Exercises for Lectures 12 and 13

1. A particle is attached to the end A of a light inelastic string of length ℓ. The other end, B, is fixed. The particle moves in a horizontal circle a distance d below B. Show that the speed of the particle, v, is given by

$$v^2 = \frac{g\ell \sin^2\theta}{\cos\theta}$$

   where $\cos\theta = d/\ell$.

2. A particle rests at the top of a smooth sphere. It is given a small nudge so that it slides down the surface of the sphere. Use conservation of energy to find the speed of the particle when the radius to the particle makes an angle θ with the vertical. Find the normal reaction of the sphere on the particle and deduce that the particle leaves the surface of the sphere when $\cos\theta = 2/3$.

3. A simple pendulum consists of a particle attached to one end of a light rod, the other end of which is fixed so that the rod can swing without friction. Use Newton's second law (not conservation of energy) to find the equation of motion of the particle using the angular displacement θ of the rod from the vertical as the variable. Show that the motion, for small θ, is approximately simple harmonic and write down the period.

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