On these sheets, no attempt is made to "model" real-life situations: no trains, cars, cyclists, lifts, etc. It is assumed that there are no "real" forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, prefering general algebraic formulae with consistent dimensions.

Exercises for Lectures 2 and 3

1. A small block of mass m rests on a rough horizontal plane. The coefficient of friction between the block and the plane is μ. The block is in limiting equilibrium under the action of a force of magnitude P inclined at an angle θ to the upward normal to the plane ($0 < \theta < \pi/2$). Show that
$$P = \frac{\mu mg}{\sin\theta + \mu\cos\theta}\,.$$

2. A small block of mass m rests on a rough plane inclined at an angle α to the horizontal. It is subject to a force of magnitude P acting at an angle θ to the plane, which *just* prevents the block from slipping *down* the plane (so that it is in limiting equilibrium). Show that
$$P = \frac{mg\sin(\alpha - \lambda)}{\cos(\theta + \lambda)}$$
where λ is the acute angle satisfying $\tan\lambda = \mu$ (the so-called *angle of friction*) and $\alpha > \lambda$. What is the corresponding result if P is such that the block is on the point of slipping *up* the plane?

3. A horizontal light* rod AB has length ℓ and is supported by vertical strings fixed to its end-points. A mass m is placed on the rod at point A and a mass M is placed at the centre of the rod. Find the tension in each of the strings.

   *"Light" in these types of problems means you can neglect the mass of the rod.

4. The foot of a light ladder rests on a smooth horizontal floor and, at the other end, rests against a smooth vertical wall. The foot of the ladder is held by a horizontal string attached to the wall, and the angle between the ladder and the horizontal is α. A mass m is placed at the midpoint of the ladder. Show that the normal reaction of the wall at the top of the ladder is given by $\frac{1}{2}mg\cot\alpha$, and find the tension in the string.

5. A uniform rod AB of mass m rests with end A in contact with a smooth vertical wall and end B attached to a string BC where C is a point on the vertical wall above A (the angle CAB is obtuse). Show that the reaction of the wall on the rod is $\frac{1}{2}mg\tan\phi$, where ϕ is the acute angle between the string and the vertical. Show also that $\tan\theta = 2\tan\phi$ where θ is the acute angle between the wall and the rod.

   [You can regard the rod as being light with a mass m placed at the midpoint.]

6. The foot of a uniform ladder of weight mg rests on a rough floor and the other end rests against a smooth wall. The ladder makes an angle α to the horizontal, and the coefficient of friction between the ladder and the floor is μ. Show that $\frac{1}{2}\cot\alpha \leq \mu$.

Comments or queries to M.Wingate@damtp.cam.ac.uk
Course website: http://www.damtp.cam.ac.uk/user/wingate/Mechanics