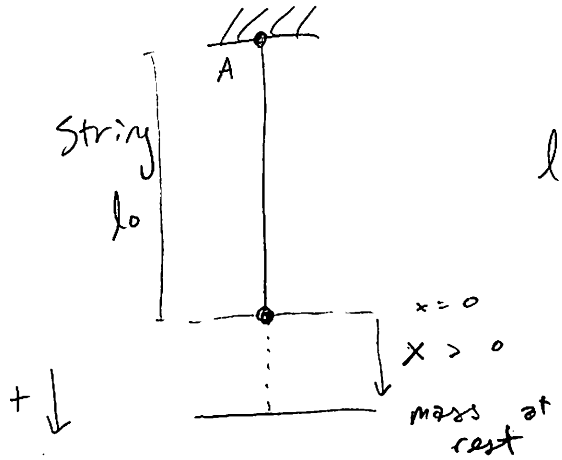


Modulus of elasticity λ

l_0: length of unstretched string (w/o mass attached)

Conservation of energy:

Initially: $E = T + U_g + U_s$

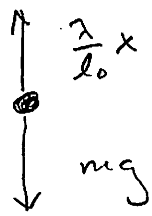
$= 0 + 0 + 0$

Finally: $E = 0 - mgx + \frac{1}{2}\frac{\lambda}{l_0}x^2$

Cons. energy $\Rightarrow$ $x\left(-mg + \frac{1}{2}\frac{\lambda}{l_0}x\right) = 0$

$\Rightarrow$ $x = 0$ or $x = \dfrac{2mg\,l_0}{\lambda}$

Eqn of motion



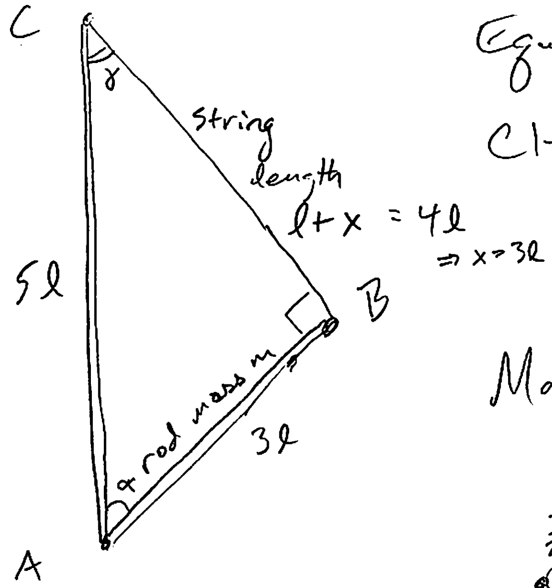
$$m\ddot{x} = mg - \frac{\lambda}{l_0}x$$

Integrate w.r.t. t $\Rightarrow$ $m\dot{x}\Big|_i^f = \left(mgx - \frac{\lambda}{2l_0}x^2\right)\Big|_i^f$

$$0 = mg(x-0) - \frac{\lambda}{2l_0}(x-0)^2$$

$\Rightarrow$ $x\left(mg - \frac{\lambda x}{2l_0}\right) = 0$ as before.

So the answer is that the particle comes to rest a distance $l_0 + x = \frac{l_0}{\lambda}(\lambda + 2mg)$ below A. //

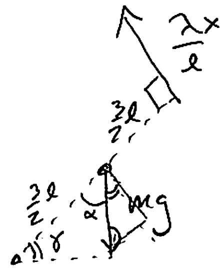


Equilibrium.

Claim: modulus of elasticity is

$$\lambda = \frac{2mg}{15}$$

Moments of force about point A:



$$0 = 3l\left(\frac{\lambda x}{l}\right) - \frac{3l}{2} mg \sin\alpha$$

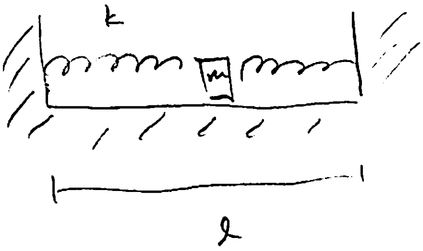
$$= 9\lambda l - \frac{3}{2} l m g \sin\alpha$$

From triangle $\sin\alpha = \frac{4}{5}$

$$= 9\lambda l - \frac{6}{5} l m g$$

$$\Rightarrow \lambda = \frac{2mg}{15}$$

3



Initial displacement from equilibrium at midpoint is x_0.

Newton's 2nd law:

$$m\ddot{x} = -kx$$

2nd order ODE w/ const. coeff : $\ddot{x} + \omega^2 x = 0$

with $\omega = \sqrt{\frac{k}{m}}$

$$\Rightarrow \quad x(t) = A\cos\omega t + B\sin\omega t$$

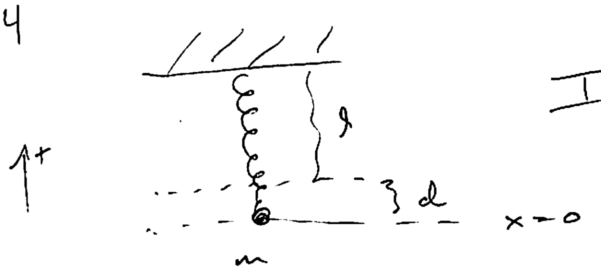
$$\dot{x}(t) = -\omega(A\sin\omega t - B\cos\omega t)$$

$$\dot{x}(0) = 0 \Rightarrow B = 0$$

$$x(0) = x_0 \Rightarrow A = x_0$$

$$x(t) = x_0\cos\omega t \quad , \quad \text{period} = \frac{2\pi}{\omega}$$

//



In equilibrium w/ just m

$$0 = kd - mg$$

$$\Rightarrow \quad k = \frac{mg}{d}$$

~~Mass~~ M is added & motion ensues

$$(m+M)\ddot{x} = -k(x-d) - (m+M)g$$

$$\ddot{x} + \omega^2 x = \frac{kd}{(m+M)} - g = -\frac{Mg}{m+M}$$

with $\omega^2 = \frac{k}{m+M} = \frac{mg}{d(m+M)}$. Period $T = \frac{2\pi}{\omega}$

So $\ddot{x} + \omega^2 x = -\frac{Md}{m}\omega^2$

$$x(t) = A\cos\omega t + B\sin\omega t - \frac{Md}{m}$$

$$\dot{x}(t) = -\omega(A\sin\omega t - B\cos\omega t)$$

$$\dot{x}(0) = 0 \Rightarrow B = 0$$

$$x(0) = 0 \Rightarrow A = \frac{Md}{m}$$

$$x(t) = \frac{Md}{m}(\cos\omega t - 1)$$

↑ Amplitude.

Also, conservation of energy $\Rightarrow E_i = E_f$

$$\frac{1}{2}kd^2 = \frac{1}{2}k(x-d)^2 + (m+M)gx + \frac{1}{2}(m+M)\dot{x}^2$$

At $x = x_{max}$, $\dot{x}\big|_{x_{max}} = 0$:

$$\frac{1}{2}kx^2 - kxd + (m+M)gx\Big|_{x_{max}} = 0 \Rightarrow x = 0 \text{ or } x = -\frac{2Md}{m}$$

$$\Rightarrow \text{amplitude} = \frac{Md}{m} \quad //$$