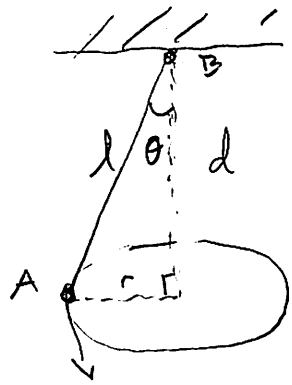
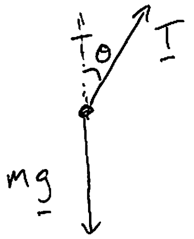


Uniform circular motion $\Rightarrow$ acceleration

$$a = \frac{v^2}{r} = \frac{v^2}{l\sin\theta}$$

Forces on particle :

Acceleration a horizontal $\Rightarrow$



$$ma = T\sin\theta$$

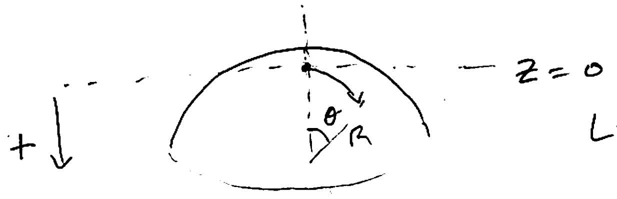
Vertically in equilib.

$$0 = T\cos\theta - mg$$

$$\Rightarrow \quad T = \frac{mg}{\cos\theta}$$

$$\Rightarrow \quad v^2 = a\, l\sin\theta$$

$$= \frac{g\, l\sin^2\theta}{\cos\theta} \qquad //$$



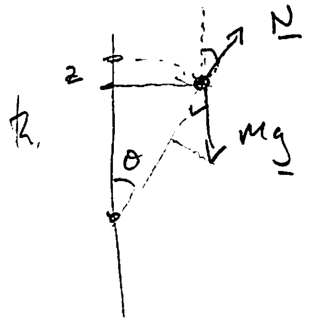
Initially, start at $z = 0$

$$E = T + U = 0$$

Later

$$R - z = R\cos\theta \qquad U = -mgz$$

$$\Rightarrow z = R(1 - \cos\theta)$$



$$E = 0 = \frac{1}{2}mv^2 - mgz$$

$$0 = \frac{1}{2}mv^2 - mgR(1 - \cos\theta)$$

$$\Rightarrow v^2 = 2gR(1 - \cos\theta) \quad //$$

Centripetal force = Net radial force inward

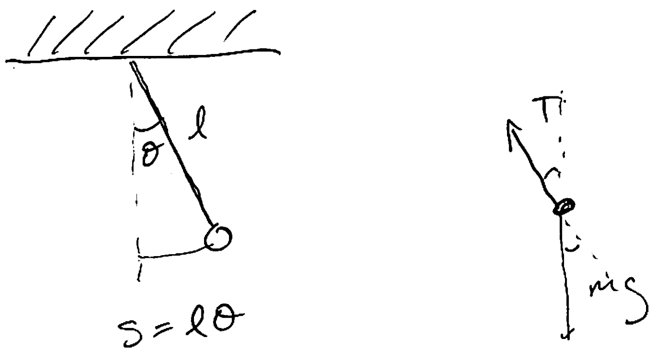
$$\frac{mv^2}{R} = mg\cos\theta - N$$

$$2mg(1 - \cos\theta) = mg\cos\theta - N$$

or $N = (3\cos\theta - 2)\, mg$

$N = 0$ when $\cos\theta = \frac{2}{3}$ indicating the particle has lost contact w/ surface.

3.



arc length $\quad s = l\theta$

Radial component : $\quad T - mg\cos\theta = \dfrac{mv^2}{l}$

$$= \frac{m\dot{s}^2}{l}$$

$$= ml\dot{\theta}^2$$

Tangential component : $\quad mg\sin\theta = m\ddot{s}$

$$= ml\ddot{\theta}$$

$$\ddot{\theta} - \frac{g}{l}\sin\theta = 0$$

Small $\theta \Rightarrow \sin\theta \simeq \theta$

$$\ddot{\theta} - \frac{g}{l}\theta = 0$$

$$\Rightarrow \quad \theta(t) = A\cos\omega t + B\sin\omega t$$

with $\omega = \sqrt{\dfrac{g}{l}}$. Period $\dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{l}{g}}$

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