Exercises for Lectures 9 and 10

1. Two particles each of mass m collide and coalesce (stick together). The velocities of the particles before impact were $(u, 0)$ and $(v\cos\theta, v\sin\theta)$. Use conservation of momentum to find an expression for the speed of the particle after the collision and find also the loss of kinetic energy.

2. Particles of mass m are attached to the ends of a light rigid rod. The rod lies along the x-axis. One particle of the rod is hit by a blow that would cause it, were it not attached to the rod, to move with velocity $(u\cos\theta, u\sin\theta)$. What is the impulse of the blow?

   This particle actually moves with initial velocity $(v\cos\phi, v\sin\phi)$. Assuming that total momentum is conserved, show that $\tan\phi = 2\tan\theta$. (Note that the rod is rigid so the other particle's initial velocity is necessarily in the direction of the rod.)

3. Two particles of masses m_1 and m_2 moving along the same line *in the same direction* with speeds u_1 and u_2 collide (i.e. the faster one runs into the slower one). The collision is perfectly elastic ($e = 1$). If the speeds after collision are v_1 and v_2, show that
$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}\,.$$
   Verify in the case $m_1 = m_2$ that kinetic energy in conserved in the collision.

4. A particle of mass m strikes a surface with speed u, its trajectory making an angle θ with the normal to the surface. It rebounds with speed v. Find the coefficient of restitution between the surface and the particle.

   Show that the impulse on the surface is $mu(1 + e)\cos\theta$.

   [Note: the component of velocity parallel to the surface is unaffected by the collision but the vertical component obeys Newton's experimental law.]

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