Exercises for Lecture 11

1. Consider a string which has a natural length ℓ_0 when hung vertically, unstretched from a point A. The modulus of elasticity of the string is λ, thus the restoring force is $\lambda x/\ell_0$ and the potential energy in the stretched string is $\frac{1}{2}\lambda x^2/\ell_0$, where x is the extension from its natural length. A particle of mass m is attached to the lower end of the string. Given that the particle is released from rest at a point ℓ_0 vertically below point A, find the distance below A at which the particle first comes to rest.

   [You can do this either by conservation of energy, or by using Newton's second law to find and solve the equation of motion.]

2. The end A of a uniform rod AB of mass m and length 3ℓ is hinged on a vertical pole so that it can move in a vertical plane. A light elastic string BC of natural (unstretched) length ℓ is attached to the rod at B. The end C of the string is fixed to the pole a distance 5ℓ above A. The rod rests in equilibrium and the angle ABC is a right angle. Show that the modulus of elasticity of the string is $2mg/15$.

   [Again, you need to know that the tension in a stretched string is modulus of elasticity times extension / unstretched length. Consider taking moments about a suitably chosen point.]

3. A spring with spring constant k and natural length ℓ lies on a smooth horizontal table and is fixed at its ends to two points a distance ℓ apart. A particle of mass m is attached to its midpoint. The particle is displaced a distance x_0 along the line of the spring and released. Find the period of the subsequent simple harmonic motion and the position of the particle a time t after it was released.

4. A particle of mass m is suspended from a light spring with spring constant k of natural length ℓ causing an extension d. It rests in equilibrium with the spring hanging vertically. A particle of mass M is added to the particle of mass m. Find the period of the ensuing motion in terms of the masses, d, and the gravitational acceleration g. What is the amplitude of the oscillation? Check that you get the same amplitude using conservation of energy.

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