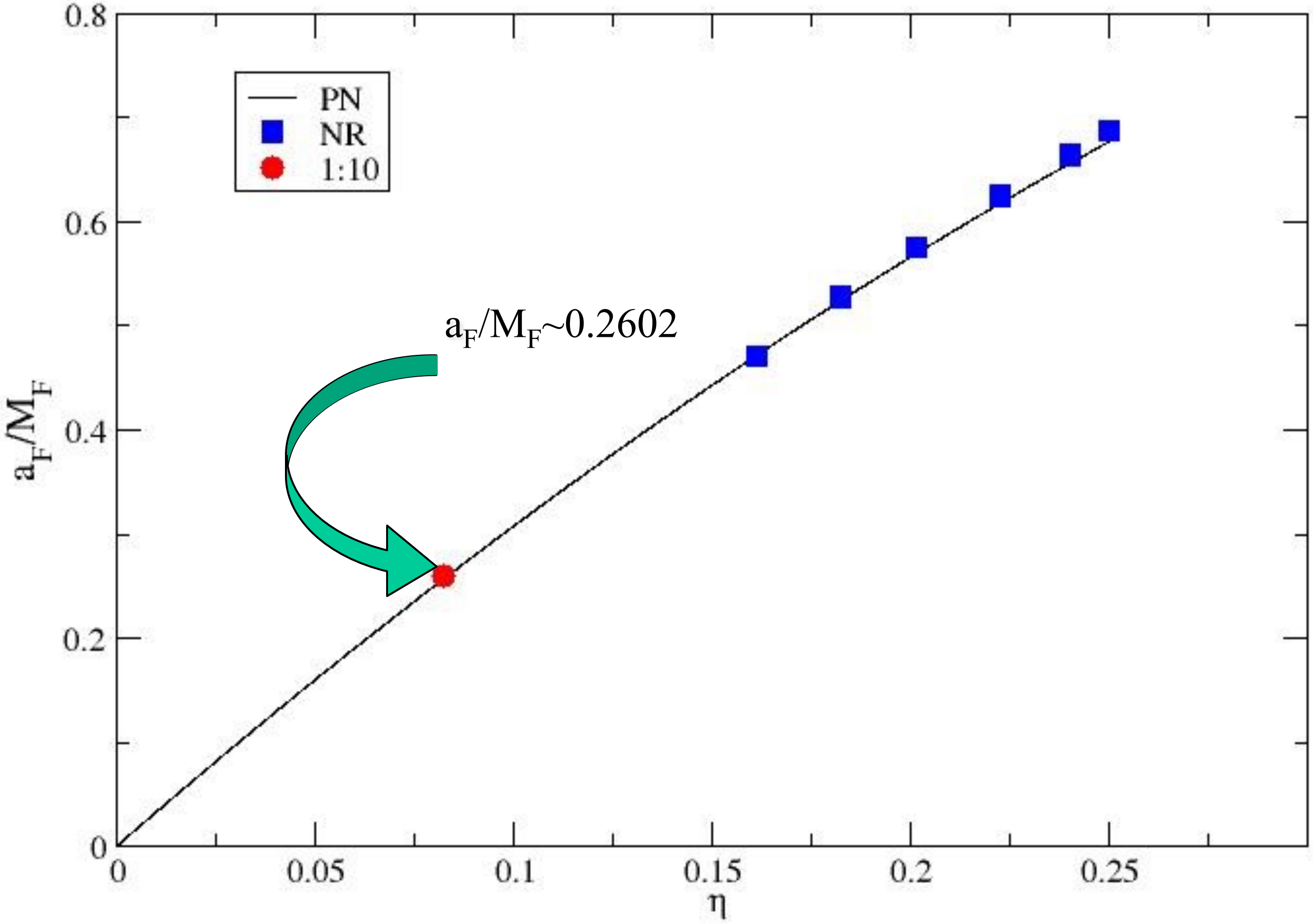
Kick: $v = 1.2 \times 10^4 \eta^2 \sqrt{1-4\eta}\,(1-0.93\eta)$ (Fitchett ‘83 Gonzalez et al. ’07)



Radiated energy: $\frac{\Delta E}{M} = 0.5802\eta^2$ (Berti et al. '07)



Final spin: $a_F/M_F = 3.32\eta - 2.45\eta^2$ (Damour and Nagar 2007)

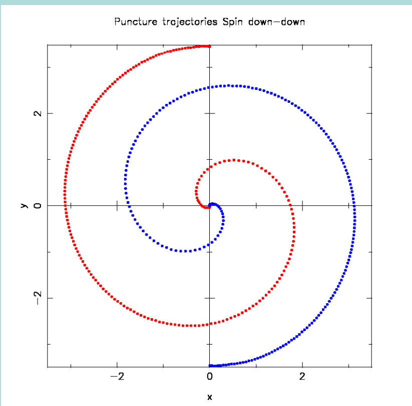


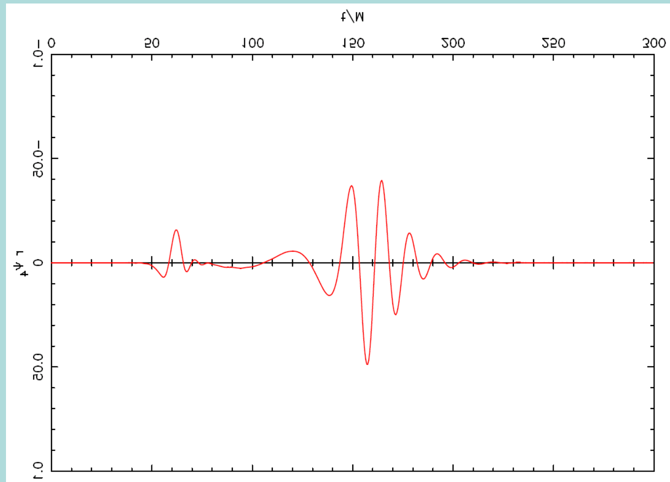
7.3. Spinning black holes

Spinning holes: The orbital hang-up

- ↑↑↑ Spins parallel to $L \Rightarrow$ more orbits, E_{rad}, J_{rad} larger

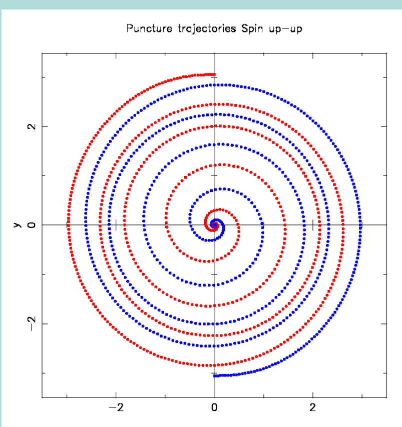
  ↓↑↓ Spins anti-par. to $L \Rightarrow$ fewer orbits E_{rad}, J_{rad} smaller

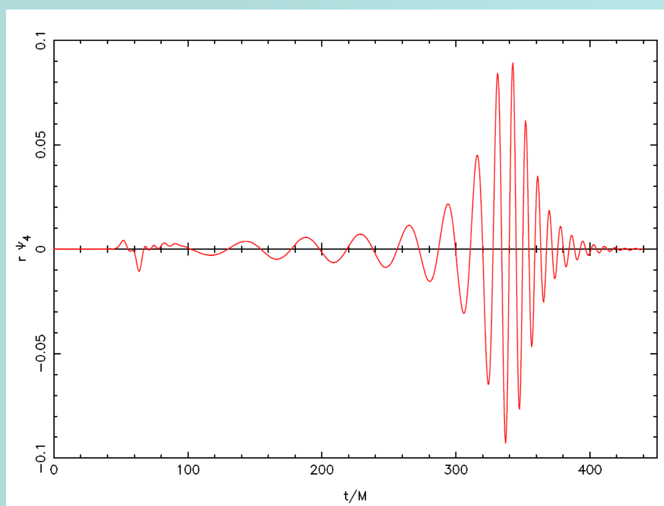




UTB/RIT '07

- no extremal Kerr BHs





Spin precession and flip

- X-shaped radio sources
  Merritt & Ekers '07
- Jet along spin axis
- Spin re-alignment
  ⇒ new + old jet
- Spin precession $98°$
  Spin flip $71°$
  UTB, Rochester '06

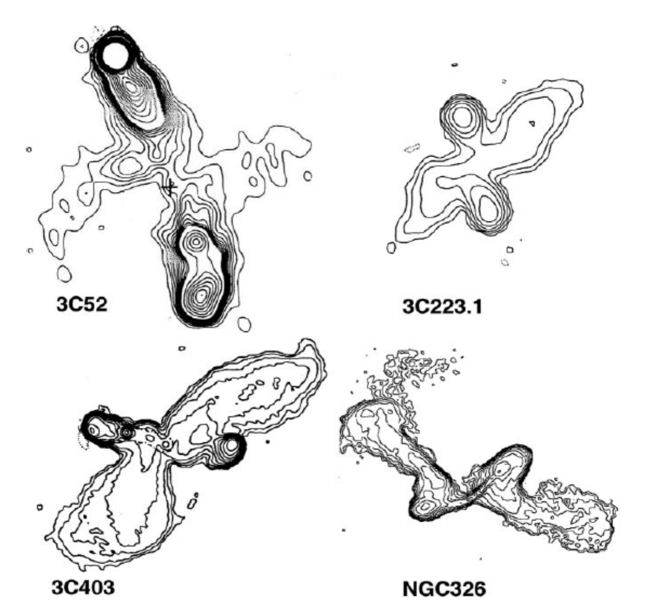
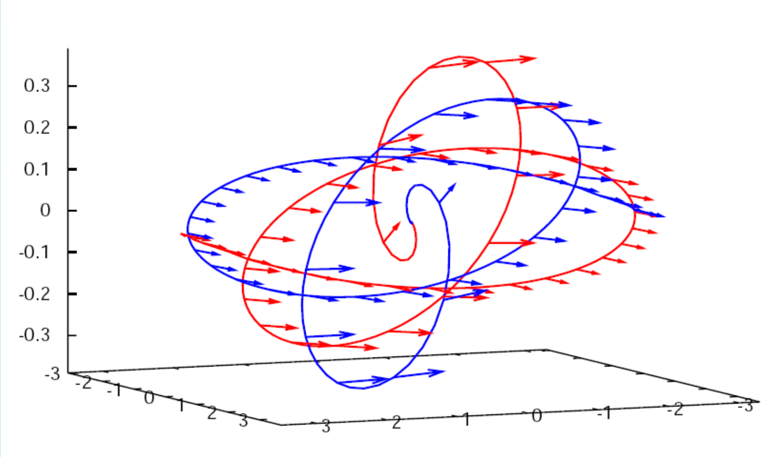


Fig. 2. A composite of four clear examples of the X-shape morphology. VLA radio observations of 3C52 (28), 3C223.1 (29), 3C403 (29), and NGC 326 (12).

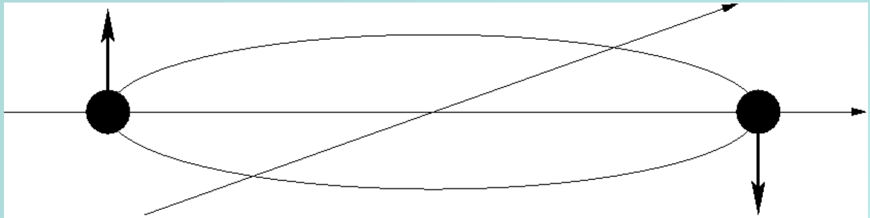


Recoil of spinning holes

- Kidder '95: PN study with Spins

$$\frac{d\mathbf{P}}{dt} = \dot{\mathbf{P}}_{\rm N} + \dot{\mathbf{P}}_{\rm SO},$$ = "unequal mass" + "spin(-orbit)"

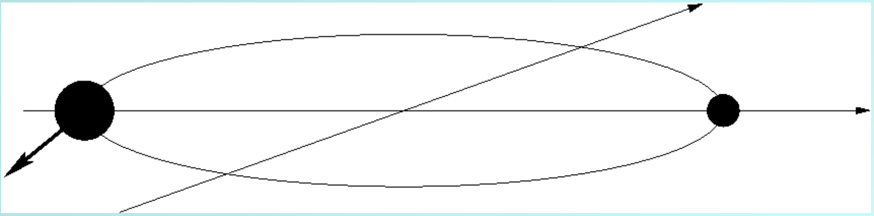
- Penn State '07: SO-term larger



$$\frac{a}{m} = 0.2,\ldots,0.8$$

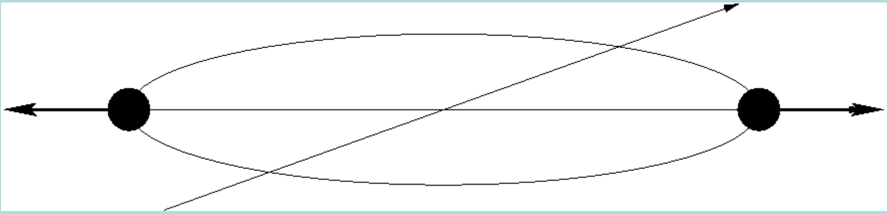
extrapolated: $v = 475\ \text{km/s}$

- AEI '07 : One spinning hole, extrapolated: $v = 440\ \text{km/s}$
- UTB-Rochester: $v = 454\ \text{km/s}$



Super Kicks

- Side result RIT '07, Kidder '95: maximal kick predicted for



$v \approx 1300\,\mathrm{km/s}$

- Test hypothesis

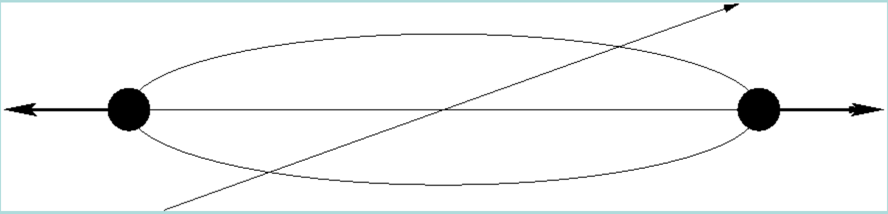
  González, Hannam, US, Brügmann & Husa '07

  Use two codes: Lean, BAM

- Generates kick $v = 2500\,\mathrm{km/s}$ for spin $a \approx 0.75$

Super Kicks

- Side result RIT '07, Kidder '95: maximal kick predicted for



$v \approx 1300\,\mathrm{km/s}$

- Test hypothesis

  González, Hannam, US, Brügmann & Husa '07

  Use two codes: Lean, BAM

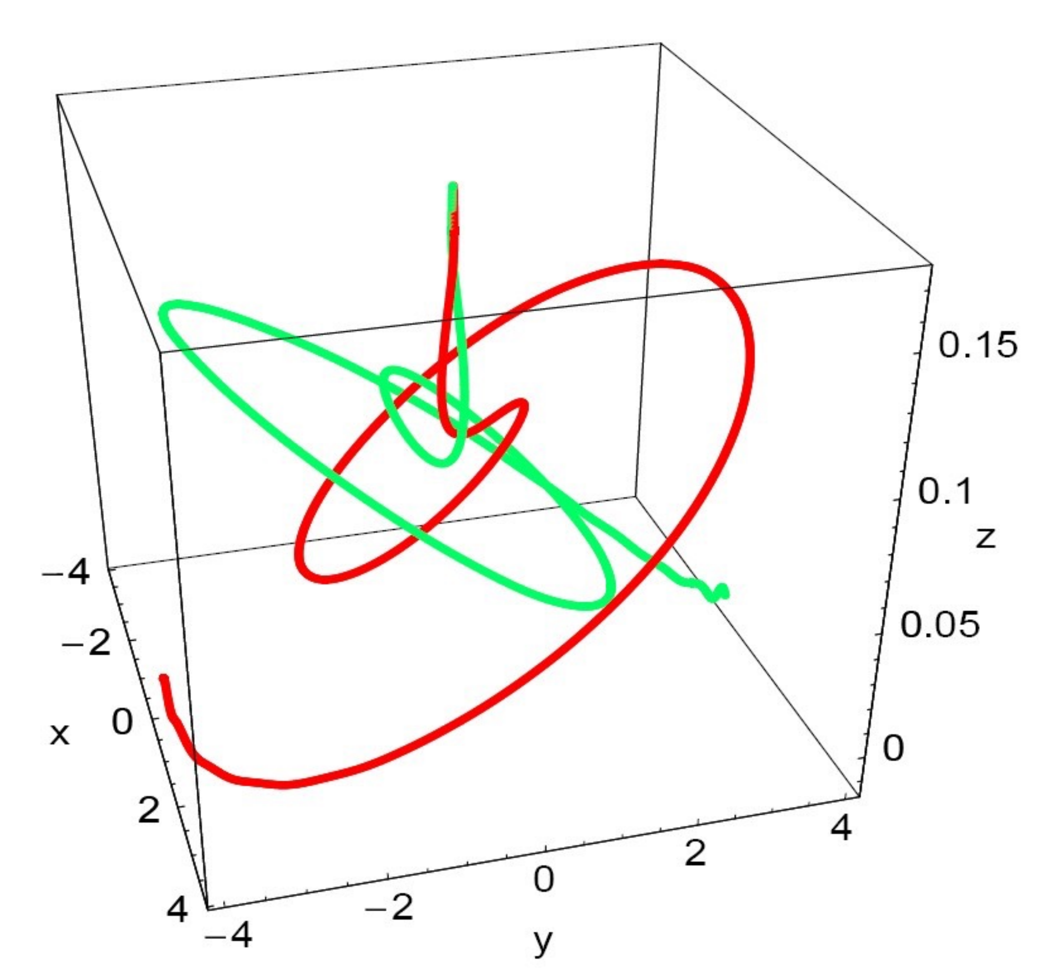
- Generates kick $v = 2500\,\mathrm{km/s}$ for spin $a \approx 0.75$

- Extrapolated to maximal spin $v = 4000\,\mathrm{km/s}$
  RIT '07

- Highly eccentric orbits $v = 10000\,\mathrm{km/s}$
  PSU '08

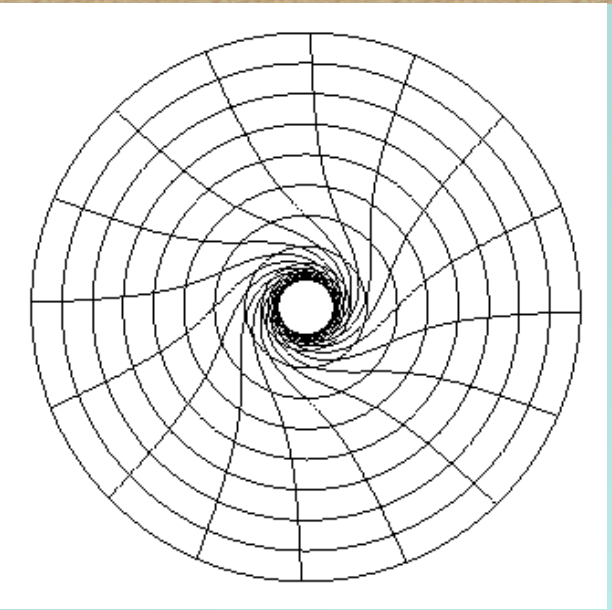
What's happening physically?

- Black holes "move up and down"



A closer look at super kicks

- Physical explanation:
  "Frame dragging"
- Recall: rotating BH drags objects along with its rotation



A closer look at super kicks

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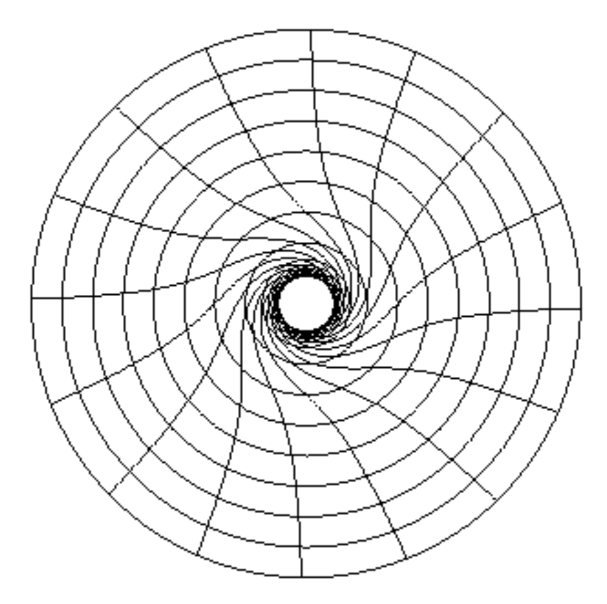
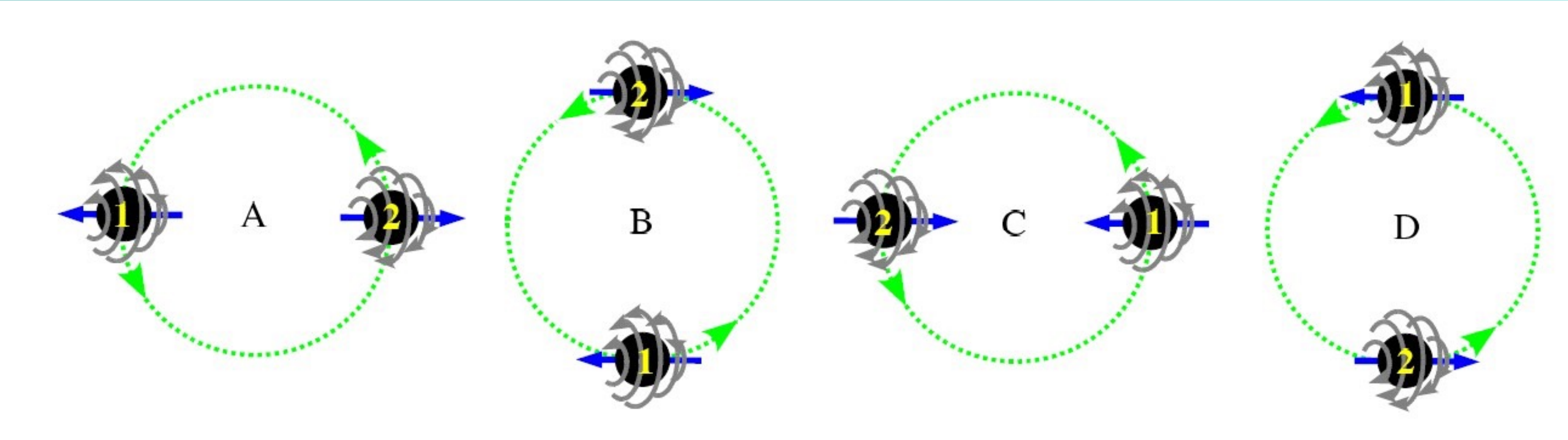




Thanks to F. Pretorius

How realistic are superkicks?

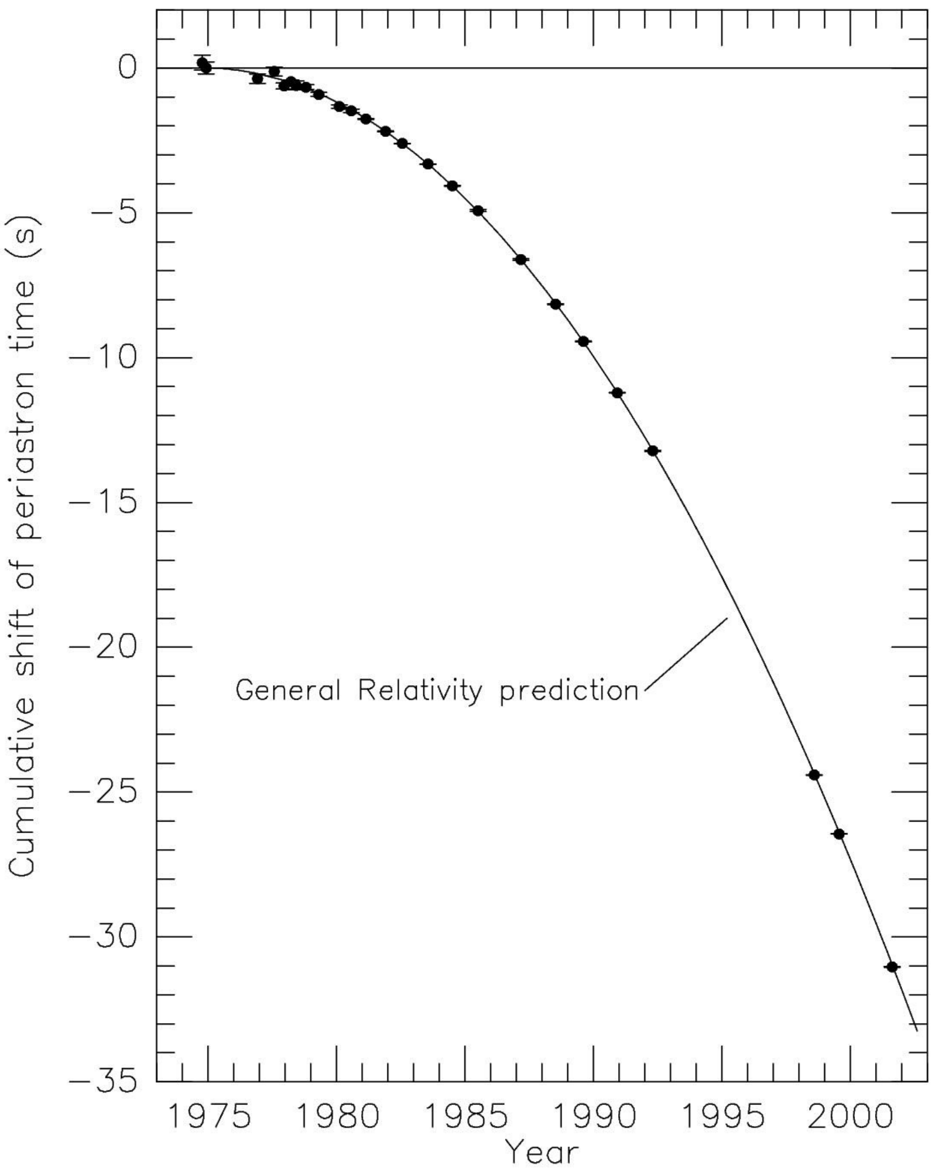
- Observations $\Rightarrow$ BHs are not generically ejected!
- Are superkicks real?
- Gas accretion may align spins with orbit Bogdanovic et al.
- Kick distribution function: $v_{\mathrm{kick}} = v_{\mathrm{kick}}\left(\vec{S}_1, \vec{S}_2, M_1 / M_2\right)$
- Analytic models and fits: Boyle, Kesden & Nissanke, AEI, RIT, Tichy & Marronetti,…
- Use numerical results to determine free parameters
- 7-dim. Parameter space: Messy! Not yet conclusive…
- EOB study $\Rightarrow$ only 12% of all mergers have $v > 500\ \mathrm{km/s}$ Schnittman & Buonanno '08

7.4. Numerical relativity and data analysis

The Hulse-Taylor pulsar

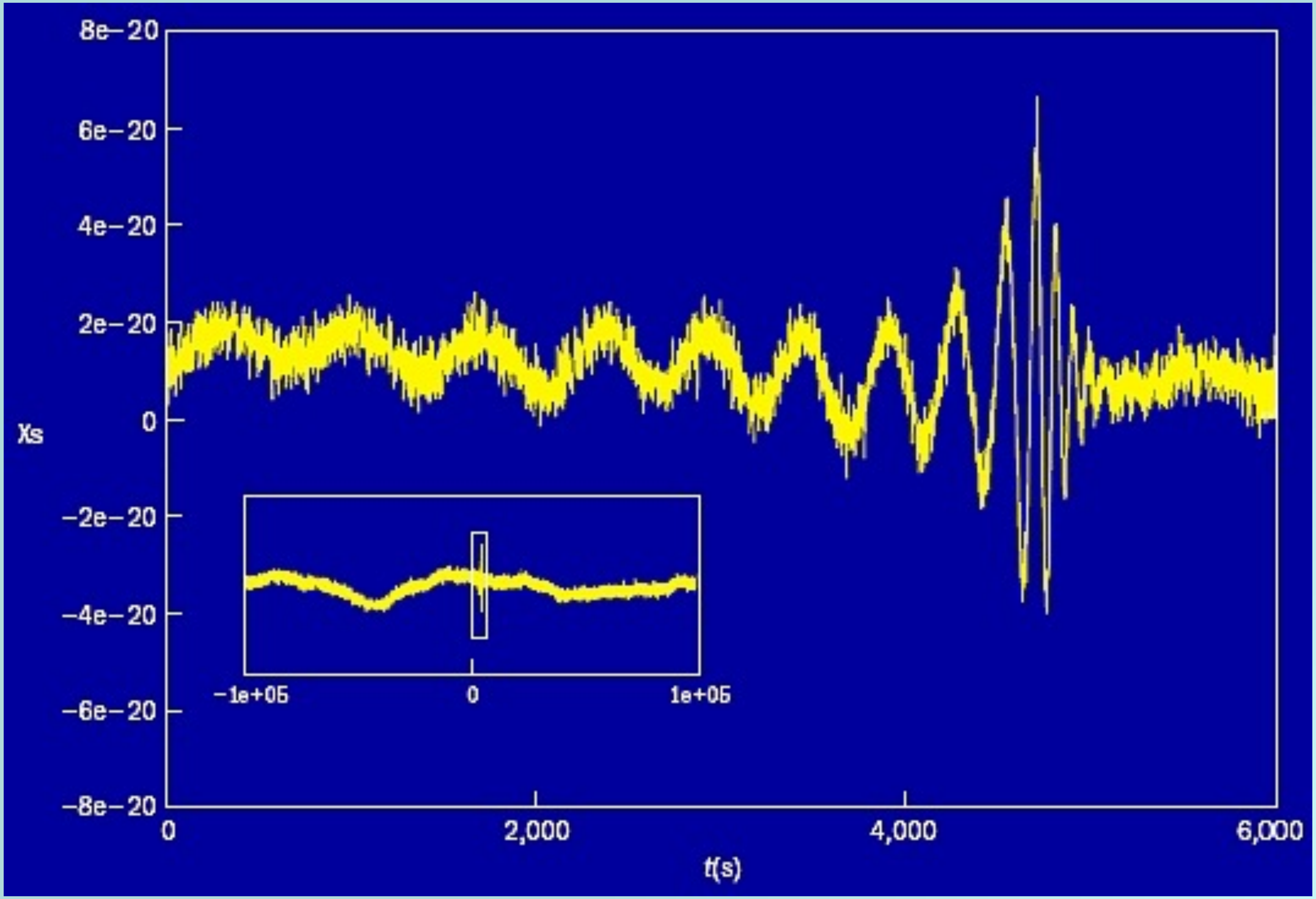
Hulse, Taylor '93

- Binary pulsar 1913+16
- GW emission
- Inspiral
- Change in period
- Excellent agreement with relativistic prediction



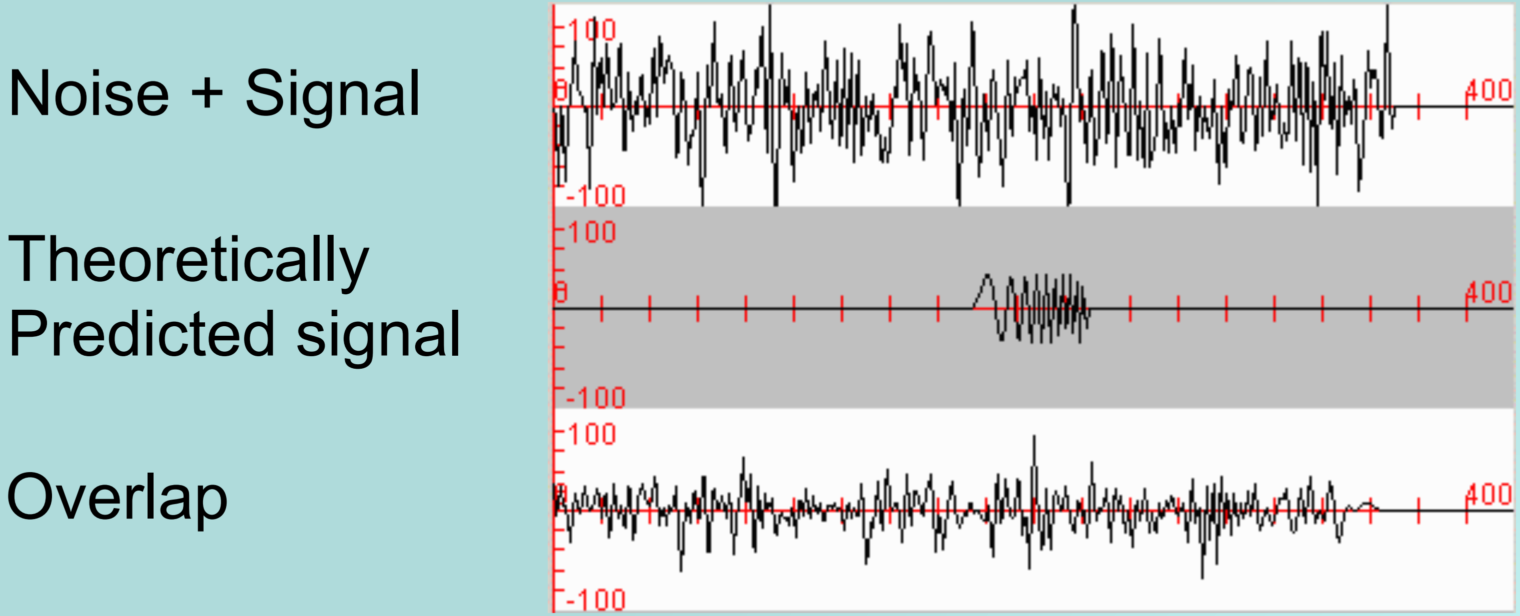
The data stream: Strong LISA source

- SMBH binary



The data stream: Matched filtering

- Matched filtering (not real data)

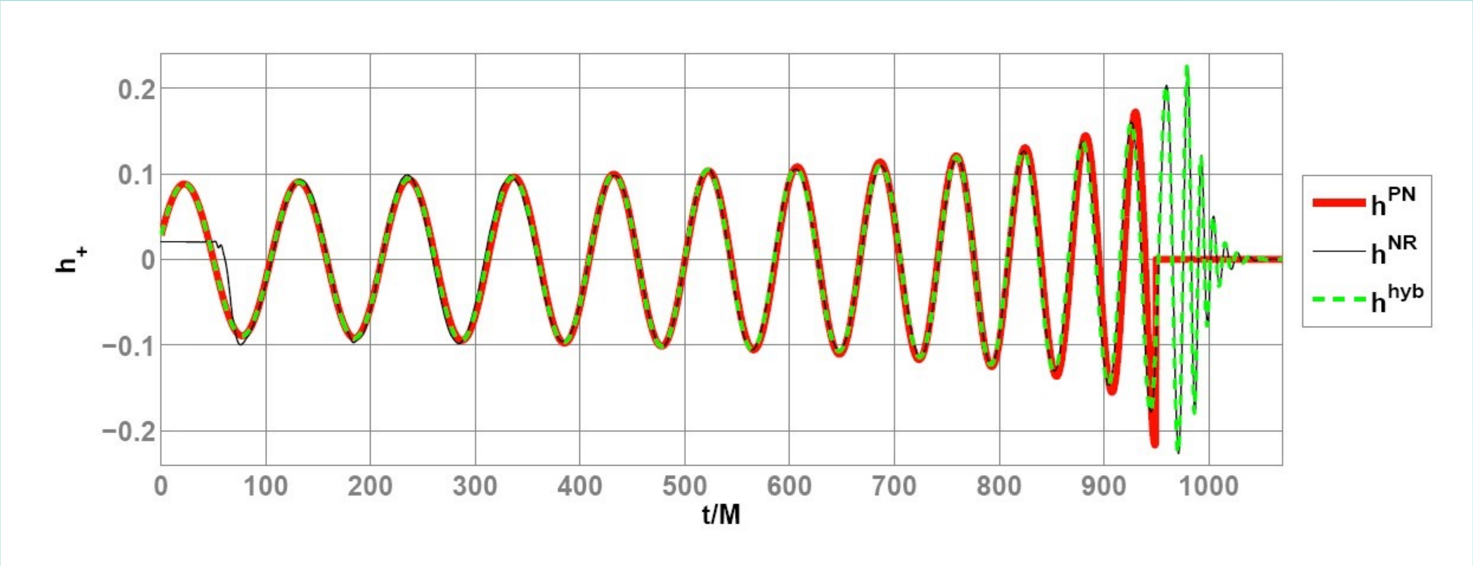


- Filter with one waveform per parameter combination
- Problem: 7-dim parameter space
- We need template banks!

Numerical relativity meets data analysis

Ajith et al. '07

- PN, NR $\Rightarrow$ hybrid waveforms



- Approximate hybrid WFs with phenomenological WFs
- Fitting factors: 0.99
- Create look-up tables to map between phenomenological and physical parameters

Numerical relativity meets data analysis

PSU '07

- Investigate waveforms from spinning binaries
- Detection of spinning holes likely to require inclusion of higher order multipoles

Cardiff '07

- Higher order multipoles important for parameter estimates

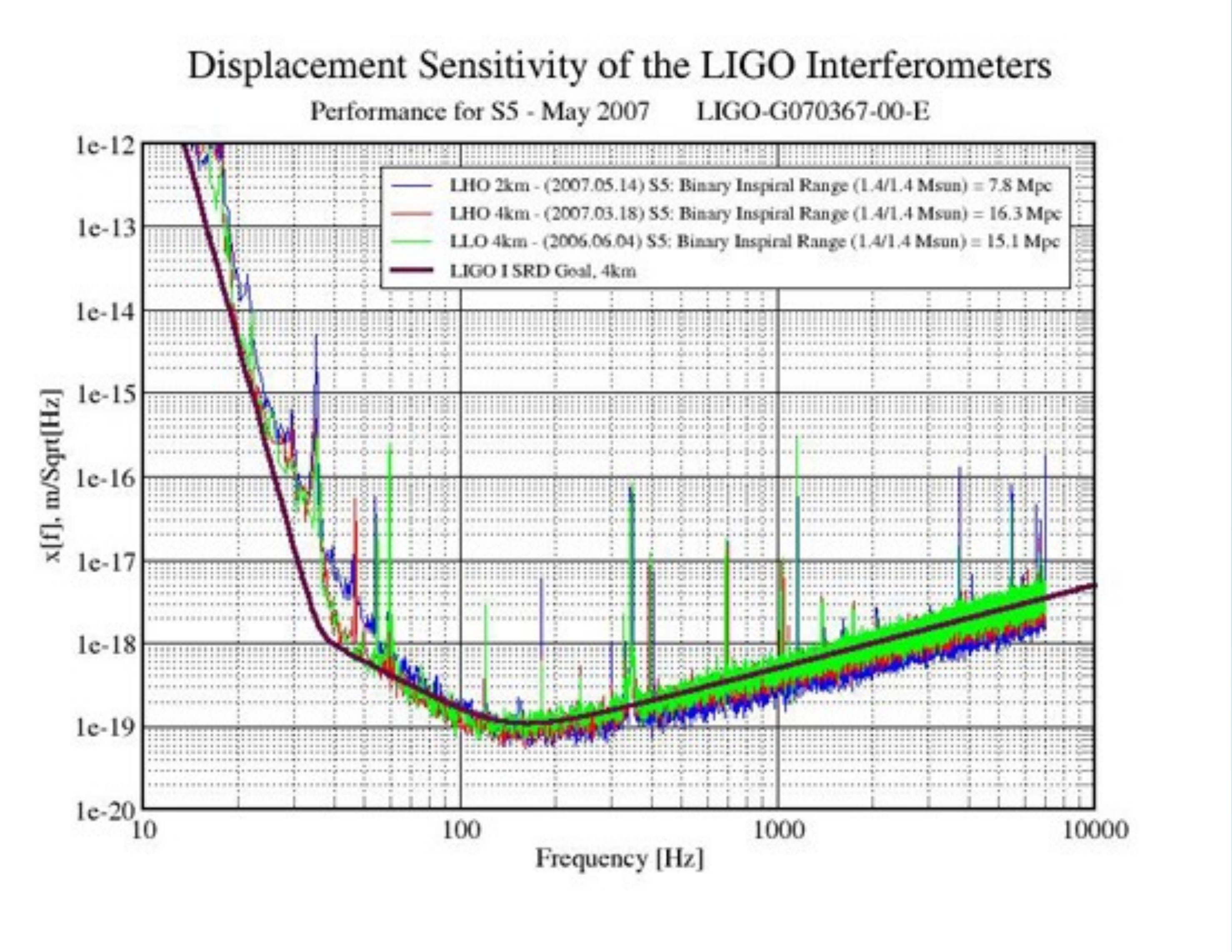
Pan et al. '07

- Equal-mass, non-spinning binaries
- Plot combined waveforms for different masses

Ninja

- Large scale effort to use NR in DA

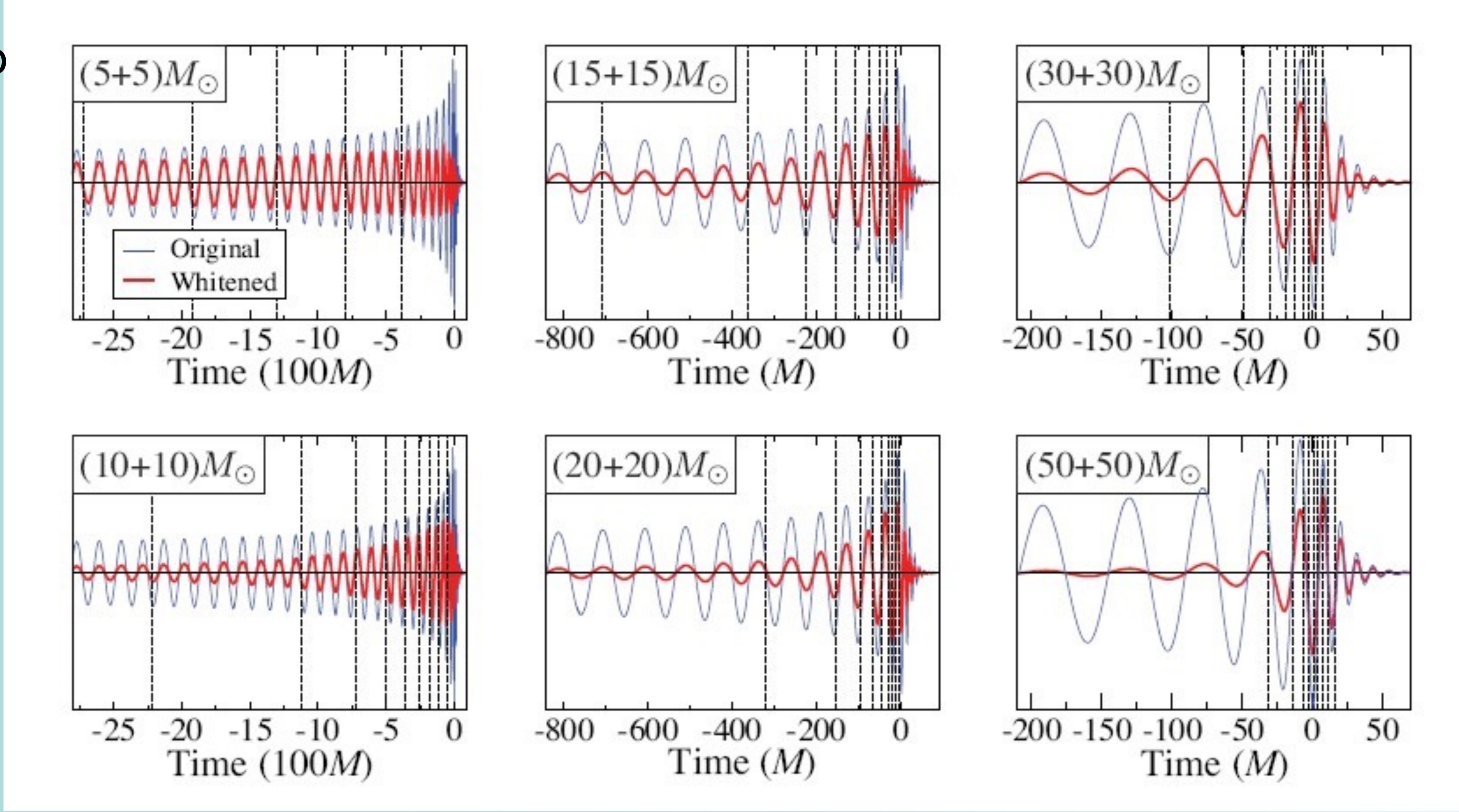
Noise curves



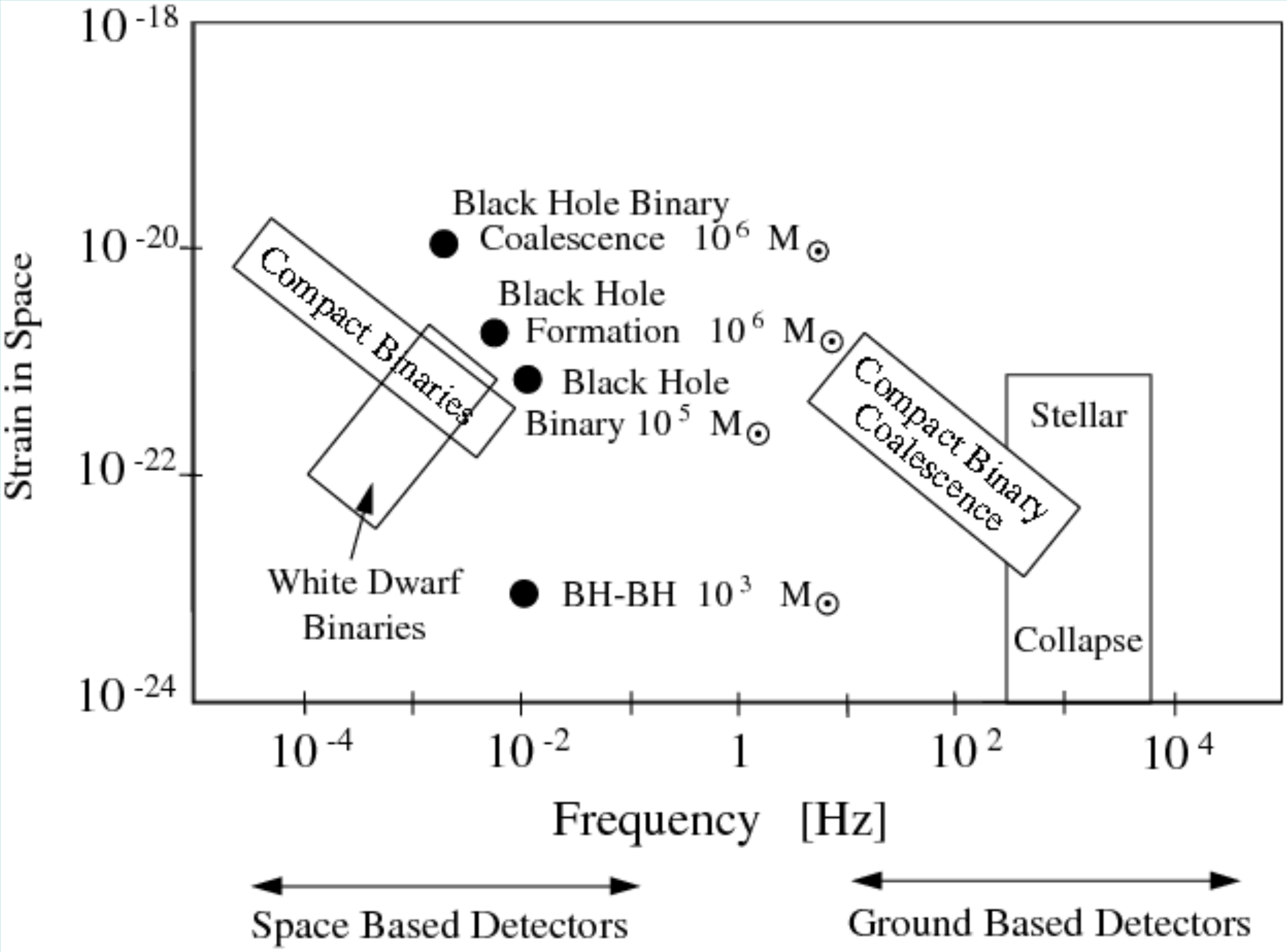
Size doesn't matter… or does it?

- $10\,M_{\rm sol}$ Only $50\,\%$ in last 25 cycles plus Merger and RD
- $20\,M_{\rm sol}$ $> 90\,\%$ in last 23 cycles + MRD
- $30\,M_{\rm sol}$ $> 90\,\%$ in last 11 cycles + MRD NR can do that!
- $100\,M_{\rm sol}$ $> 90\,\%$ in last cycle + MRD Burst!

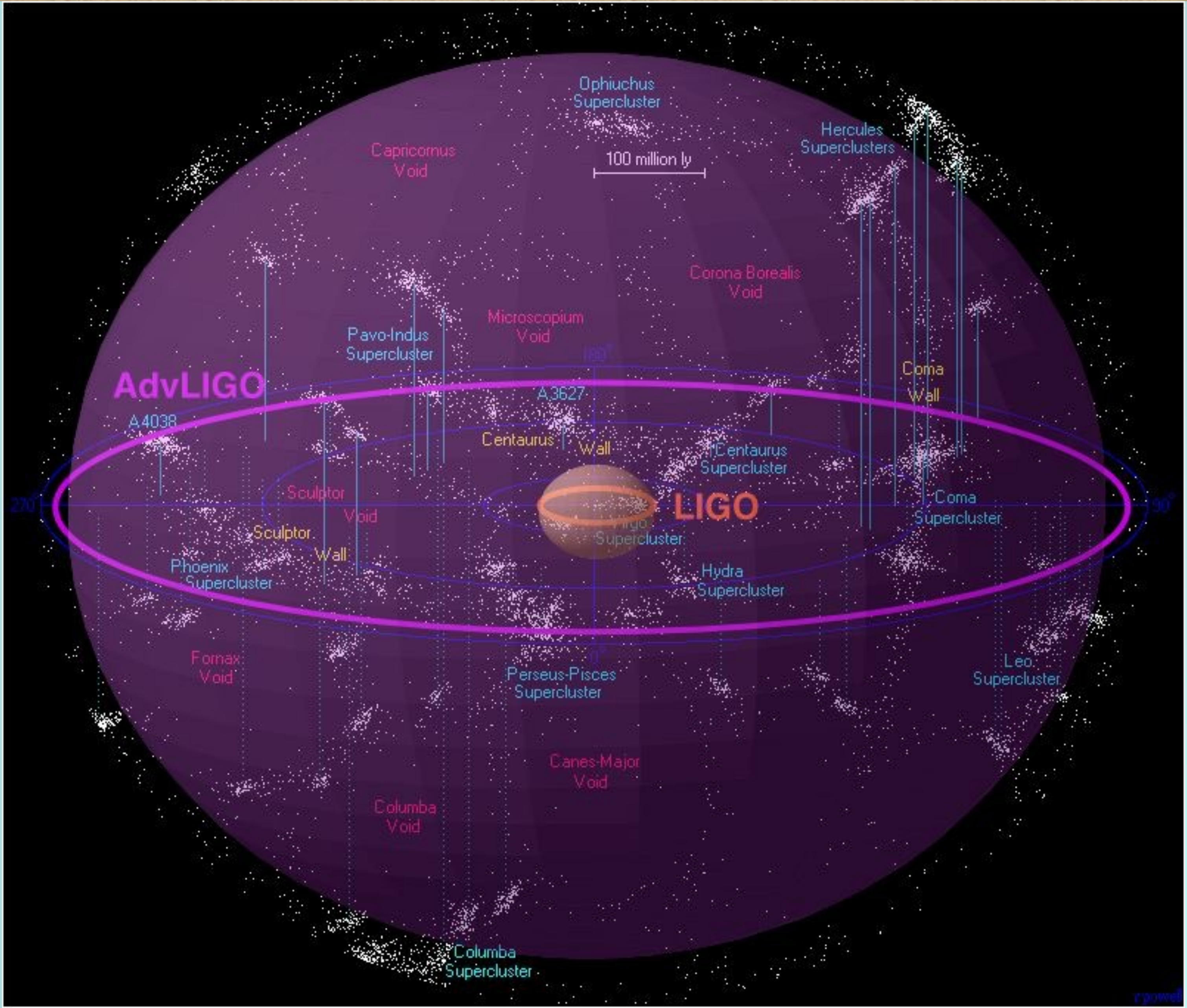
Buonanno et al.'07



Expected GW sources



How far can we observe?



7.4. High energy collisions

Motivation

US, Cardoso, Pretorius, Berti & González '08

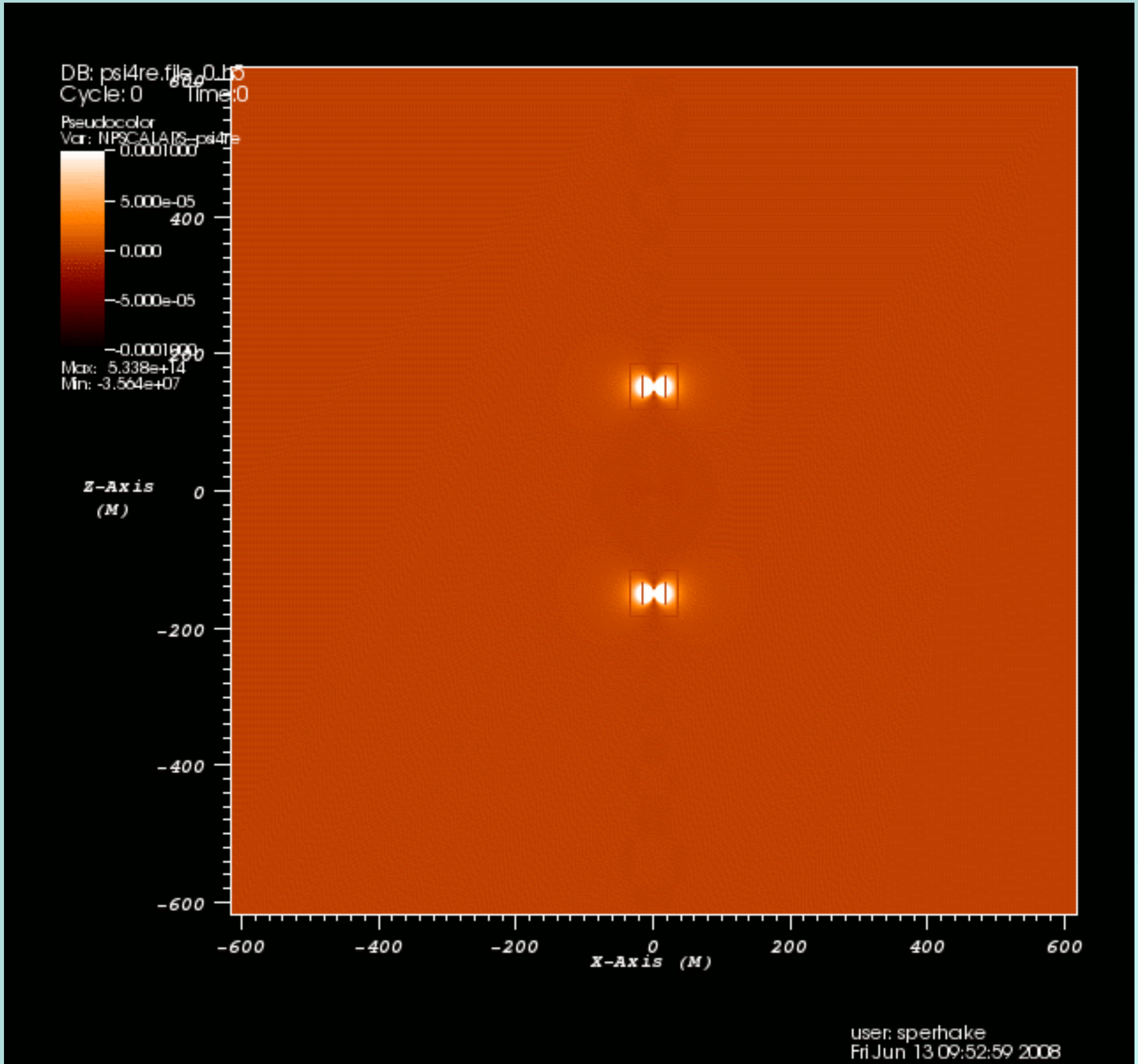
- Head-on collision of BHs near the speed of light
- Test cosmic censorship
- Maximal radiated energy
- First step to estimate GW leakage in LHC collisions
- Model GR in most violent regime
- Numerically challenging

  Resolution, Junk radiation

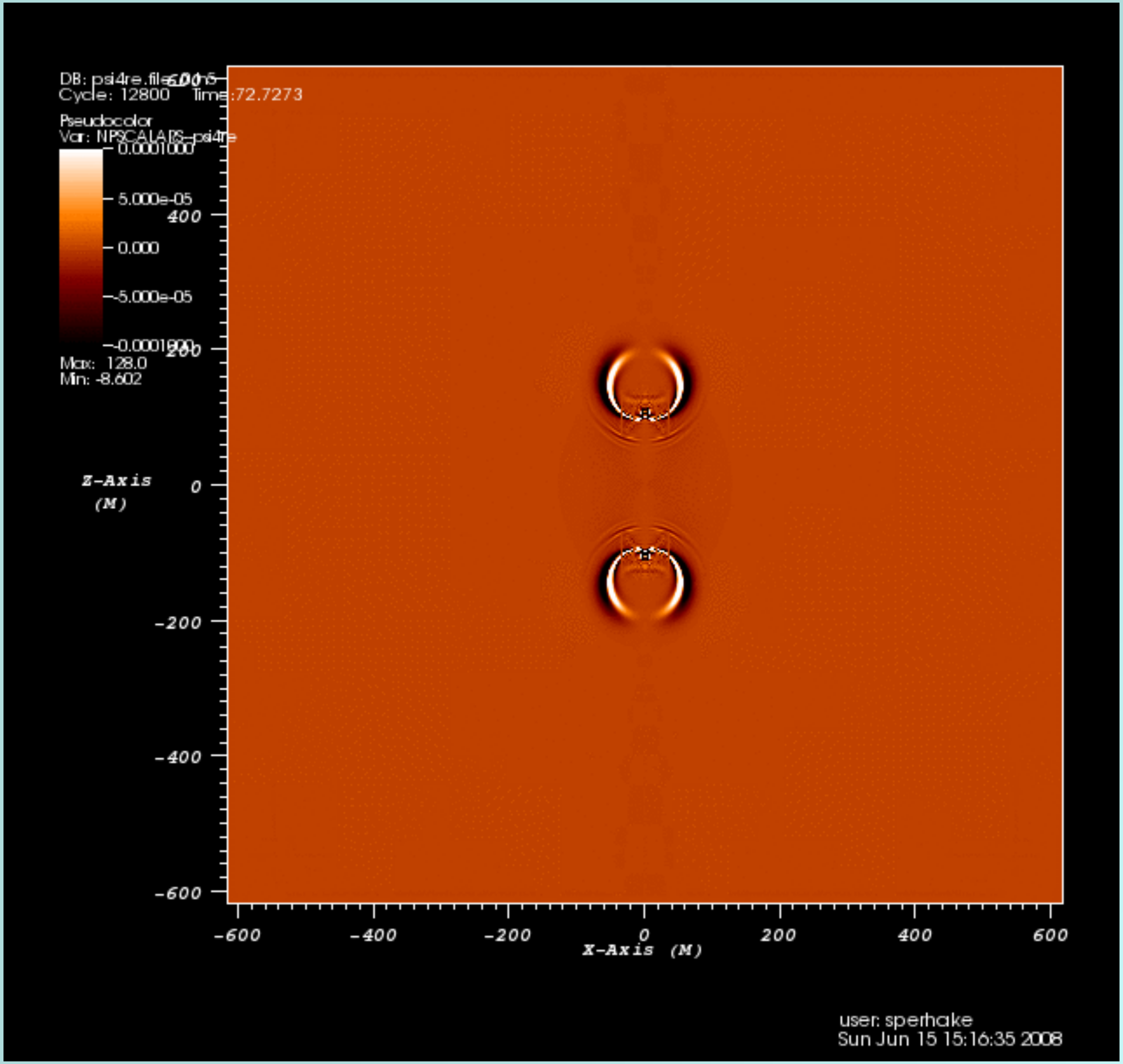
Shibata et al. '08

- Grazing collisions, cross sections
- Radiated energy even larger

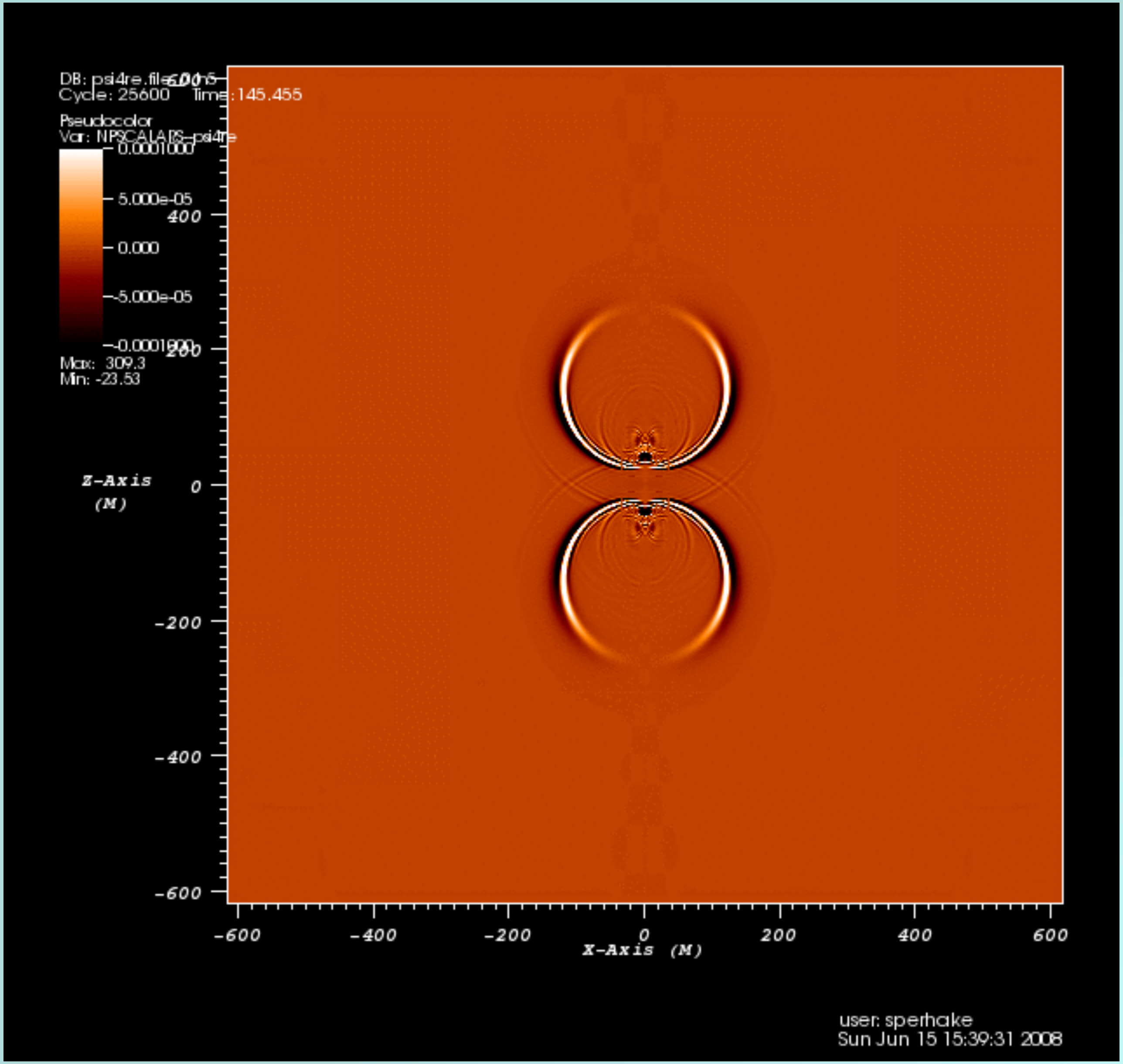
Example: Head-on with $\gamma = 2.75$



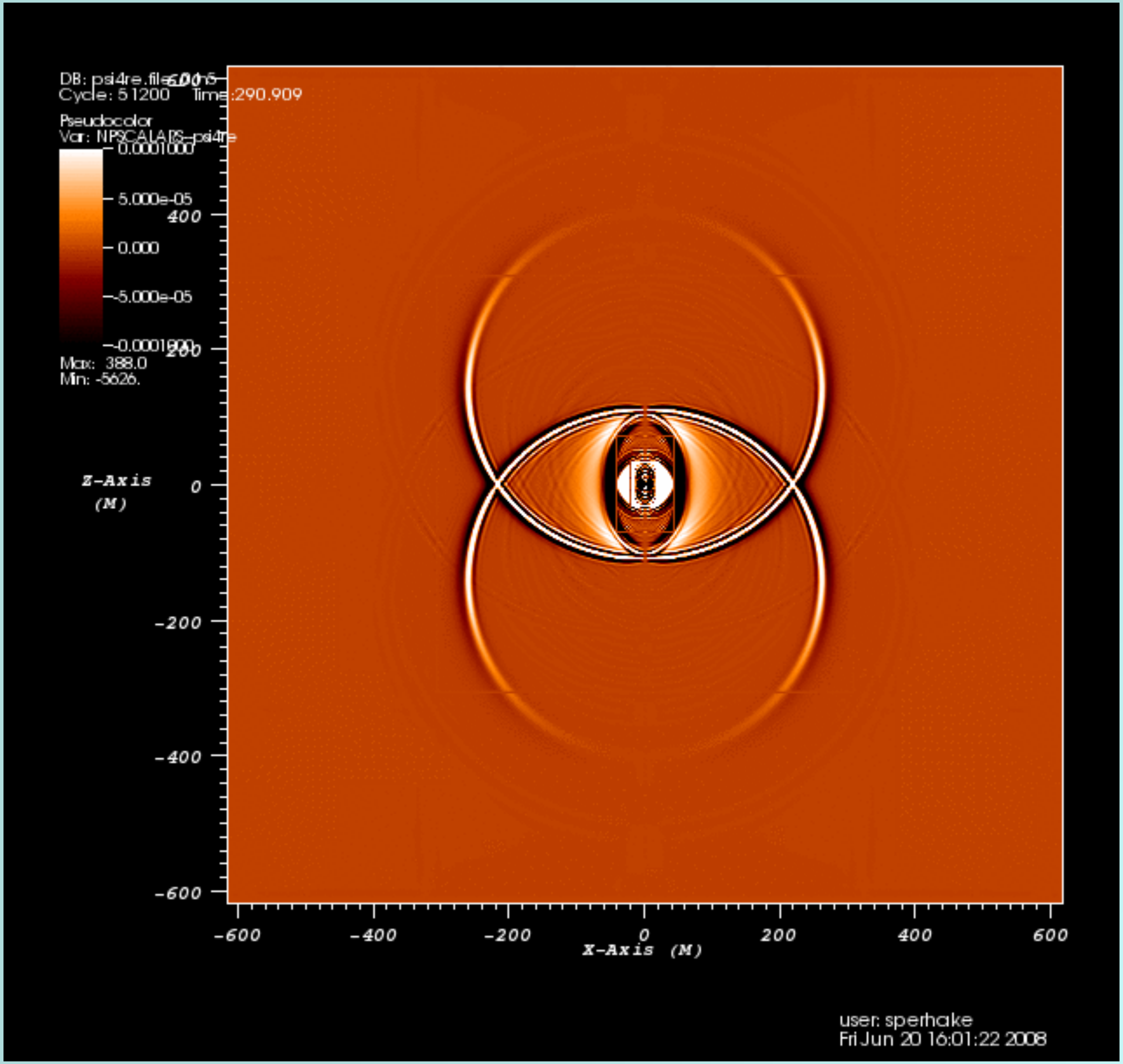
Example: Head-on with $\gamma = 2.75$



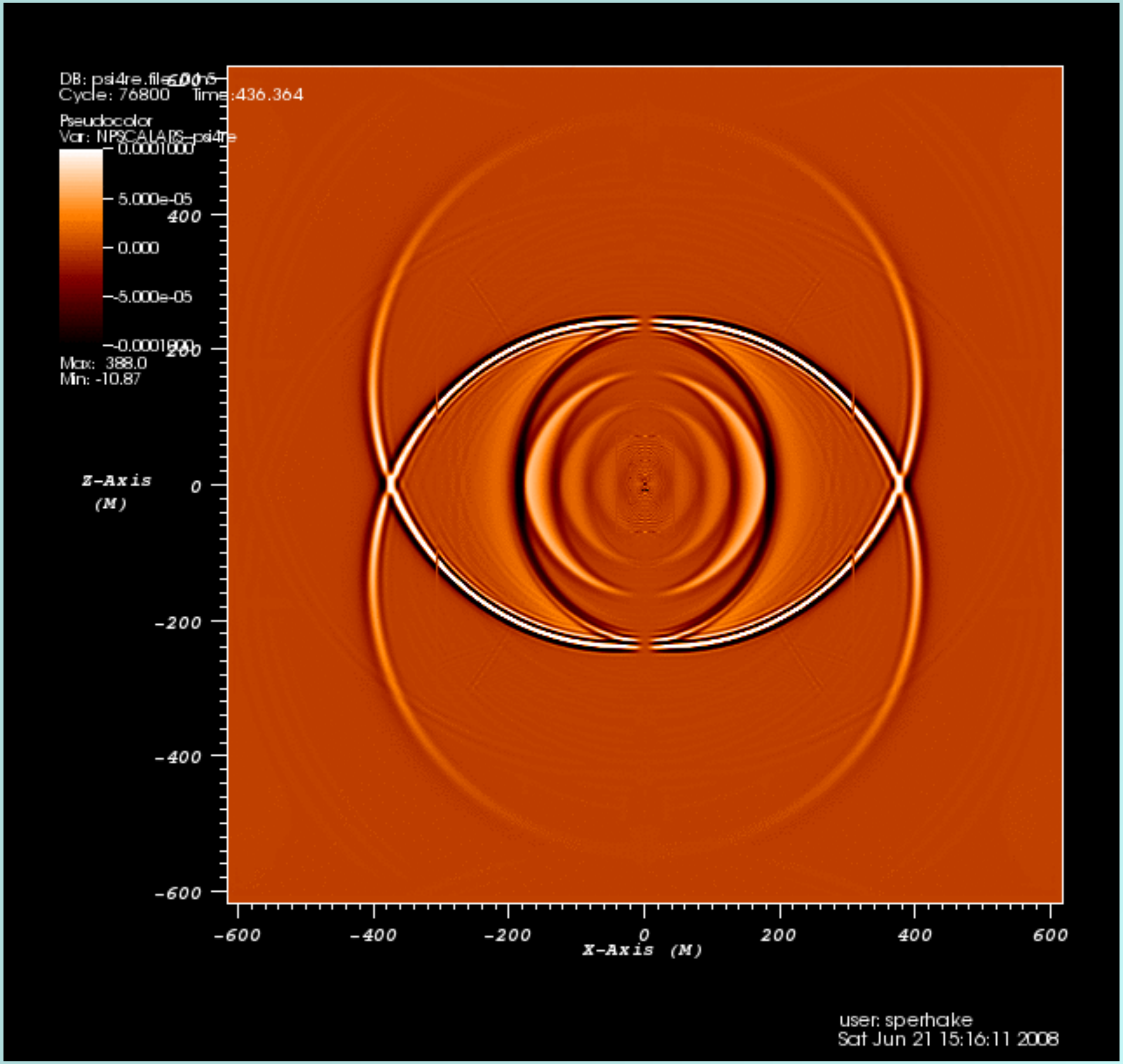
Example: Head-on with $\gamma = 2.75$



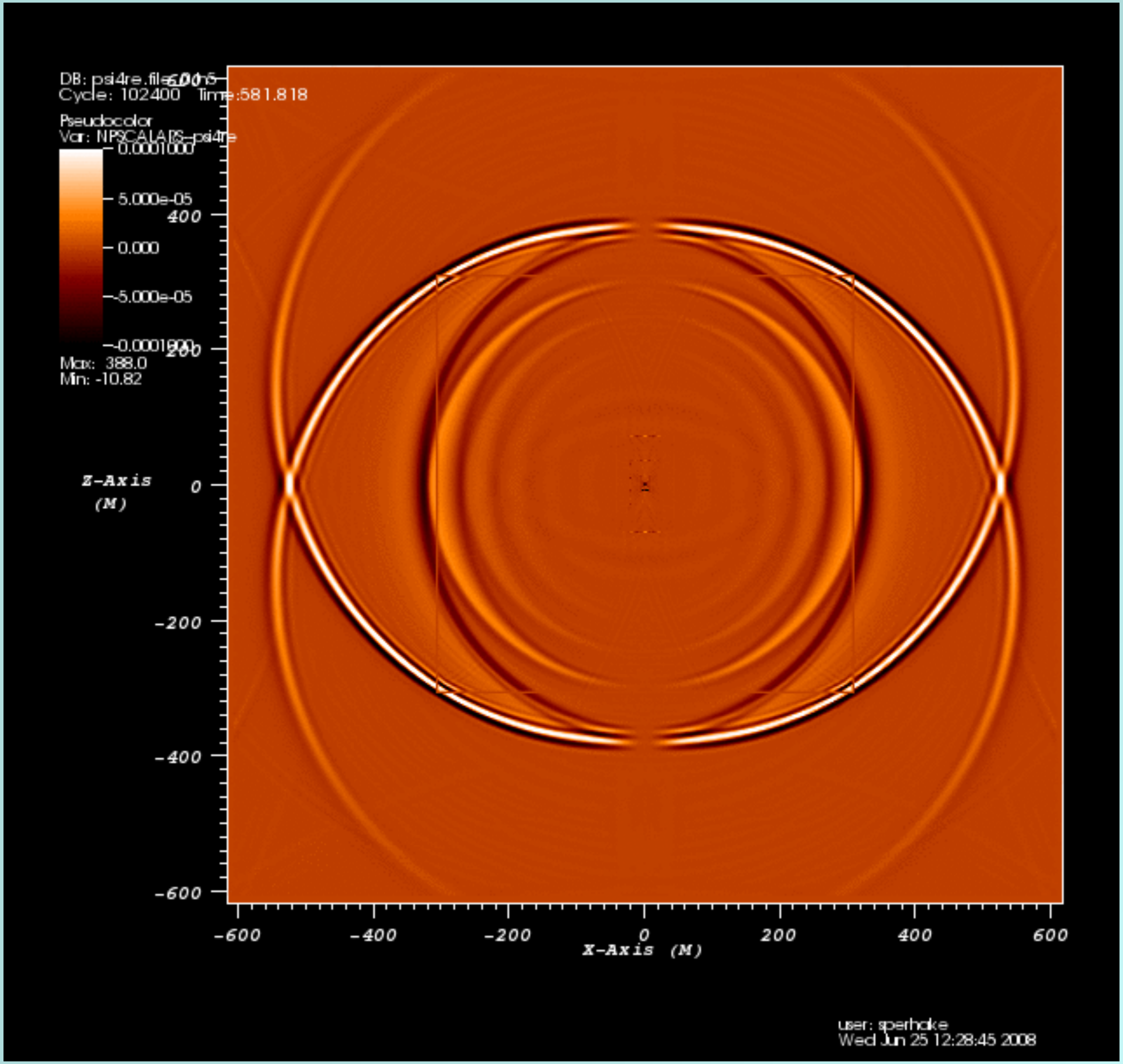
Example: Head-on with $\gamma = 2.75$



Example: Head-on with $\gamma = 2.75$

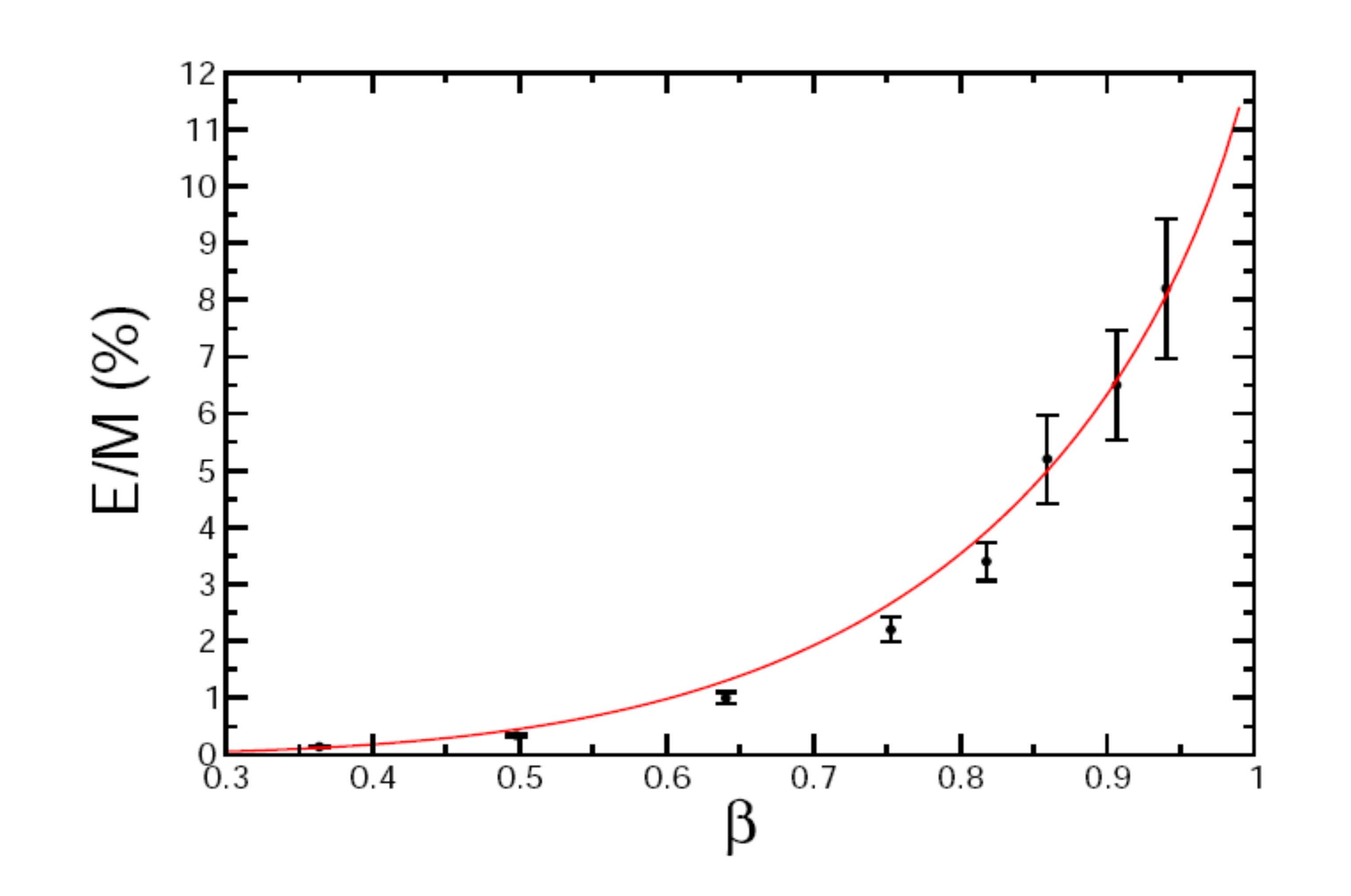


Example: Head-on with $\gamma = 2.75$



Total radiated energy

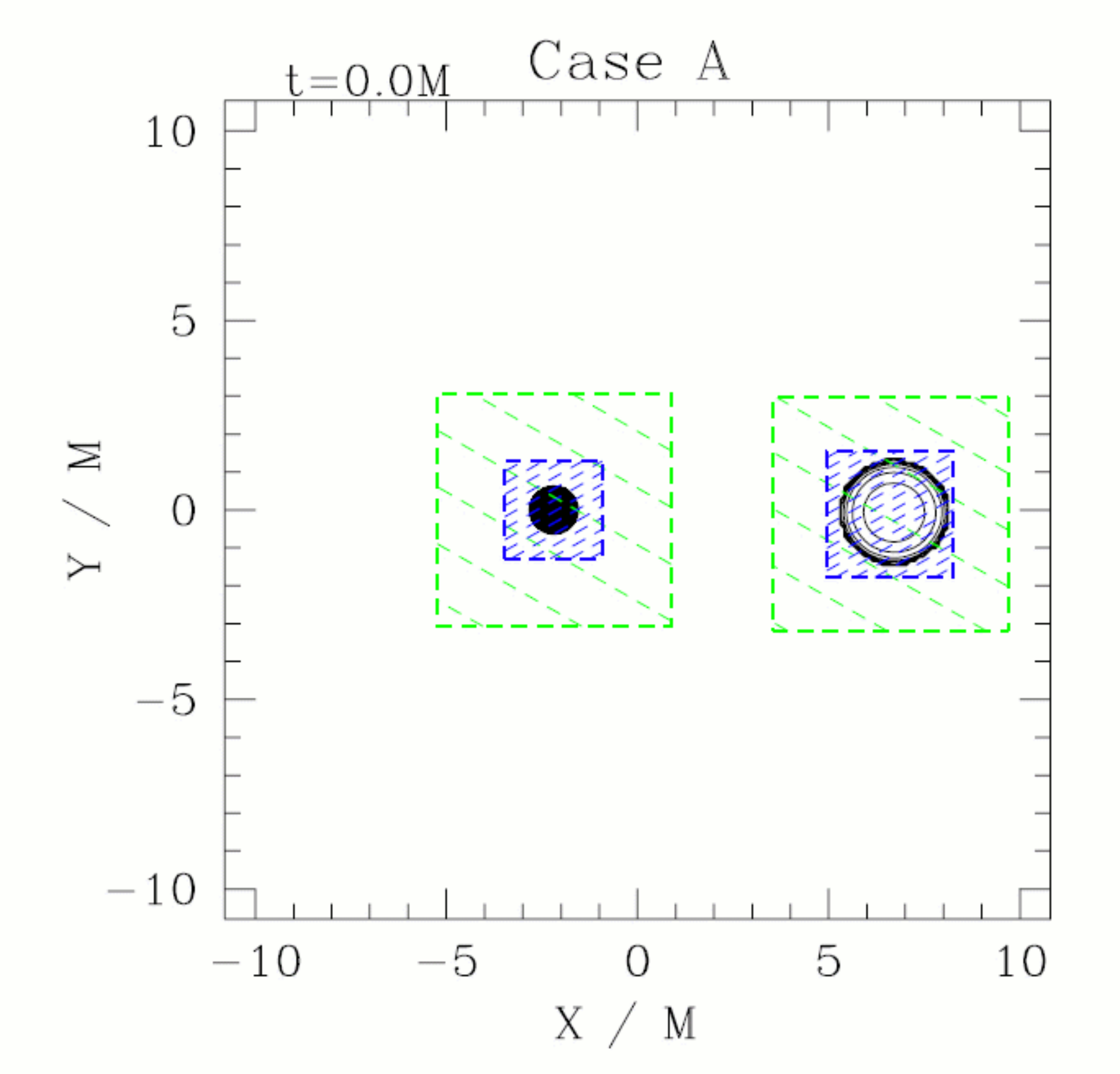
- Total radiated energy: $14 \pm 3\,\%$ about half of Penrose's limit



7.5. Neutron star – BH binaries

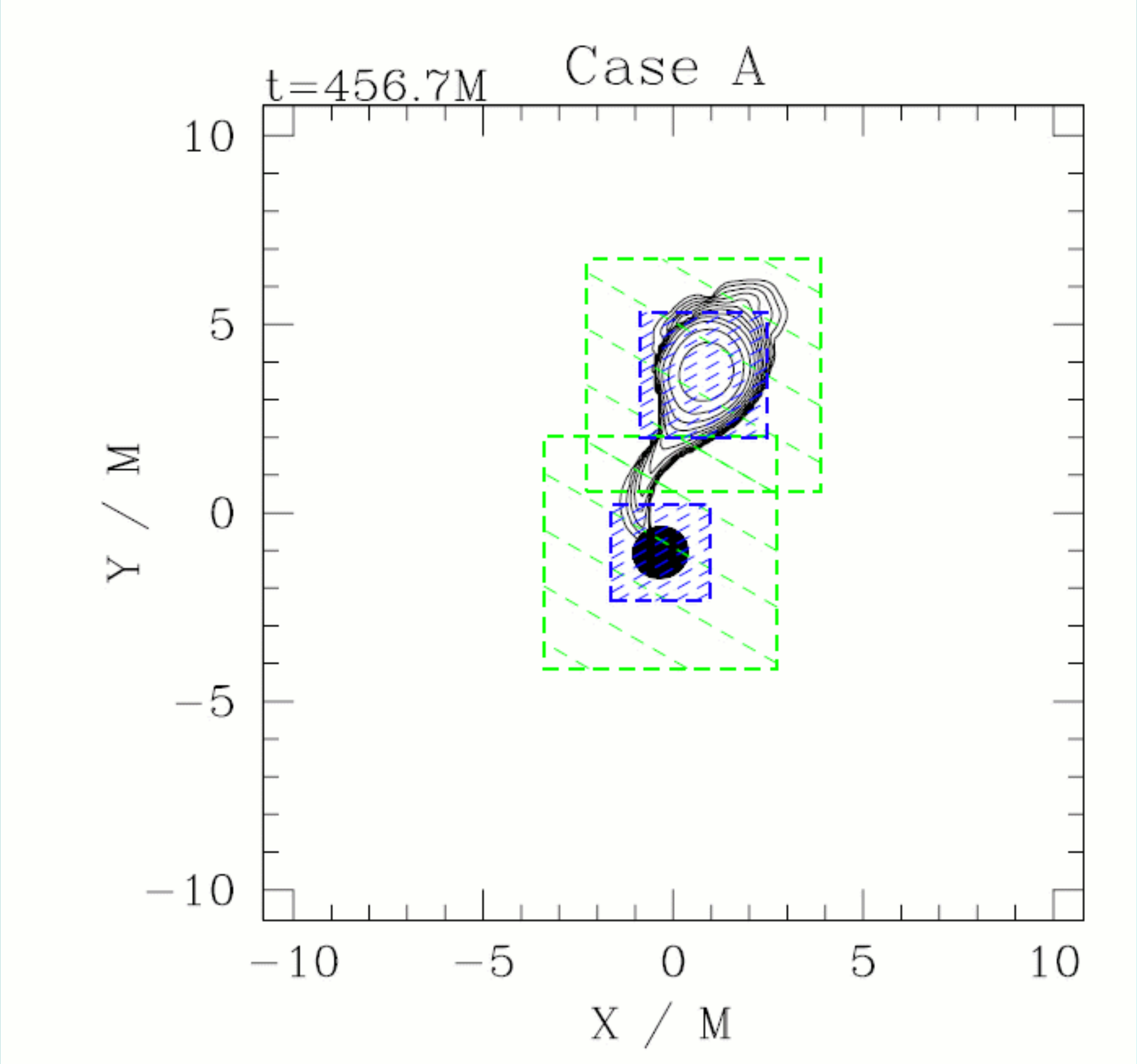
Neutron star is disrupted

Etienne et al. '08



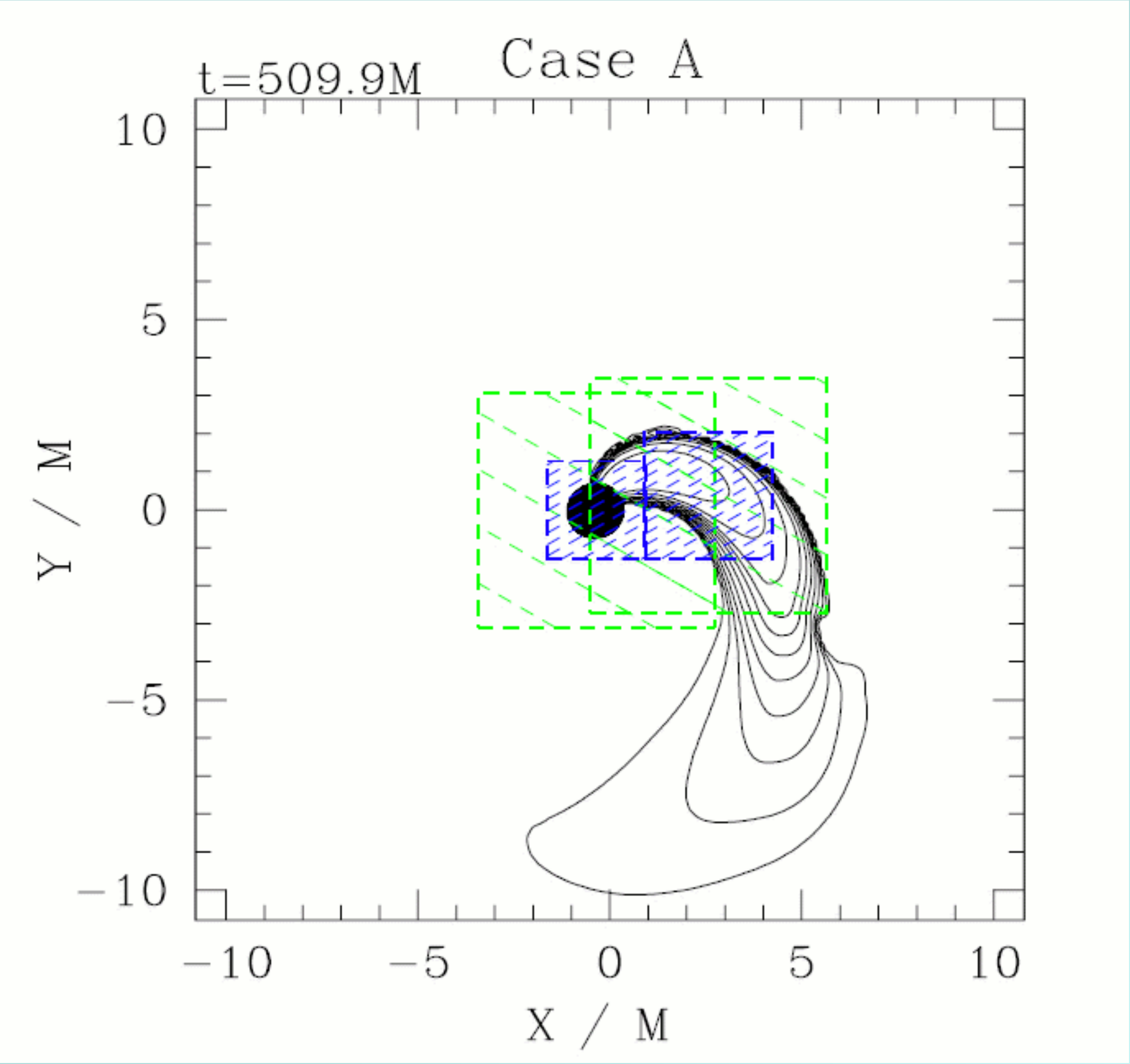
Neutron star is disrupted

Etienne et al. ‘08



Neutron star is disrupted

Etienne et al. '08



Waveforms

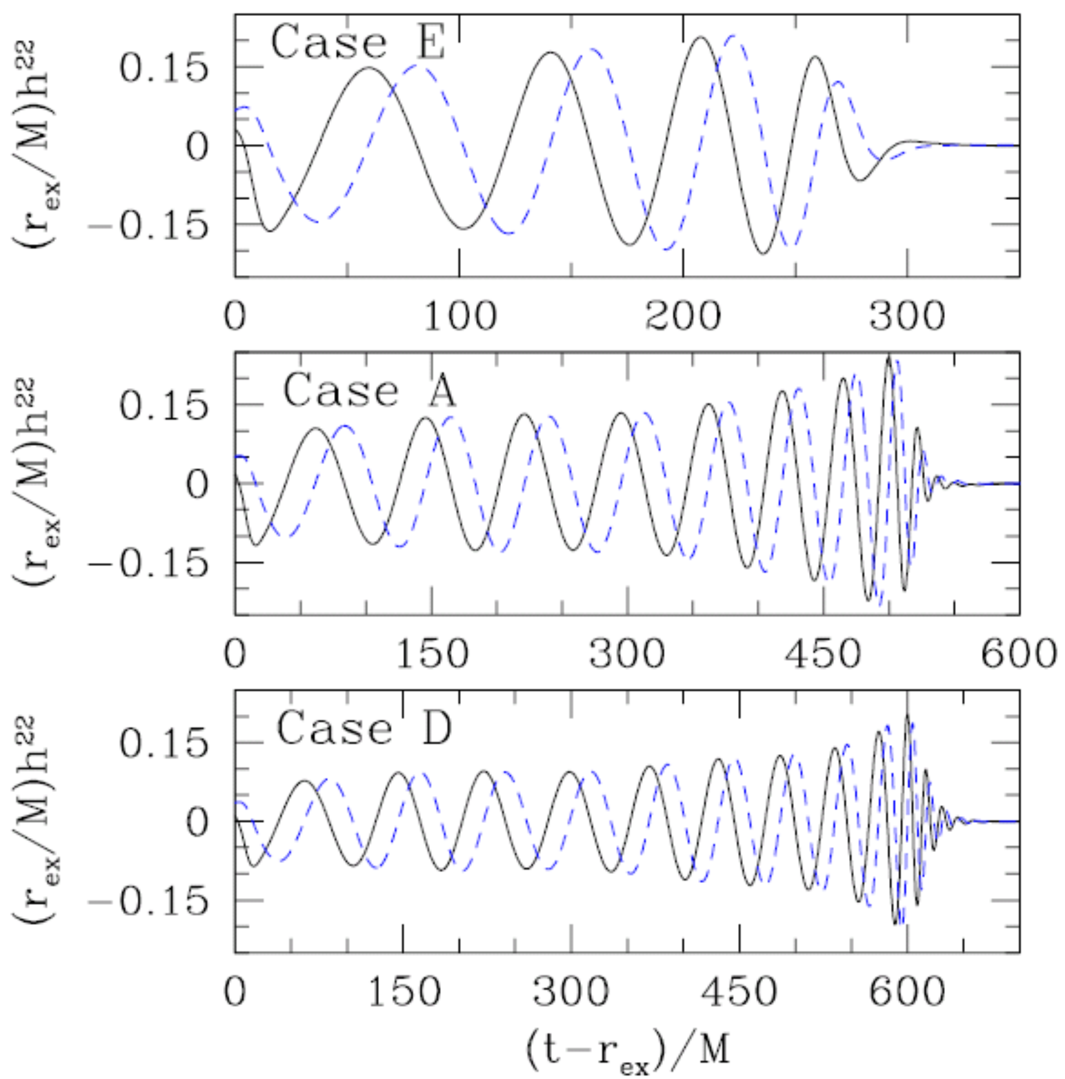
Etienne et al. ‘08

- Ringdown depends on mass ratio

  $q = 1, 3, 5$

- Active research area:

  UIUC, AEI, Caltech/Cornell



Future research

Main future research directions

- Gravitational wave detection
  - PN comparisons with spin
  - Generate template banks
  - Understand how to best generate/use hybrid wave forms
  - Simulate extreme mass ratios
- Astrophysics
  - Distribution functions for Kick, BH-spin, BH-mass
  - Improve understanding of Accretion, GW bursts,…
- Fundamental physics
  - High energy collisions: radiated energy, cross sections
  - Higher dimensional BH simulations