Mechanics (non-examinable) | Mathematical Tripos, Part IA
Dr M B Wingate | Michaelmas Term 2013

On these sheets, no attempt is made to "model" real-life situations: no trains, cars, cyclists, lifts, etc. It is assumed that there are no "real" forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, prefering general algebraic formulae with consistent dimensions.

Exercises for Lecture 5

1. A particle moving in a straight line has initial speed u and accelerates with constant acceleration a, where $a < 0$, until it comes to rest. How long does this take and how far has the particle travelled?

2. A particle moving in a straight line accelerates from rest at constant acceleration b and then decelerates to rest again with constant deceleration $b/2$, moving a total distance d in time t. Draw a graph of velocity against time. Show that $d = \frac{1}{6}bt^2$.

3. The acceleration of a particle moving in a straight line, at time t after starting from rest is $5 + 4t - t^2$ for $0 \leq t \leq 5$ and zero for other values of t. Find:

   (i) the maximum acceleration;

   (ii) the greatest speed;

   (iii) the distance travelled until the particle attains its greatest speed.

4. A particle is projected vertically upwards from the ground at an initial speed of $25\,\mathrm{m/s}$. Taking $g = 10\,\mathrm{m/s^2}$ (actually, g is more like $9.8\,\mathrm{m/s^2}$), find the amount of time that the particle spends at a height greater than or equal to $30\,\mathrm{m}$.

5. A particle, originally at the origin and moving at velocity $\mathbf{u}$, accelerates with constant acceleration $\mathbf{a}$ until, at displacement $\mathbf{x}$ from the origin, it attains velocity $\mathbf{v}$. Show that

$$v^2 - u^2 = 2\mathbf{a} \cdot \mathbf{x}$$

   where u and v are the initial and final speeds of the particle.

6. A particle is projected at speed V from ground level at an angle of α above the horizontal. Show that its maximum height is $\frac{1}{2}V^2 \sin^2 \alpha / g$ and that the particle lands a distance $V^2 \sin 2\alpha / g$ from the point of projection. Deduce that the maximum range is obtained by firing the particle at $\alpha = \pi/4$.

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