Exercises for Lectures 7 and 8

1. A particle is released from rest and falls under the action of gravity. Use the kinematic equations of Lecture 5, calculate its speed after falling a distance h and verify that the principle of conservation of energy holds in this situation.

2. A particle of mass m is projected with initial speed v_0 up a rough plane inclined at α to the horizontal. The kinetic coefficient of friction is μ (assume the magnitude of the frictional force is μN, where N is the magnitude of the normal reaction force). Show that the acceleration down the plane is $g(\sin\alpha + \mu\cos\alpha)$ and hence calculate the distance s up the plane at which the particle comes to rest. Use the conservation of energy to show that the amount of work done against the frictional force is
$$\frac{1}{2}mv_0^2\frac{\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}.$$

3. Use the principle of conservation of energy (carefully!) to find the maximum height of a particle projected with speed v_0 at an angle of α to the horizontal, noting that the horizontal velocity is constant.

4. A light inelastic string passes over two small smooth pulleys A and B at the same horizontal level a distance $2a$ apart. Particles of mass m are attached to either end and a particle of mass M (where $M < 2m$) is attached to the midpoint. The system is released from rest with the particle of mass M at the midpoint of AB. Using conservation of energy, show that the system next comes to rest when the particle of mass M has fallen a distance
$$\frac{4amM}{4m^2-M^2}.$$

5. Water is pumped to the surface of the Earth from a depth d and issues from a pipe of cross-sectional area A at a speed of v. The density of the water is ρ. Using energy considerations, find the power of the pump.

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