Core collapse in scalar-tensor theory of gravity

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arXiv:1505.07462

D. Gerosa, C. Ott, US work in preparation

Gravity @ all scales

Nottingham, 27^{th} August 2015

Science & Technology Facilities Council

MARIE CURIE ACTIONS

NSF

Overview

- Introduction
- Formalism
- Neutron stars in bi-STT
- Core collapse in STT
- Conclusions

1. Introduction

Scalar-tensor theories of gravity

- Extra degree(s) of freedom ϕ^A additionally to $g_{\mu\nu}$
  - Appear in low-energy limit of string theories
  - Kaluza-Klein like models
  - Braneworld scenarios
- Historically: time-space dependent $G_{\rm Newton}$

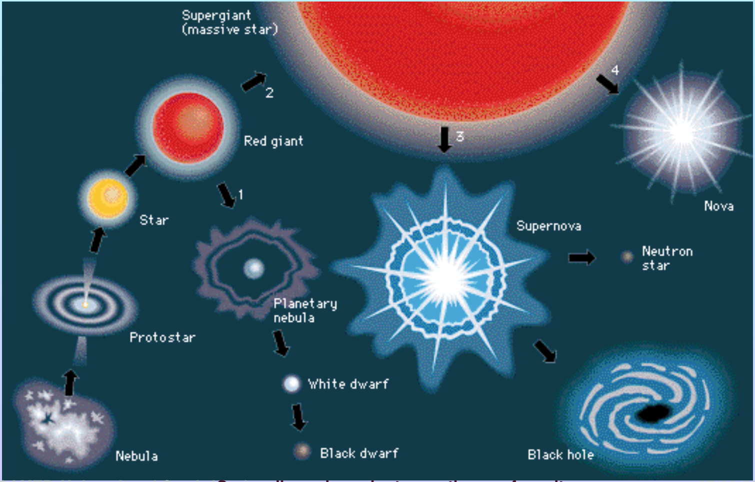
  Jordan '59, Fierz '56, Brans & Dicke '61
- Candidate for explaining the dark sector in cosmology, inflation
- Many alternative theories can be formulated as ST theories
- No-hair theorems for BHs

  $\Rightarrow$ matter sources often more sensitive to ST effects

  E.g. spontaneous scalarization Damour & Esposito-Farese '93

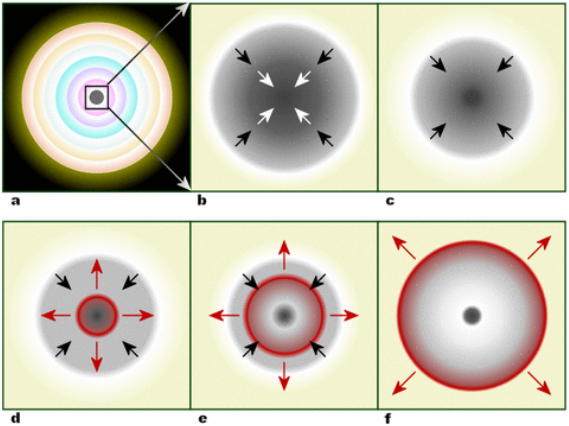
The end of stellar evolution

- Nuclear fusion above iron: energy consuming
- Stars with $M_{\rm ZAMS} \gtrsim 8\ M_{\odot}$ explode as SN $\rightarrow$ NS, BH



Core-collapse scenario (0^{th}-order)

- Ni-Fe core reaches Chandrasekhar mass $\rightarrow$ Collapse
- EOS stiffens at $\rho \gtrsim \rho_{\text{nuc}}$ $\rightarrow$ Bounce
- Outgoing shock, re-invigorated by ν_e $\rightarrow$ Outer layers blast away



2. Formalism

Notation

φ^A	Scalar field(s)
γ_{AB}	Target space metric
γ^A_{BC}	Target space Christoffel symbols
$g_{\mu\nu}$	Physical spacetime metric
$\bar{g}_{\mu\nu}$	Spacetime metric in the Einstein frame
ds^2	Physical line element
$d\bar{s}^2$	Line element in the Einstein frame
$a(\varphi^A)^2$	Conformal factor: $g_{\mu\nu} = a^2(\varphi^A)\bar{g}_{\mu\nu}$
∂_A	$\equiv \frac{\partial}{\partial\varphi^A}$

In general: bar $\rightarrow$ Einstein frame variable

no bar $\rightarrow$ Jordan frame variable

Action and equations

cf. Damour & Esposito-Farese CQG **9**, 2093

$$S = \frac{c^4}{4\pi\bar{G}}\int\frac{dx^4}{c}\sqrt{-\bar{g}}\left[\frac{\bar{R}}{4}-\frac{1}{2}\bar{g}^{\mu\nu}\gamma_{AB}(\partial_\mu\varphi^A)(\partial_\nu\varphi^B)+W(\varphi^A)\right]$$

$$+S_m[\psi_m, a^2(\varphi^A)\bar{g}_{\mu\nu}]$$

Energy momentum tensor:

$$T^{\mu\nu} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta S_m(\psi_m, g_{\mu\nu}}{\delta g_{\mu\nu}}$$

$$\bar{T}^{\mu\nu} = a^6 T^{\mu\nu}$$

$$\Rightarrow \quad \bar{R}_{\mu\nu} = 2\gamma_{AB}(\partial_\mu\varphi^A)(\partial_\nu\varphi^B)+2W(\varphi^A)\bar{g}_{\mu\nu}+\frac{8\pi\bar{G}}{c^4}\left(\bar{T}_{\mu\nu}-\frac{1}{2}\bar{T}\bar{\mu\nu}\right)$$

$$\bar{\Box}\varphi^A = -\gamma^A_{BC}\,\bar{g}^{\mu\nu}(\partial_\mu\varphi^B)(\partial_\nu\varphi^C)-\frac{4\pi\bar{G}}{c^4}\gamma^{AB}\frac{1}{a}(\partial_B a)\bar{T}+\gamma^{AB}\partial_B W$$

$$\bar{\nabla}_\nu\bar{T}^{\mu\nu} = \frac{1}{a}(\partial_A a)\bar{T}\bar{\nabla}^\mu\varphi^A$$

Spherically symmetric stars

- Line element

$$ds^2 = -\alpha^2 dt^2 + X^2 dr^2 + a^2 r^2 d\Omega^2 \,, \qquad d\bar{s}^2 = -\bar{\alpha}^2 dt^2 + \bar{X}^2 dr^2 + r^2 d\Omega^2$$

Auxiliary variables

$$\bar{m} \equiv \frac{r}{2}\left(1 - \frac{a^2}{X^2}\right) \,, \qquad \bar{\Phi} \equiv \ln\left(\frac{\alpha}{a}\right) \,,$$

$$\eta^A = \frac{\partial_r \varphi^A}{X} \,, \qquad \psi^A = \frac{\partial_t \varphi^A}{\alpha} \,, \qquad \Xi = \gamma_{AB}(\eta^A \eta^B + \psi^A \psi^B)$$

- Matter

$$T_{\alpha\beta} = (\epsilon + \rho + p) u_\alpha u_\beta + P g_{\alpha\beta}$$

$$u^\alpha = \frac{1}{\sqrt{1 - v^2}} \left[\frac{1}{\alpha},\ \frac{v}{X},\ 0,\ 0\right]$$

$J^\alpha = \rho u^\alpha$ “baryonic flow” satisfies $\nabla_\mu J^\mu = 0$

Spherically symmetric stars

- Equation of state $P = K\rho^{\Gamma}\,, \quad \epsilon = \frac{P}{\rho(\Gamma-1)}$

  We typically use: $\Gamma = 2.34\,, \quad K = 1187 \quad (c = M_{\odot} = 1)$

  EOS1 in Novak gr-qc/9707041

- Flux conservative variables

  $$\bar{D} = \frac{a^3 \rho X}{\sqrt{1-v^2}}$$

  $$\bar{S}^r = \frac{a^4[\rho(1+\epsilon)+P]v}{1-v^2}$$

  $$\bar{\tau} = \frac{a^4[\rho(1+\epsilon)+P]}{1-v^2} - a^4 P - \bar{D}$$

The equations: Metric and scalar field

- $$\partial_r \Phi = \frac{X^2}{a^2}\left[\frac{\bar{m}}{r^2} + 4\pi r\left(\bar{S}^r\, v + a^4 P\right) + \frac{a^2 r}{2}\Xi\right],$$

  $$\partial_r \bar{m} = 4\pi r^2 (\bar{\tau} + \bar{D}) + \frac{a^2 r^2}{2}\Xi\,,$$

- $$\partial_t \phi^A = \alpha \psi^A\,,$$

  $$\partial_t \eta^A = -\eta^A \frac{\partial_t X}{X} + \frac{\alpha}{X}\left(\partial_r \psi^A + \psi^A \frac{\partial_r \alpha}{\alpha}\right),$$

  $$\partial_t \psi^A = \frac{\alpha}{X}\left[\partial_r \eta^A + \frac{2}{r}\eta^A + \eta^A \frac{\partial_r \alpha}{\alpha}\right] - \psi^A \frac{\partial_t X}{X} - \alpha \gamma^A_{BC}(\psi^B \psi^C - \eta^B \eta^C)$$
  $$-4\pi\alpha\left(\bar{\tau} - \bar{S}^r\, v + \bar{D} - 3a^4 P\right)\gamma^{AB}\frac{\partial_B a}{a^2}$$

The equations: Matter variables

- $\partial_t \bar{D} + \frac{a}{r^2}\partial_r\left(r^2 \frac{\alpha}{aX} f_{\bar{D}}\right) = s_{\bar{D}}\,,\qquad f_{\bar{D}} = \bar{D} v\,,$

  $\partial_t \bar{S}^r + \frac{1}{r^2}\partial_r\left(r^2 \frac{\alpha}{X} f_{\bar{S}^r}\right) = s_{\bar{S}^r}\,,\qquad f_{\bar{S}^r} = \bar{S}^r v + a^4 P\,,$

  $\partial_t \bar{\tau} + \frac{1}{r^2}\partial_r\left(r^2 \frac{\alpha}{X} f_{\bar{\tau}}\right) = s_{\bar{\tau}}\,,\qquad f_{\bar{\tau}} = \bar{S}^r - \bar{D} v\,.$

- Flux conservative form!

  The source terms $s_{\bar{D}},\ s_{\bar{S}^r},\ s_{\bar{\tau}}$ contain no derivatives.

- Suitable for high-resolution shock-capturing methods

  extension of GR1D O'Connor & Ott 0912:2393 [astro-ph]

The static limit $\rightarrow$ TOV models, initial data

- All time derivatives vanish
- Relation $(\bar{D},\ \bar{S}^r,\ \bar{\tau}) \leftrightarrow (\rho,\ \epsilon,\ v)$ trivial as $v = 0$
- Gives system of 5 ODEs for $(\alpha,\ X,\ P,\ \varphi^A,\ \eta^A)$
- Boundary conditions

  At $r = 0$: $X = 1,\ \rho = \rho_c,\ \eta^A = 0$

  At $r = r_S$: $P = 0$

  At $r \rightarrow \infty$: $\varphi^A = 0$ (wlog)

3. Neutron stars in multi-ST theories

Specifying the theory

- Target geometry: maximally symmetric

$$\gamma_{AB} = \delta_{AB}\left[1 + \frac{(\varphi^1)^2 + (\varphi^2)^2}{4\boldsymbol{r}^2}\right]$$

spherical: $\boldsymbol{r}^2 > 0$

hyperbolic: $\boldsymbol{r}^2 < 0$

flat: $\boldsymbol{r}^2 \to \infty$

- Conformal factor

$$\log a = 2\alpha_0\varphi^1 - 2\alpha_1\varphi^2 + \frac{1}{2}(\beta_0 + \beta_1)(\varphi^1)^2 + \frac{1}{2}(\beta_0 - \beta_1)(\varphi^2)^2$$

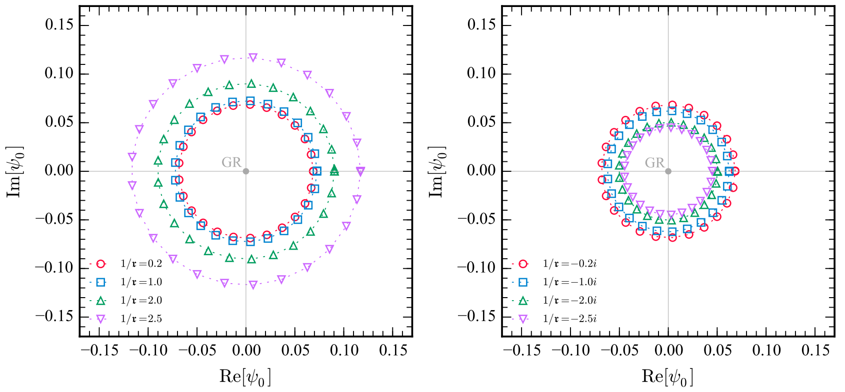
- Complex scalar field: $\varphi = \varphi^1 + i\varphi^2$

- Free parameters: $\alpha_0,\ \alpha_1,\ \beta_0,\ \beta_1,\ \boldsymbol{r}$

Case 1: $\alpha_0 = \alpha_1 = \beta_1 = 0, \quad \beta_0 = -5$

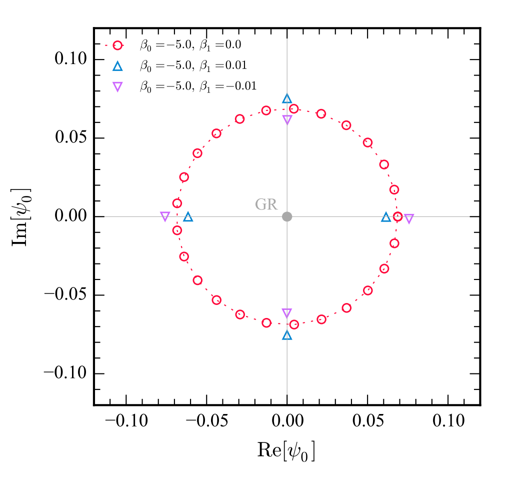
- $O(2)$ symmetry: Invariance under rotation in φ^1, φ^2 plane
- Spherical (hyperbolic) target geometry

  $\Rightarrow$ scalarization strengthened (weakened)



Case 2: $\alpha_0 = \alpha_1 = 0, \quad \beta_0 = -5, \ \beta_1 \neq 0$

- No bi-scalarized solutions! "Circle" $\rightarrow$ "Cross"
- $r = 5$



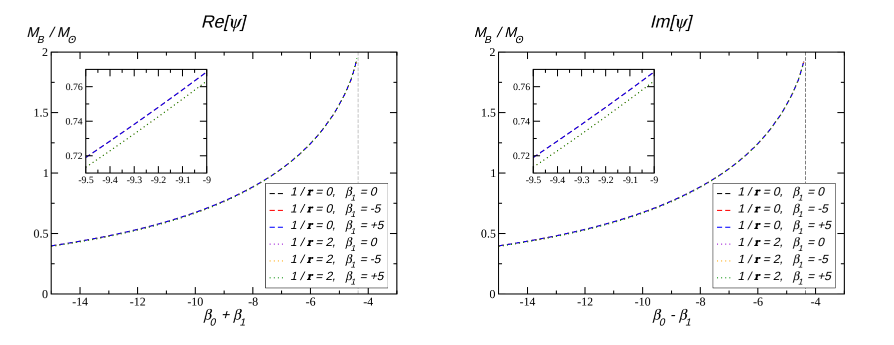
Case 2: Scalarization for $\beta_0 \pm \beta_1 \lesssim -4.35$

- Spontaneous scalarization for single-STT if $\beta \lesssim -4.35$

  Damour & Esposito-Farese '93

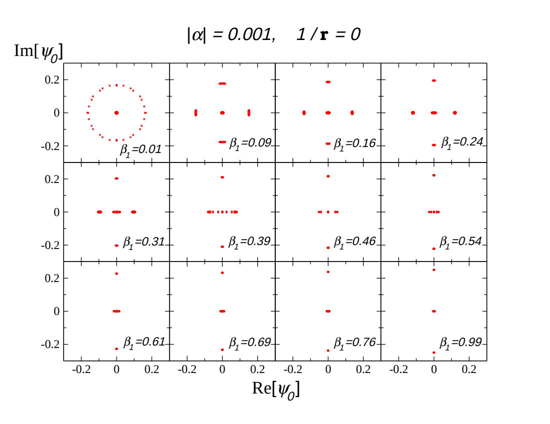
- Here: $\log a = 2\alpha_0\varphi^1 - 2\alpha_1\varphi^2 + \frac{1}{2}(\beta_0 + \beta_1)(\varphi^1)^2 + \frac{1}{2}(\beta_0 - \beta_1)(\varphi^2)^2$

  $\rightarrow$ Like single-STT with $\beta \rightarrow \beta_0 \pm \beta_1$



Case 3: $(\alpha_0, \alpha_1) \neq 0, \quad \beta_0 = -5$

- $\alpha \equiv |(\alpha_0, \alpha_1)|$ constrained; But phase not!
- $\alpha \neq 0$ facilitates bi-scalar solutions



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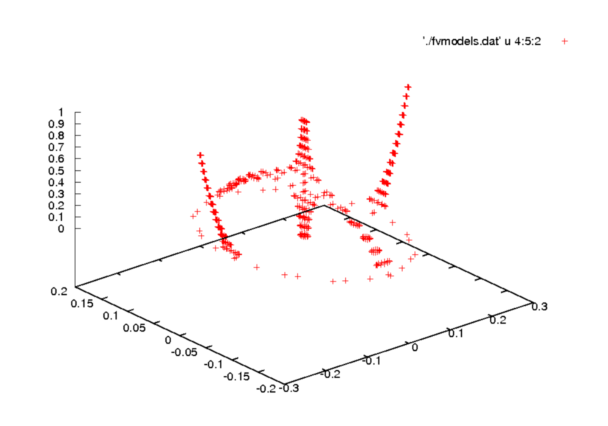
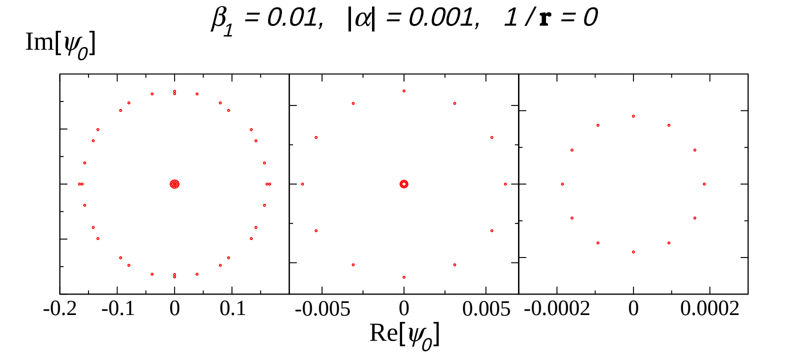


Case 3: $(\alpha_0, \alpha_1) \neq 0, \quad \beta_0 = -5$

- Zoom into upper left panel of above ($\beta_1 = 0.01$)
- Fine structure of (weakly) scalarized solutions



4. Core collapse in single-ST theories

Core collapse

- Massive stars: $M_{\rm ZAMS} \approx 8 \ldots 100 M_\odot$
- Core compressed from $\sim 1500~\mathrm{km}$ to $\sim 15~\mathrm{km}$

  $\sim 10^{10}~\mathrm{g/cm^3}$ to $\sim> 10^{15}~\mathrm{g/cm^3}$
- Released gravitational energy: $\mathcal{O}(10^{53})~\mathrm{erg}$

  $\sim 99~\%$ in neutrinos, $\sim 10^{51}~\mathrm{erg}$ in outgoing shock, explosion
- Explosion mechanism: still uncertainties...
- Failed explosion $\Rightarrow$ BH formation

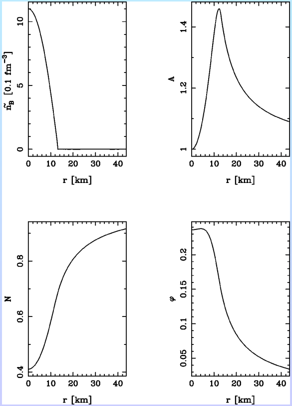
  Collapsar possible engine for long-soft GRB

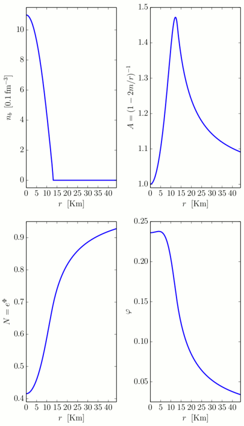
Code test: Static NS models

- $\bar{M} = 2.4\ M_\odot, \quad \bar{R} = 13.1\ \mathrm{km}$ model with $\alpha_0 = 0, \quad \beta_0 = -6$

  Novak gr-qc/9707041

- Baryon density, metric functions, scalar field

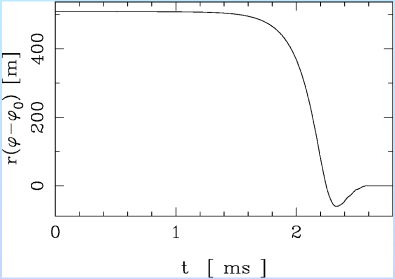


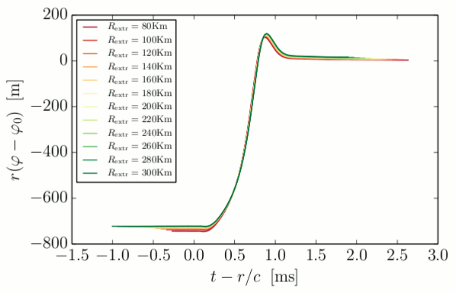


Code test: NS collapse to BH

- Intial model: $\bar{R} = 11.8\ \mathrm{km}, \quad \bar{M} = 2.07\ M_\odot$
- $\alpha_0 = 0.0025, \quad \beta_0 = -5$

Novak gr-qc/9707041



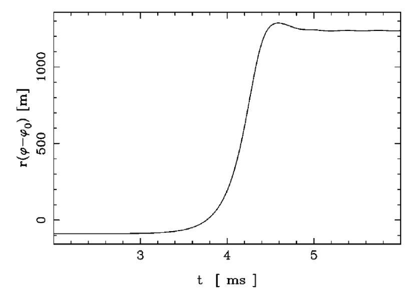


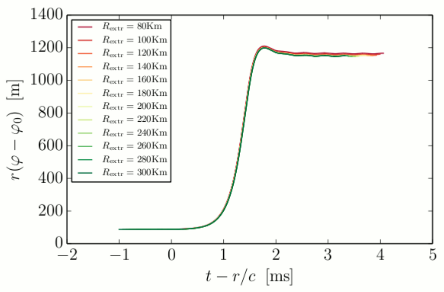
- Discrepancy due to sign error in α_0 in Novak
- As $\alpha_0 \to 0$, we agree with Novak

Code test: Transition from GR to scalarized star

- Unstable GR-like model: $\bar{R} = 13.2\ \mathrm{km}, \quad \bar{M} = 1.378\ M_{\odot}$
- ... migrates to scalarized model: $\bar{R} = 13.0\ \mathrm{km}, \quad M = 1.373\ M_{\odot}$
- Here: $\alpha_0 = 0.01\ , \quad \beta_0 = -6$

Novak gr-qc/9806022





Core collapse: Hybrid EOS

- Model stiffening of EOS through change in polytropic index

- $P = P_{\text{cold}} + P_{\text{thermal}}$ where

  P_{cold} = Polytrope(Γ_1, Γ_2) matched at $\rho = \rho_{\text{nuc}}$

  $P_{\text{thermal}} = (\Gamma_{\text{th}} - 1)\rho(\epsilon - \epsilon_0)$

- Before shock formation: $\epsilon = \epsilon_0$

  $\rightarrow$ P_{thermal} models non-adiabatic shock flow

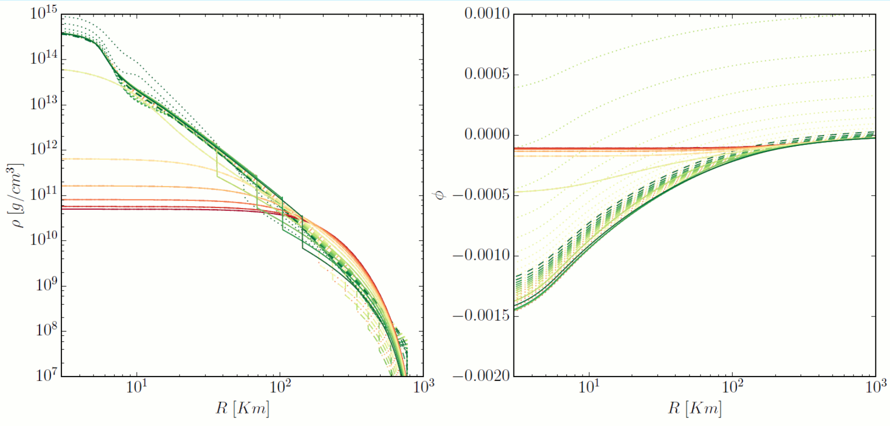
- We use $\Gamma_1 = 1.3,\ \ \Gamma_2 = 2.5,\ \ \Gamma_{\text{th}} = 1.25 \ldots 1.5$

Presupernova model: s12WH2007

- From stellar evolution codes up to the onset of core collapse
  Woosley & Heger Phys.Rep.**442**, 269
- Solar metalicity
- ZAMS mass $M = 12\ M_{\odot} \gtrsim M_{\mathrm{pre-SN}}$
- Generated with Newtonian gravity
- Set $\alpha_0 \neq 0$ to trigger scalar field

Core bounce

- ρ, φ profiles at different *t*
- Core bounce $\Rightarrow$ outgoing shock



Wave signal: Varying $\Gamma_{\rm th}$; $\alpha_0 = 0.01$, $\beta_0 = -5$

- Extra pressure $\propto \Gamma_{\rm th}$
- Small $\Gamma_{\rm th}$

  $\Rightarrow$ more massive NS

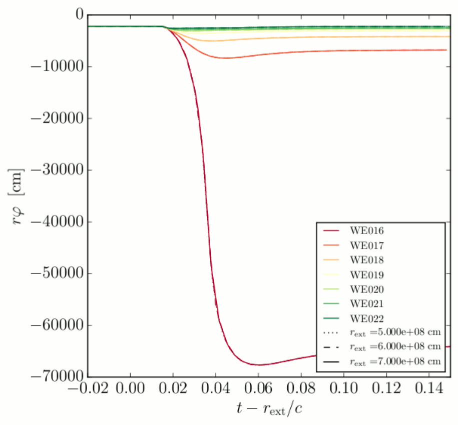
  $\Rightarrow$ Spontaneous scalarization

- Similar to WD collapse

  Novak & Ibañez astro-ph/9911298

- Detectable to $\sim$ 1 Mpc

  But depends on α_0!!



5. Conclusions

Conclusions

- Formalism for multi-scalar theories similar to single ST
- For $\alpha_0 = 0$, effectively like ST
- Bi-scalarized solutions require $\alpha_0 \neq 0$
- Complex structure in $(\varphi_c^1, \varphi_c^2)$ plane
- Core collapse in spherical symmetry
- Code tested successfully; identified few typos in literature
- Core bounce dynamics so far similar to GR
- Scalar waveforms strongly dependent on EOS

  Detectability $\propto \alpha_0$